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—Research Paper—

Entropical Risk Analysis Method For Managing Project Disruptions

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ABSTRACT

This paper is an attempt at developing a method for the analysis and estimation of the effects of project disruptions due to uncertainties. Such uncertainties may result from design changes in large-scale, complex, research and development, or construction projects.

An entropical risk analysis method is developed. The method is able to estimate the project capacity to handle equivocation due to design changes and the effects of project disruptions. In an attempt to evaluate the predictive capability of the method, it is compared with the results obtained by a computer Monte Carlo simulation program. It is shown that the entropical risk analysis method may be suggested as an expedient means of evaluating project status for management in the different stages of project execution.

1. Introduction

Most project managers in industry have experienced unexpected and randomly occuring design changes that gravely affect the progress of the project. As a result, an experienced project manager recognizes that the design changes do occur and takes provisions to avoid serious project cost overrun and the slippage of project completion date over the original estimates. In spite of these precautions, project management is often placed in a serious condition of uncertainty in carrying out normal project operations due to the large variability of design modification activities and the random characteristics of occurrences of design changes. The design changes affect significantly the cost overrun and the slippage of the construction project completion time. This phenominon has been experimented by computer simulation (Inoue, 1977). Although tbe cause-and-effect relationship is established between design changes and project slippage, project management still is placed in "uncertain" situations in a project execution.

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It is surprising that little effort has been made to study about the evaluation of disruption effects in project management. The entropical risk analysis method is therefore proposed.

2. Development of the method

Followings are the list of basic premises we make to develop the method

1) The activity distribution, including that of design modification activity, follow a triangular distribution
2) The original resource requirement for each activity can be doubled
3) Each activity duration consists of the fixed (uncompressible) portion and the compressible portion
4) The 30% fixed portion of activity duration is adopted
5) An activity duration can be shortened by one half when the fixed portion is equal to zero and the required resources are doubled.

Information, Entropy and Uncertainty

We introduce uncertainty, information and entropy show their relationships to each other. When we want to measure the information from a message in terms of the probability p_i that prevailed prior to the arrival of the message, the decreasing function should be selected. The function proposed by Shannon (1948, 1949) is:

$$h = \log_2(1/p_i) = -\log_2 p_i$$

The value of h decreases from ∞ (infinite information when the probability p_i prior to the message is zero) to 0 (zero information when the probability p_i is one). The function is illustrated in Figure 1. The unit of information is determined by the base of the logarithm. Two is used as a base in this paper.

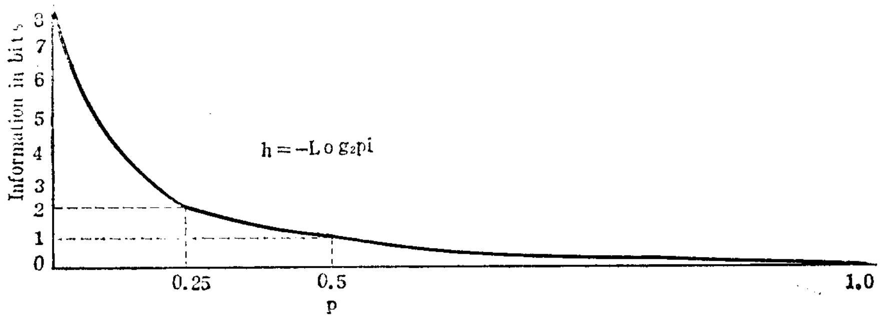

Figure 1. Information measured in bits.

Raisbeck (1963) states that a choice of base two of logarithms as a unit of measurement gives consistent results. More rigorous justifications for the use of base two are given by Shannon and Weaver (1949), Watanabe (1969) and others.

A way of reasoning about uncertainty

We desire logical means for reasoning about uncertainty of events. Probability theory as a general way for reasoning about uncertainty has been advocated by two approaches: One approach is a personalistic approach developed by Ramsey (1950) and Savage (1954) and the other is a logical approach supported by Keynes (1921) and Jefferys (1939). An essential insight of both the personalistic and logical approaches is that probability must reflect the knowledge upon which they are based. Probability theory may provide a way of reasoning about one state of knowledge in the light of other given information. We need thus an assumption which may support the concept of translating a state of knowledge into probability assignments.

Assumption: A probability assignment is the reflection of our state of knowledge.

For the logical derivation of probability distribution, it would be highly desirable to have a formal principle by which a state of knowledge may be translated to reach a unique choice of probability distribution. Such a principle is the maximum entropy principle originated by Jaynes (1957a, 1957b, 1962, 1963, 1965, 1967, and 1968).

The maximum entropy principle

Suppose we have to assign a probability distribution which is consistent with the information that we may collect under certain circumstances.

If there are several probability distribution which are consistent with the information that we have collected, we shall choose that distribution for which the entropy function $\{H(x) = -\sum p_i \log p_i\}$ is largest. This criterion is the maximum entropy principle (Jaynes, 1957; North, 1970).

Entropy as a measure of uncertainty

A measure of uncertainty (in the stage of knowledge before the experiment) with regard to the outcomes of the experiment, is usually expressed in the form of negative entropy (Goldman, 1953; Tribus, 1961, 1962, 1969; Raisbeck, 1963):

$$H(x) = -\sum_{i=1}^{n} p_i \log_2 p_i,$$

for $i = 1, 2, 3, \cdots\cdots, n$

where H is the entropy in bits, p_i is the individual probability of each outcome of an experiment which must satisfy $\sum_{i=1}^{n} p_i = 1$ and $p_i \geqq 0$.

Suppose that we have the p_i before the experiment, and the corresponding uncertainty is the entropy above mentioned. After knowing the result of the experiment, the probability $p_j = 1$, $p_i = 0$, $i \neq j$. The uncertainty, $H'(x)$, after the experiment therefore becomes:

$$H'(x) = 0$$

The reduction in uncertainty caused by the experiment can be considered as the information furnished by the experiment:

Information = reduction in uncertainty

$$=H(x)-H'(x)=-\sum_{i=1}^{n} p_i \log_2 p_i$$

Therefore, the entropy may be considered not only as a measure of the uncertainty associated with a distribution whose probabilities are p_1, p_2, ……, p_n, but also as the expected information from the message.

Derivation of an entropy formula of a triangular distribution

The objective of this part is to provide a working vehicle for developing the entropical risk analysis method. Figure 2 shows a typical triangular distribution with minimum value, a, most likely value, m, and maximum value, b. The probability density function of a triangular distribution, $f(x)$, can be given:

$$f(x)=\frac{2}{(b-a)(m-a)}(x-a)+\frac{2(b-x)}{(b-a)(b-m)} \tag{1}$$

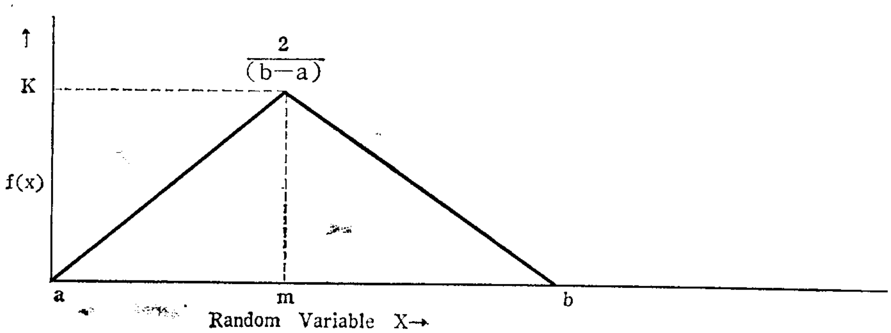

Figure 2. A typical triangular distribution

The entropy of a continuous distribution with density function, $f(x)$, is defined

$$H(x)=-\int_{-\infty}^{\infty} f(x)\log_e f(x)dx \tag{2}$$

Let $f(x)$ in Equation 1 be divided into two parts, $f_1(x)$ and $f_2(x)$:

$$f_1(x)=\frac{2}{(b-a)(m-a)}(x-a) \tag{3}$$

$$f_2(x)=\frac{2(b-x)}{(b-a)(b-m)} \tag{4}$$

Substitute Equation 3 and Equation 4 into Equation 2:

$$\begin{aligned} H(x)&=-\int_a^m f_1(x)\log_e f_1(x)dx-\int_m^b f_2(x)\log_e f_2(x)dx \\ &=-\int_a^m \frac{2(x-a)}{(b-a)(m-a)}\log_e \frac{2(x-a)}{(b-a)(m-a)}dx \\ &\quad-\int_m^b \frac{2(b-x)}{(b-a)(b-m)}\log_e \frac{2(b-x)}{(b-a)(b-m)}dx \end{aligned} \tag{5}$$

Let

$$v=\frac{2(x-a)}{(b-a)(m-a)} \tag{6}$$

$$dv=\frac{2dx}{(b-a)(m-a)} \tag{7}$$

$$dx = \frac{(b-a)(m-a)}{2} dv \tag{8}$$

$$w = \frac{2(b-x)}{(b-a)(b-m)} \tag{9}$$

$$dw = \frac{-2dx}{(b-a)(b-m)} \tag{10}$$

$$dx = -\frac{(b-a)(b-m)}{2} dw \tag{11}$$

Substitute Equation 6 through 11 into Equation 5:

$$\begin{aligned} H(x) &= -\frac{(b-a)(m-a)}{2}\int_a^m v \log_e v dv + \frac{(b-a)(b-m)}{2}\int_m^b w \log_e w dw \\ &= -\frac{(b-a)(m-a)}{2} \left| \frac{v^2}{2}\log_e v - \frac{v^2}{4} \right|_a^m \\ &\quad + \frac{(b-a)(b-m)}{2} \left| \frac{w^2}{2}\log_e w - \frac{w^2}{4} \right|_m^b \end{aligned} \tag{12}$$

Substitute the values of v and w in Equation 6 and 9 into Equation 12:

$$\begin{aligned} H(x) = &-\frac{(b-a)(m-a)}{2} \left| \frac{4(x-a)^2}{2(b-a)^2(m-a)^2} \log_e \frac{2(x-a)}{(b-a)(m-a)} \right. \\ &\left. - \frac{1}{4}\frac{4(x-a)^2}{(b-a)^2(m-a)^2} \right|_a^m \\ &+ \frac{(b-a)(b-m)}{2} \left| \frac{4(b-x)^2}{2(b-a)^2(b-m)^2} \log_e \frac{2(b-x)}{(b-a)(b-m)} \right. \\ &\left. - \frac{1}{4}\frac{4(b-x)^2}{(b-a)^2(b-m)^2} \right|_m^b \end{aligned}$$

The entropy of a triangular distribution can be simplified:

$$H(x) = \log_e(b-a) - 0.193 \text{ bits}$$

Since

$$\log_a b = \log_a e \log_e b$$

Then

$$\log_2 b = \log_2 e \log_e b = 1.4427 \log_e b$$

$$H(x) = 1.4427\{\log_e(b-a) - 0.193\} \text{ bits} \tag{13}$$

The derived entropy formula of the distribution will be used as a working vehicle for the estimation of activity capacity and project capacity.

Concept of 'capacity'

The 'capacity' concept of an activity in a project is devised from Kunisawa's (1959) transformation function. He suggested that the transformation function can be used to evaluate the capability of a system. He defined the transformation function as the transformation of a statistically distributed input into statistically distributed output. If the input and the output are statistically distributed, the input has statistical equivocation and the output also must have statistical equivocation.

The transformation function (T.F.) was expressed in terms of equivocation by Kunisawa as the difference between input equivocation and output equivocation.

T.F. = input equivocation − output equivocation

The concept of the transformation function is used to express the capacity of an activity in a project. The capacity of an activity is defined as the capability of reducing equivocation when the original resources for the activity are increased. In other words, the capacity of an activity can be measured as the difference between the original equivocation and the crashed equivocation. The original equivocation and the crashed equivocation are calculated:

$$H_0(x) = 1.4427\{\log(b-a) - 0.193\}\text{ bits}$$
$$H_c(x) = 1.4427\{\log(b'-a') - 0.193\}\text{ bits}$$

where $H_c(x)$ and $H_0(x)$ designate the original equivocation and the crashed equivocation respectively. Minimum value, a, and maximum value, b, are the original time estimates. Minimum value, a', and maximum value, b', are the crased time estimates.

The capacity of an activity (AC) can be therefore calculated:

$$\text{AC} = H_0(x) - H_c(x)$$

Compressibility of an activity duration

The problem of reducing a project duration arises frequently in industrial projects when a project schedule is delayed. It is believed that we may define an activity duration in a similar manner to the approach to set the standard time for a work element. An activity duration may be divided as shown in Figure 3. The fixed portion consists of fatigue allowance, personal allowance, unavoidable delays, extra allowance (contingency), and machine controlled time.

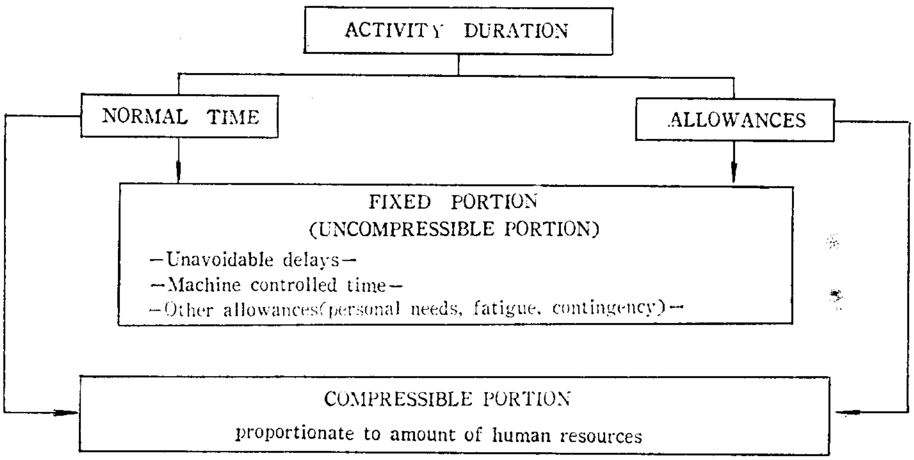

Figure 3. Relationship between fixed portion and compressible portion in activity duration.

The findings of literature survey can briefly be presented as follows:

Allowance factor	Range	Average
fatigue allowance	(5—10%)	7.5%
Personal allowance	(2—10%)	6.0%
unavoidable allowance	(3—13%)	8.0%
extra allowance (Contingency)		8.0%
Total		29.5%~30%

The average allowance for fatigue, personal needs and unavoidable delays is approximately 22%. An extra allowance must be provided to compensate for the effects of the accumulative fatigue brought on by strenuous situations. Such is often the case in delayed project schedules while trying to meet the original project completion date, work interference effects due to design changes, and engineering drawing delays. A reasonable allowance for these effects would be 8%. Thus, the activity duration of the 30% fixed portion can be explained by the average allowance for the uncompressible factors of 22% and the extra allowance of 8%.

The relationship between fixed portion of an activity duration and its compressibility

When the fixed portion is 0%, the minimum activity duration is shortened to 50% of the original figure. When the fixed portion is 100%, the minimum activity duration is equal to the original activity duration. In such a case, the original activity can not be compressed at all. The crashed activity duration can be calculated:

$$a' = (\alpha \times a) + \frac{1}{2}(1-\alpha) \times a$$

$$m' = (\alpha \times m) + \frac{1}{2}(1-\alpha) \times m$$

$$b' = (\alpha \times b) + \frac{1}{2}(1-\alpha) \times b$$

If $\alpha = 0.30$ is subsituted into the above Equations, then we have:

$$a' = 0.65 \times a$$

$$m' = 0.65 \times m$$

$$b' = 0.65 \times b$$

α designates a compression factor. a' designates crashed minimum activity duration, b', crashed maximum value and, m', crashed most likely value. a designates minimum original activity duration, b, maximum original value and m, most likely original value.

The maximum capacity of a project

The maximum capacity of a project is defined as the maximum capability of reducing the equivocation when the original resources; requirements are doubled. In order to estimate the maximum capacity in a project, the activities that have design changes and the successors of the activities are involved. Then, the maximum capacity, MC, can be defined:

$$\mathrm{MC} = \sum_{i=1}^{s} \{H_{io}(x) - H_{ic}(x)\} \text{bits, for } i = 1, 2, \cdots\cdots, s$$

where $H_{io}(x)$ and $H_{ic}(x)$ designate the original equivocation and the reduced equivocation repectively. s designates number of activities involved.

Comparison of maximum capacity with equivocation due to design modification activity

We can easily check if a project has enough capacity to cover the equivocation due to design modification activity. When the equivocation is larger than the maximum capacity of the project, the project slippage might be experienced. On the other hand, when the maximum capacity is larger than the equivocation due to design modification activity, the project would not experience the delay of project completion. The necessary amount of resources can be estimated to resolve the contingent situation in the project. Let X be the necessary amount of resources to absorb the disruption effects, then X can be expressed:

$$X = \frac{\text{equivocation due to design modification}}{\text{maximum project capacity}} \times 100$$

Suppose that the maximum project capacity is 3.72 bits and the equivocation due to design modification is 2.04 bits. Then, the necessary amount of resource, x, is about 55%. Thus, we know that if the project can increase the original requirments of resources by 55% on the related activities the project may be completed by the due date with 50% chance of success.

Estimation of disruption effects, project slippage due to design changes

We have considered the case in which the maximum capacity of a project is large enough to handle the equivocation due to design modification. When the equivocation can not be covered by the maximum capacity of a project, it is inevitable to prolong the original project schedule. We might experience the effects of disruption, project slippage, in the project execution.

In an attempt to estimate project slippage in the project, the average unit equivocation, AUE, is defined:

$$\text{AUE} = \sum_{i=1}^{s} \text{UE}_i / s, \text{ for } i = 1, 2, 3, \cdots\cdots, s$$

where AUE designates the average unit equivocation and UE_i designates the unit equivocation of the ith activity, which will be defined below and s is the number of activities involved.

The unit equivocation, UE_i, is defined:

$$\text{UE} = H_{ic}(x) / m_i'$$

where $H_{ic}(x)$ and m'_i designate the reduced equivocation and the crashed most likely activity duration respectively.

The project slippage therefore is defined:

$$\text{Project slippage} = \frac{\text{equivocation} - \text{maximum capacity}}{\text{AUE}}$$

3. Comparison of the results of entropical risk analysis method with those of Computer Monte Carlo simulation program

In an attempt to evaluate the predictive capability of the method we compare the results produced by the method and those obtained by the computer Monte Carlo simulation program which has been used successfully in five super oil tankers construction projects. Consider a fictious project which includes 17 activities. Its project network is shown in Figure 4. Suppose that design changes occurred on activity A, B, F and I and that the time estimates for the design modification activities are given by the expertise in the project owner team in table 1. The four design modification activities are designated by A*, B*, F*, and I* respectively.

Table 1. Time estimates and equivocation for design modification activities

Activity	Time estimates			Equivocation in bits
	a	*m*	*b*	
A*	3	7	8	2.04
B*	8	14	22	3.53
F*	7	8	15	2.72
I*	10	12	19	2.89
Total				11.18 bits

Maximum capacity estimation of project

In an attempt to check if the project is able to handle the contingent situation resulting from the four design changes, we first compute the amount of equivocation which is generated by the four design modification activities. The equivocation due to the design changes is 11.18 bits in table 1. Then, the maximum capacity of the project

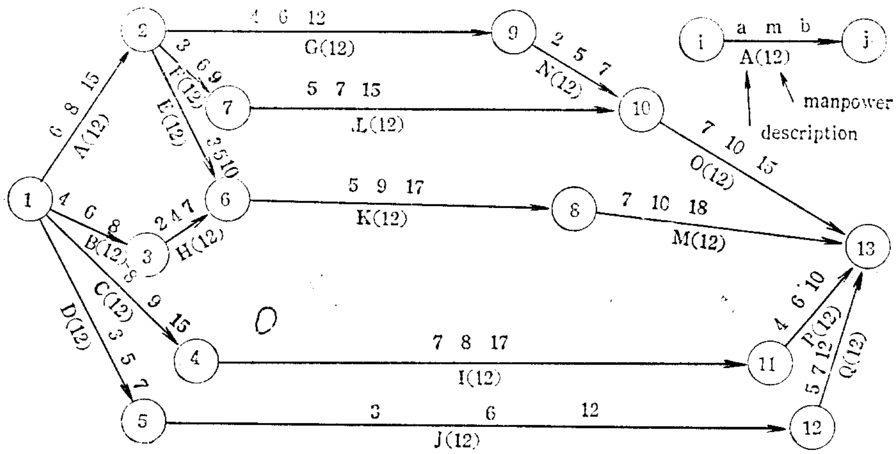

Figure 4. Project Network

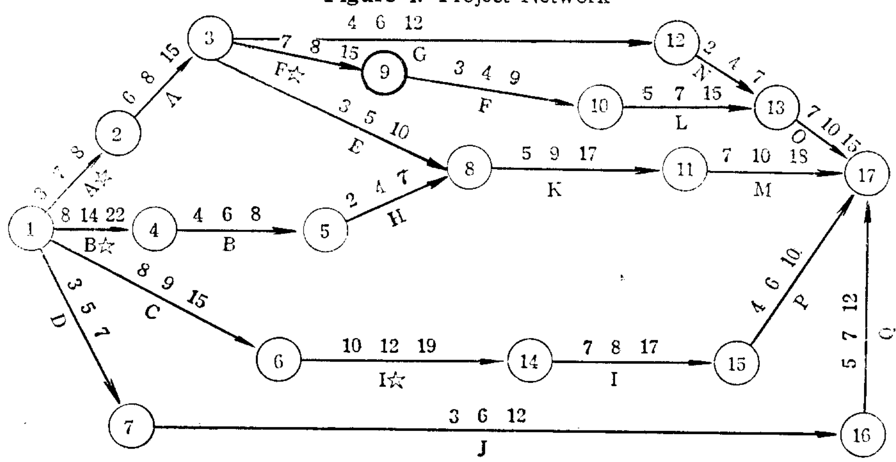

Figure 5. Project Network with A*, B*, F*, and I*

needs to be estimated. Since the four design changes occurred on the activities A, B, F, and I, the successors of the four activities including the activities, A, B, F, and I themselves, are included to resolve the equivocation, 11.18 bits, in the project; they are 13 activities, A, B, E, F, G, H, I, K, L, M, N, O, and P in the project network in Figure 4 and Figure 5. So the maximum capacity, MC, can be estimated:

$$MC=\sum_{i=1}^{13}\{H_{io}(x)-H_{ic}(x)\}=0.62 \text{ bits}\times 13=8.06 \text{ bits}$$

The maximum capacity of each activity is 0.62 bits and the 13 activities as shown above are involved. Now, we can easily see that the project can not handle the equivocation due to design changes, since we know that the equivocation is larger than the maximum estimated project capacity. In other words, the maximum project capacity can not cover the amount of equivocation, 3.12 bits, which is the difference between the equivocation due to the four design changes, 11.18 bits and the maximum capacity, 8.06 bits. The difference equivocation, 3.12 bits, will appear as the disruption effects of the four design changes, project slippage. Then, we need to show that the equivocation, 3.12 bits, is equivalent to how many man-days in project schedule.

Conversion of the Equivocation, 3.12 bits to Man-Day

As defined in the previous section, the average unit equivocation (AUE) can be computed as:

$$AUE=\sum_{i=1}^{13} UE_i/13$$
$$=\frac{(0.44+0.28+0.59+0.43+0.54+0.44+0.47}{13}$$
$$\frac{+0.46+0.53+0.39+0.44+0.32+0.43)}{}=\frac{5.76 \text{ bits}}{13 \text{ weeks}}$$
$$=0.443 \text{ bits/week}$$

Table 2. The changed equivocation and the minimum most likely duration, the unit equivocation (bits) for the 13 associated activities.

Activity description	Changed Equivocation in bits, $H_c(x)$	Most likely duration in bits, m'	Unit Equivocation $H_c(x)/m'$ in bits
A	2.27	5.2	0.44
B	1.10	3.9	0.28
E	1.91	3.25	0.59
F	1.69	3.9	0.43
G	2.10	3.9	0.54
H	1.42	3.25	0.44
I	2.42	5.2	0.47
K	2.69	5.85	0.46
L	2.42	4.55	0.53
M	2.56	6.5	0.39
N	1.42	3.25	0.44
O	2.10	6.5	0.32
P	1.69	3.9	0.43

The unit equivocations for the associated 13 activities are shown in Table 2.

Then, the project slippage can be computed as:

$$\text{Project slippage} = \frac{3.12 \text{ bits}}{0.443 \text{ bits/week}}$$

$$= 7.043 \text{ weeks} = 35.2 \text{ man-days (for 5-day week)}$$

X^2 'goodness-of-fit' test for comparison of the results by computer simulation program with those of the method

The computer Monte Carlo simulation program is used to evaluate the predictive capability of the method. Ten different runs are performed. Table 3 shows the project slippages obtained by the computer Monte Carlo simulation program and the method.

Table 3. The Project slippages obtained by the computer Monte Carlo simulation program and those by the method

Experiment i	O_i	E_i	$(O_i-E_i)^2$
1	7.2	7.0	0.04
2	7.6	7.0	0.36
3	7.4	7.0	0.16
4	7.8	7.0	0.64
5	7.6	7.0	0.36
6	7.8	7.0	0.64
7	7.4	7.0	0.16
8	6.8	7.0	0.04
9	7.8	7.0	0.64
10	7.0	7.0	0.00
Total			3.04

X^2 'goodness-of-fit' test's statistic is given as:

$$X^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i designates the project slippages obtained by the computer simulation program and E_i the estimated project slippages by the method. A X^2 'goodness-of-fit' test is conducted. The results of the test shows that the estimated project slippages do not significantly differ from those obtained from the computer simulation program at the 0.005 significance level.

4. Conclusion

The entropical risk analysis method may be suggested as an expedient management tool for the analysis and estimation of disruption effects due to uncertainties in large scale, complex construction projects.

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