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論 文
29—10—3

Optimal and Decentralized Control of Power System Frequency

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Abstract

A new approach for optimal decentralized load-frequency control in a multi-area interconnected power system is presented, which includes the optimal determination of decentralized load-frequency controller, observer for unmeasurable local states and load disturbances, quadratic estimator for tie-line power flow information transmitted at intervals.

The optimal design of the decentralized controller is based on a modified application of the singular perturbation theory, and the decentralized Luenberger observer uses techniques of state augmentaton for exponential disturbance functions and the representation of tie-line power flow states as non-directly-controlled inputs.

The approach presented herein is numerically tested through Elgerd's two-area load-frequency system model, and the results demonstrate remarkable advantages over the conventional ones.

1. Introduction

The first work based on the modern control theory in power system load frequency control (LFC) was done by Fosha and Elgerd [3], assuming that the entire states be completely available for measurement and that system load disturbances be known a priori. But the control is not feasible because of the unrealistic assumptions, although mathematically optimal.

Cavin et al. [4] performed the disturbance identification by a modified Kalman filter, and their work is still unrealistic by assuming that the tie-line power flows be known and also that statistical noise data be available.

Lately, Miniesy and Bohn [5] suggested an attractive method for the use of a Luenberger observer to identify unmeasurable states and disturbances, but their work needs further improvements in a disturbance model.

The author's first work [1], based on the use of the same observer as above, achieved some improvements in the representation of a disturbance model and in the derivation of the optimal control law, but the optimal parameter determination of the observer system resorted intuitively to a trial-and-error method and heuristic reasoning.

The second work by the author [2] extended to a two-area system, suggestd an algorithm for the observer design which could virtually determine its optimal parameter, although theoretically not optimal in a strict sense.

So far as the decentralized load frequency control of a multi-area interconnected power system is concerned, the recent main streams have been towards applications of aggregation methods, perturbation theory, coordination pri-

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接受日字：1980年 7月 29日

nciples, etc. for mathematical model establishments and the use of linear-quadratic-Gaussian approaches, robust control theory, pole-placement techniques, etc. for the design of decentralized feedback controllers. Particularly, applications of aggregation methods [13] as well as robust decentralized control [8,9] in conjunction with the dominant eigenvalue of the closed-loop system to the load-frequency control problems are currently receiving increasing attention. The results so far are very attractive in practical viewpoints, but it appears that no definite conclusion could be drawn, since no clear-cut notion is caught in defining the dominant eigenvalues in an optimal sense.

On the other hand, the recent results based on the singular perturbation theory [10, 11, 12], are thought to have a lot of possibilities for the optimal decentralized control of a multi-area interconnected power system due to its inherent structural characteristics.

The principal problems in conjuction with the optimal decentralized control of a multi-area load-frequency control system can be categorized into two parts: 1) the optimal design of the decentralized load-frequency controller and 2) the optimal design of the decentralized observer for identifying local states and disturbances.

The purpose of this paper, as an extention of the former works [1,2], is to present an improved approach for the design of optimal decentralized controller and observer for a multi-area load-frequency control system, using the singular perturbation theory and the Luenberger observer, such that

i) A performance measures for the optimal control slightly different from the standard one used in linear-quadratic-regulator problems is introduced in consideration of relatively long duration of load disturbances which the controller can regulate.

ii) More accurate method is suggested for deriving the slow component of decentralized feedback gain matrix than the conventional one.

iii) An exponential disturbance model is suggested to identify the unmeasurable load disturbances, and the disturbances are treated as states in the observer.

iv) Tie-line power flows are actually states, but are treated as uncontrollable inputs in the observer system in order to isolate the local observer from other area's states in the system.

v) A quadratic extapolation algorithm is suggested to estimate uninformed tie-line power flow changes.

2. Near-Optimal Decentralized Load-Frequency Controller

The basic load-frequency system dynamics for a multi-area interconnected power system is approximated by

$$\dot{x}=Ax+Bm+Dp \tag{1}$$

where x, m and p are, respectively, the state, control and disturbance vectors, such that

$$x \triangleq [\underset{n_0}{x^{0'}} \vdots \underset{n_1}{x^{1'}} \vdots \underset{n_2}{x^{2'}} \vdots \cdots\cdots \vdots \underset{n_i}{x^{i'}} \vdots \cdots\cdots \vdots \underset{n_r}{x^{r'}}]'\ 1 = [\underset{n_0}{n^{0'}} \vdots \underset{\sum n_i}{x^{a'}}]' 1 \tag{2}$$

$$m \triangleq [\underset{1}{m^1} \vdots \underset{1}{m^2} \vdots \cdots\cdots \vdots \underset{1}{m^i} \vdots \cdots\cdots \vdots \underset{1}{m^r}]' 1 \tag{3}$$

$$p \triangleq [\underset{1}{p^1} \vdots \underset{1}{p^2} \vdots \cdots\cdots \vdots \underset{1}{p^i} \vdots \cdots\cdots \vdots \underset{1}{p^r}]' 1 \tag{4}$$

where n_i : total number of local states within area i

n_0 : total number of interconnecting states among areas

r : total number of areas

and A, B and D are constant matrices which have the following structures:

$$A \triangleq \begin{array}{c} \left(\begin{array}{c:c:c:c:c} A^0 & A^{01} & A^{02} & \longleftrightarrow & A^{0r} \\ A^{10} & A^1 & O & \longleftrightarrow & O \\ A^{20} & O & A^2 & \longleftrightarrow & O \\ \updownarrow & \updownarrow & \updownarrow & \leftarrow\!\!\updownarrow\!\!\rightarrow & \updownarrow \\ A^{r0} & O & O & \longleftrightarrow & A^r \end{array}\right) \begin{array}{l} n_0 \\ n_1 \\ n_2 \\ \vdots \\ n_r \end{array} \\ \begin{array}{ccccc} n_0 & n_1 & n_2 & \cdots\cdots & n_r \end{array} \end{array} \triangleq \begin{array}{c} \left(\begin{array}{c:c} A^0 & A^{0a} \\ \hdashline A^{a0} & A^a \end{array}\right) \begin{array}{l} n_0 \\ \sum n_i \end{array} \\ \begin{array}{cc} n_0 & \sum n_i \end{array} \end{array} \tag{5}$$

$$B \triangleq \begin{array}{c} \left(\begin{array}{c:c:c:c} O & O & \longleftrightarrow & O \\ \hdashline B^1 & O & \longleftrightarrow & O \\ \hdashline O & B^2 & \longleftrightarrow & O \\ \hdashline \updownarrow & \updownarrow & \leftarrow\!\!\updownarrow\!\!\rightarrow & \updownarrow \\ \hdashline O & O & \longleftrightarrow & B^r \end{array}\right) \begin{array}{l} n_0 \\ n_1 \\ n_2 \\ \vdots \\ n_r \end{array} \\ \begin{array}{cccc} 1 & 1 & \cdots\cdots & 1 \end{array} \end{array} \triangleq \begin{array}{c} \left(\begin{array}{c} B^0 \\ \hdashline B^a \end{array}\right) \begin{array}{l} n_0 \\ \sum n_i \end{array} \\ r \end{array} \tag{6}$$

$$D \triangleq \begin{array}{c}\left[\begin{array}{c:c:c:c} O & O & \longleftrightarrow & O \\ \hdashline D^1 & O & \longleftrightarrow & O \\ \hdashline O & D^2 & \longleftrightarrow & O \\ \hdashline \updownarrow & \updownarrow & \leftarrow\!\!+\!\!\rightarrow & \updownarrow \\ \hdashline O & O & \longleftrightarrow & D^r \end{array}\right]\begin{array}{l} n_0 \\ n_1 \\ n_2 \\ \vdots \\ n_r \end{array} \\ \begin{array}{cccc} 1 & 1 & \cdots\cdots & 1 \end{array}\end{array} \triangleq \left[\begin{array}{c} D^0 \\ \hdashline D^a \end{array}\right]\begin{array}{l} n_0 \\ \sum n_i \end{array} \quad (7)$$
$$\qquad\qquad r$$

If the final equilibrium states ($x_f \triangleq x|_{t\to\infty}$) and controls ($m_f \triangleq m|_{t\to\infty}$) are known, the performance measure J for Eq.(1) is taken to have such a quadratic form

$$J=\frac{1}{2}\int_0^\infty \{(x-x_f)'Q(x-x_f)+(m-m_f)'R(m-m_f)\}dt \qquad (8)$$

where Q and R are, respectively, positive definite weighting matrices with appropriate dimensions.

It is noted that the performance measure above is somewhat different in the second term from the standard one, since in the conventional linear regulator problem, $(m-m_f)'R(m-m_f)$ is to be replaced by $m'Rm$. The reason is based on the fact that the load disturbances regulated by the controller is mainly of long duration.

On the other hand, an asymptotically stable and controllable case ensures the existence of constant matrices E and S, such that [Appendix I]

$$x_f=Ep_f \qquad (9)$$

$$m_f=Sp_f \qquad (10)$$

where

$$E \triangleq \begin{array}{c}\left[\begin{array}{c:c:c:c} O & O & \longleftrightarrow & O \\ \hdashline E^1 & O & \longleftrightarrow & O \\ \hdashline O & E^2 & \longleftrightarrow & O \\ \hdashline \updownarrow & \updownarrow & \leftarrow\!\!+\!\!\rightarrow & \updownarrow \\ \hdashline O & O & \longleftrightarrow & E^r \end{array}\right]\begin{array}{l} n_0 \\ n_1 \\ n_2 \\ \vdots \\ n_r \end{array} \\ \begin{array}{cccc} 1 & 1 & \cdots\cdots & 1 \end{array}\end{array} \qquad (11)$$

$$S \triangleq \begin{array}{c}\left[\begin{array}{c:c:c:c} S^1 & O & \longleftrightarrow & O \\ \hdashline O & S^2 & \longleftrightarrow & O \\ \hdashline \updownarrow & \updownarrow & \leftarrow\!\!+\!\!\rightarrow & \updownarrow \\ \hdashline O & O & \longleftrightarrow & S^r \end{array}\right]\begin{array}{l} 1 \\ 1 \\ \vdots \\ 1 \end{array} \\ \begin{array}{cccc} 1 & 1 & \cdots\cdots & 1 \end{array}\end{array} \qquad (12)$$

$$p_f \triangleq p|_{t\to\infty}$$

The variable changes as $\overline{x} \triangleq x-x_f$, $\overline{m} \triangleq m-m_f$, $\bar{p} \triangleq p-p_f \fallingdotseq 0$ result in

$$\dot{\overline{x}}=A\overline{x}+B\overline{m} \qquad (13)$$

$$J=\frac{1}{2}\int_0^\infty (\overline{x}'Q\overline{x}+\overline{m}'R\overline{m})dt \qquad (14)$$

It is also noted that the assumption for $\bar{p} \fallingdotseq 0$ or $p \fallingdotseq p_f$ is based on a rational approximation, since the unpredictability of the future p_f makes it inevitable to guess p_f as the present best knowledge p.

Accordingly, the optimal control vector $\overset{*}{m}(\triangleq \overset{*}{\overline{m}}+m_f)$ which minimizes J in Eq. (14), by solving a steady-state Riccati equation, is given by

$$\overset{*}{\overline{m}}=H\overline{x} \qquad (15)$$

or $\overset{*}{m}=Hx+(S-HE)p_f \fallingdotseq Hx+(S-HE)p \qquad (16)$

where $H: r\times(n_0+\sum_{i=1}^{r} n_i)$ constant centralized feedback gain matrix.

For deriving the near-optimal control law on each area basis, a new approach is suggested which is basically an application of the singular perturbation theory, but different from the conventional one in an algorithm for slow subsystem feedback gain matrix.

Taking into account the fact that the interconnecting states (tie-line power flow deviaitons) in a multi-area power system are relatively slow, the entire system is approximately decomposed into a slow subsystem and r fast subsystems. That is, from *Eq.* (13) and with the use of the structural properties, as shown in ***Eqs.*** (2)~(7).

$$\dot{\overline{x}}^0=A^0\overline{x}^0+\sum_{i=1}^{r}A^{0i}\overline{x}^i=\sum_{i=1}^{r}A^{0i}\overline{x}^i \qquad (17)$$

$$\dot{\overline{x}}^i=A^{i0}\overline{x}^0+A^i\overline{x}^0+B^i\overline{m}^i,\ i=1,2,\cdots r \qquad (18)$$

where $A^0=0$ and A^{i0}, A^{0i} are determined by the tie-line connection topology.

Partitioning $\overline{x}^0(\triangleq x^0-x_f^0)$, $\overline{x}^i(\triangleq x^i-x_f^i)$ and $\overline{m}^i(\triangleq m^i-m_f^i)$ as

$$\overline{x}^0=\overline{x}_S^0+\overline{x}_F^0 \qquad (19)$$

$$\overline{x}^i=\overline{x}_S^i+\overline{x}_F^i,\quad i=1,2,\cdots r \qquad (20)$$

$$\overline{m}^i=\overline{m}_S^i+\overline{m}_F^i,\ i=1,2,\cdots r \qquad (21)$$

where the subscripts S and F denote slow and fast component, respectively.

and assuming, according to the singular perturbation theory, that

$$\dot{\overline{x}}^0 \fallingdotseq 0 \qquad (22)$$

$$\overline{x}_S^i \fallingdotseq -A^{i^{-1}}A^{i0}\overline{x}_S^0-A^{i^{-1}}B^i\overline{m}_S^i \qquad (23)$$

Eq. (17) and (18) can be approximated by

$$\overline{x}_S^0=-\sum_{i=1}^{nr}A^{0i}A^{i^{-1}}A^{i0}\overline{x}_S^0-\sum_{i=1}^{n_r}A^{0i}A^{i^{-1}}B^i\overline{m}_S^i$$
$$=A^S\overline{x}_S^0+B^S\overline{m}_S^i \tag{24}$$

where $A^S\triangleq-\sum_{i=1}^{r}A^{0i}A^{i^{-1}}A^{i0}$

$$B^S\triangleq-\sum_{i=1}^{r}A^{0i}A^{i^{-1}}B^i$$

$$\overline{x}_F^i=A^i\overline{x}_F^i+B^i\overline{m}_F^i \tag{25}$$

Then, it can be shown that the fast near-optimal control $\overline{m}_F^i$ is obtained by minimizing the performance measure

$$J_F^i=\frac{1}{2}\int_0^\infty(\overline{x}_F^{i\prime}Q^i\overline{x}_F^i+\overline{m}_F^{i\prime}R^i\overline{m}_F^i)\,dt,\quad i=1,2,\cdots,r \tag{26}$$

for the i-th fast subsystem, in Eq. (25), such that

$$\overline{m}_F^i=-R^{i^{-1}}B^iK_F^i\overline{x}_F^i=H_F^i\,\overline{x}_F^i \tag{27}$$

where K_F^i is the positive definite stablizing solution of the Riccati equation.

$$O=K_F^iA^i+A^{i\prime}K_F^i-K_F^iB^iR^{i^{-1}}B^{i\prime}K_F^i+Q^i \tag{28}$$

On the other hand, two kinds of algorithms are available for the slow near-optimal control $\overline{m}_S$: the one is of the conventional type and the other suggested by the author.

The conventional one is also obtained by minimizing

$$J_S=\frac{1}{2}\int_0^\infty\Big\{\overline{x}_S^{0\prime}Q^0\overline{x}_S^0+\sum_{i=1}^{r}\overline{x}_S^{i\prime}Q^i\overline{x}_S^i+\sum_{i=1}^{r}\overline{m}_S^{i\prime}R^i\overline{m}_S^i\Big\}dt=\frac{1}{2}\int_0^\infty\Big\{\overline{x}_S^{0\prime}Q^S\overline{x}_S^0+2\overline{m}_S'C^S\overline{x}_S+\overline{m}_S'(R^i+R^S)\overline{m}_S\Big\}dt \tag{29}$$

where

$$Q\triangleq\begin{array}{cc}\left(\begin{array}{c:c:c:c}Q^0&O&\longleftrightarrow&O\\ \hdashline O&Q^1&\longleftrightarrow&O\\ \hdashline \updownarrow&\updownarrow&\leftarrow+\rightarrow&\updownarrow\\ \hdashline O&O&\longleftrightarrow&Q^r\end{array}\right)&\begin{array}{c}0\\n_1\\ \\ n_r\end{array}\\ \begin{array}{cccc}n_0&n_1&&n_r\end{array}&\end{array} \tag{30}$$

$$R\triangleq\begin{array}{cc}\left(\begin{array}{c:c:c:c}R^1&O&\longleftrightarrow&O\\ \hdashline O&R^2&\longleftrightarrow&O\\ \hdashline \updownarrow&\updownarrow&\leftarrow+\rightarrow&\updownarrow\\ \hdashline O&O&\longleftrightarrow&R^r\end{array}\right)&\begin{array}{c}1\\1\\ \\ 1\end{array}\\ \begin{array}{cccc}1&1&&1\end{array}&\end{array} \tag{31}$$

$$Q^S\triangleq Q^0+\sum_{i=1}^{r}(A^{i^{-1}}A^{i0})'Q^i(A^{i^{-1}}A^{i0}) \tag{32}$$

$$C^S\triangleq[\underset{1}{((A^{1^{-1}}B^1)'Q^1(A^{1^{-1}}A^{10}))'}\vdots\underset{1}{((A^{2^{-1}}B^2)'Q^2(A^{2^{-2}}A^{20}))'}\vdots\longleftrightarrow\vdots\underset{1}{((A^{r^{-1}}B^r)'Q^r(A^{r^{-1}}A_r^0))'}]n_0 \tag{33}$$

$$R^S\triangleq\begin{array}{cc}\left(\begin{array}{c:c:c:c}(A^{1^{-1}}B^1)'Q^1(A^{1^{-1}}B^1)&O&\longleftrightarrow&O\\ \hdashline O&(A^{2^{-1}}B^2)'Q^2(A^{2^{-1}}B^2)&\longleftrightarrow&O\\ \hdashline \updownarrow&\updownarrow&\leftarrow+\rightarrow&\updownarrow\\ \hdashline O&O&\longleftrightarrow&(A^{r^{-1}}B^r)Q^r(A^{r^{-1}}B^r)\end{array}\right)&\begin{array}{c}1\\1\\ \\ 1\end{array}\\ \begin{array}{cccc}1&1&&1\end{array}&\end{array} \tag{34}$$

for the slow subsystem in Eq.(24), such that

$$\overline{m}_S=(R+R^S)^{-1}\{C^S+B^{S\prime}K_S\}\overline{x}_S^0=H_S\overline{x}_S^0 \tag{35}$$

where K_S is the positive definite solution of the Riccati equation

$$O=K_S\{A^S-B^S(R+R^S)^{-1}C\}+\{A^S-B^S(R+R^S)^{-1}C^S\}'K_S-K_SB^S(R+R^S)^{-1}B^{S\prime}K_S+Q^S-C^{S\prime}(R+R^S)^{-1}C^S \tag{36}$$

An alternative method for Eq. (35) is to derive H_S from the centralized feedback gain matrix H in Eq. (15) at the expense of computational easiness.

That is, Eq. (15) is decomposed as

$$\overline{m}_S=H^0\overline{x}_S^0+\sum_{i=1}^{r}H^i\overline{x}_S^i \tag{37}$$

where $H=[\underset{n_0}{H^0}\vdots\underset{n_1}{H^1}\vdots\underset{n_2}{H^2}\vdots\longleftrightarrow\vdots\underset{n_r}{H^r}]r$

and substituting the approximate relation in Eq. (23) into $\overline{x}_S^i$ in the second term of Eq.(37), H_S is more accuratly derived as

$$\overline{m}_S=T(H^0-\sum_{i=1}^{r}H^iA^{i^{-1}}A^{i0})\overline{x}_S^0=H_S\overline{x}_S^0 \tag{38}$$

where

$$T\triangleq\begin{array}{cc}\left(\begin{array}{c:c:c:c}(1+H^1A^{1^{-1}}B^1)^{-1}&O&\longleftrightarrow&O\\ \hdashline O&(1+H^2A^{2^{-1}}B^2)^{-1}&\longleftrightarrow&O\\ \hdashline \updownarrow&\updownarrow&\leftarrow+\rightarrow&\updownarrow\\ \hdashline O&O&\longleftrightarrow&(1+H^rA^{r^{-1}}B^r)^{-1}\end{array}\right)&\begin{array}{c}1\\1\\ \\ 1\end{array}\\ \begin{array}{cccc}1&1&&1\end{array}&\end{array} \tag{39}$$

Comparing Eqs. (35) and (38), the H_S given by Eq. (38) is apparently more accurate than that by Eq. (35) due to the partial use of the approximation of Eq. (23), but takes more time to compute the entire system gain matrix H. For the determination of a feedback gain, accuracy is usually more important than computational easiness, because of its precalculation or non-realtime calculation nature. Consequently, it is suggested to adopt Eq. (38) for H_S rather than Eq. (35).

Then, the near-optimal composite control, with replacing $\overline{x^0}$ by $\overline{x_S}^0$ and $\overline{x^i}$ by $\overline{x_S}^i+\overline{x_F}^i$, is derived as

$$\begin{aligned}\overline{m}^i &= \overline{m}_F{}^i+\overline{m}_S{}^i = H_F{}^i\overline{x_F}^i+H_S{}^i\overline{x_S}^0 \\ &= (H_S{}^i+H_S{}^iA^{i^{-1}}B^iH_S{}^i+H_F{}^iA^{i^{-1}}A^{i0})\overline{x}^0 \\ &\quad +H_F{}^i\overline{x}^i \end{aligned} \tag{40}$$

or

$$m^i=H_0{}^ix^0+H_F{}^ix^i+(S^i-H_F{}^iE^i)p^i \tag{41}$$

where $H_S \triangleq [\underset{1}{H_S{}^{1\prime}}\vdots\underset{1}{H_S{}^{2\prime}}\vdots\longleftrightarrow\vdots\underset{1}{H_S{}^{r\prime}}]'r$

$x_f{}^0=0$ ($\because$ tie-line power flows are regulated to zeroes)

Therefore, the near-optimal gain for the i-area's decentralized controller H^i is given by

$$H^i=[\underset{n_0}{H_0{}^i}\vdots\underset{n_i}{H_F{}^i}\vdots\underset{1}{S^i-H_F{}^iE^i}]1 \tag{42}$$

and the inputs to the controller are categorized into following three parts:

i) measurable local information

The i-th area's local information x^i is composed of the two parts such that

$$x^i=[y^{i\prime}\vdots x_u{}^{i\prime}]' \tag{43}$$

but the decentralized controller has direct access to y^i (usually local frequency deviation) and to the directly-interconnected states y^{0i} (the power flow deviations in the tie-line directly connected to the i-th area) out of $y^0(=x^0)$

ii) Information on unmeasurable local states and disturbance

The unmeasurable $x_u{}^i$ in Eq. (43) and P^i in Eq. (41) are identified by the optimal decentralized observer which will be described in section 3.

iii) Information on indirectly-interconnected states

Out of $y^0(=x^0)$ the other portions $y^{0j}(j\neq i)$ not connected to the i-th area, change slowly, and information on them may be required only at intervals. So the values between samples could be extrapolated by the approach suggested in section 4.

3. Optimal Decentralized Observer for Unmeasurable Local states and Disturbance

In this section, a new approach is presented for identifying unmeasurable local states $x_u{}^i$ and disturbance p^i available as part of inputs to the i-th controller.

The Subsystem dynamics for each area can be written from the system structure of Eqs. (1)～(7), as

$$\dot{x}^i=A^ix^i+B^im^i+D^ip^i+A^{i0}x^0, \quad i=1,2,\cdots,r \tag{44}$$

But out of the interconnecting states (tie-line power flow deviations) $x^0(=y^0)$ and the local states x^i, y^0 and part of x^i are available for measurement, and the directly-connected portion y^{0i} out of y^0 to area i only is reflected as

$$A^{i0}x^0=A^{i0}y^0=\overline{A}^{i0}y^{0i} \tag{45}$$

Therefore, Eq. (44) can be rewritten as

$$\dot{x}^i=A^ix^i+B^im^i+D^ip^i+\overline{A}^{i0}y^{0i} \tag{46}$$

So far as the local observer to be presented is concerned, p^i and y^{0i} are treated as a state and non-feedback input, respectively as follows:

i) Augmented input vector

The augmented input $\tilde{m}^i$ is defined by

$$\tilde{m}^i=[m^{i\prime}\vdots y^{0i\prime}]' \tag{47}$$

ii) Exponential disturbance model

A new model for representing the load disturbance p^i as a state, which may improve the conventional disturbance identification, is based

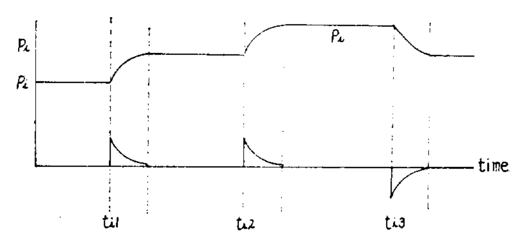

Fig. 1. Disturbance and its time derivative.

on an exponential approximation, as shown in Fig. 1 or given by Eq. (48), instead of an stepchange approximation done by others.

That is, p^i is approximated by

$$p^i \triangleq \sum_k k_{ik}[1-\exp\{-\alpha(t-t_{ik})\}]u(t-t_{ik}) \quad (48)$$

$$\dot{p}^i=\alpha\sum_k k_{ik}\exp\{-\alpha(t-t_{ik})\}u(t-t_{ik}) \quad (49)$$

$$\ddot{p}^i=-\alpha\dot{p}^i+\alpha\sum_k k_{ik}\delta(t-t_{ik})\fallingdotseq-\alpha\dot{p}^i \quad (50)$$

where $u(t),\delta(t)$: unit step and unit impulse time function, respectively

t_{ik} : the k-th occuring time of load disturbance in area i

k_{ik} : disturbance magnitude

α : properly selected positive number sufficiently large

It should be noted that the error contribution due to the negligence of $\alpha\sum_k k_{ik}\delta(t-t_{ik})$ is of little significance since its effect is reflected on $\dot{p}^i$ ($\fallingdotseq 0$).

The inclusion of additional states p^i and $\dot{p}^i$ in x^i leads to the augmented states

$$\tilde{x}^i=[x^{i\prime}\vdots p^i\vdots\dot{p}^i]' \quad (51)$$

Consequently, Eq. (46) is also augmented as

$$\dot{\tilde{x}}^i=\tilde{A}^i\tilde{x}^i+\tilde{B}^i\tilde{m}^i \quad (52)$$

where

$$\tilde{A}^i\triangleq\begin{array}{cc}\left(\begin{array}{c:c:c} A^i & D^i & O \\ \hdashline O & O & 1 \\ \hdashline O & O & -\alpha \end{array}\right) & \begin{array}{l} n_i \\ 1 \\ 1 \end{array} \\ \begin{array}{ccc} n_i & 1 & 1 \end{array} & \end{array} \triangleq \begin{array}{cc}\left(\begin{array}{c:c} \tilde{A}_{11}{}^i & \tilde{A}_{12}{}^i \\ \hdashline \tilde{A}_{21}{}^i & \tilde{A}_{22}{}^i \end{array}\right) & \begin{array}{l} m_i \\ n_i-m_i+2 \end{array} \\ \begin{array}{cc} m_i & n_i-m_i+2 \end{array} & \end{array} \quad (53)$$

$$\tilde{x}^i \triangleq [\underset{m_i}{y^{i\prime}}\vdots \underset{n_i-m_i}{x_u{}^{i\prime}}\vdots \underset{1}{p^i}\vdots\underset{1}{\dot{p}^i}]'1=[\underset{m_i}{y^i}\vdots\underset{n_i-m_i+2}{\tilde{x}_u{}^{i\prime}}]'1 \quad (55)$$

y^i : measurable states (usually frequency deviation) in area i

$x_u{}^i$: unmeasurable states in area i

$$\tilde{B}^i\triangleq\begin{array}{cc}\left(\begin{array}{c:c} B^i & \bar{A}^{i0} \\ \hdashline O & O \end{array}\right) & \begin{array}{l} n_i \\ 2 \end{array} \\ \begin{array}{cc} 1 & 1 \end{array} & \end{array} \triangleq \begin{array}{cc}\left(\begin{array}{c} \tilde{B}_1{}^i \\ \hdashline \tilde{B}_2{}^i \end{array}\right) & \begin{array}{l} m_i \\ n_i-m_i+2 \end{array} \\ 2 & \end{array} \quad (55)$$

Then, assuming that the local subsystem be observable, the reduced order Luenberger Observer System [6] can be expressed as

$$\dot{z}^i=(\tilde{A}_{22}{}^i-L^i\tilde{A}_{12})z^i+\{(\tilde{A}_{22}-L^i\tilde{A}_{12}{}^i)L^i+\tilde{A}_{21}{}^i-L^i\tilde{A}_{11}{}^i\}y^i+(\tilde{B}_2{}^i-L^i\tilde{B}_1{}^i)\tilde{m}^i \quad (56)$$

$$z^i=w^i-L^iy^i \quad (57)$$

where z^i : observer states

L^i : constant matrix (observer parameter) to be optimally selected.

$$w^i \triangleq [\underset{n_i-m_i}{w_u{}^{i\prime}}\vdots \underset{1}{w_p{}'}\vdots\underset{1}{w_{\dot{p}}{}^i}]'1 \quad (58)$$

Therefore, the unmeasurable $x_u{}^i$ and p^i are estimated as

$$x_u{}^i\fallingdotseq w_u{}^i \quad (59)$$

$$p^i\fallingdotseq w_p{}^i \quad (60)$$

For the optimal determination of the observer parameter L^i, a new performance measure $J_z{}^i$ for estimation errors is defined by

$$J_z{}^i(L^i,\tilde{x}_0{}^i) \triangleq \int_0^\infty(w^i-\tilde{x}_u{}^i)'(w^i-\tilde{x}_u{}^i)dt$$
$$=\int_0^\infty\tilde{x}_0{}^{i\prime}(-L^i,I)'\exp\{(A_{22}{}^i-L^iA_{12}{}^i)'t\}\exp\{(A_{22}{}^i-L^iA_{12}{}^i)t\}(-L^i,I)\tilde{x}_0{}^idt \quad (61)$$

where $\tilde{x}_0{}^i\triangleq\tilde{x}^i|_{t=0}$

and, it is, however, necessary to avoid J_z's dependence on the initial states unknown. Therefore, introducing the usual technique of statistical averages, J_z is redefined as

$$\bar{J}_z(L^i) \triangleq E_x[J_z(L^i,\tilde{x}_0{}^i)]$$
$$=\mathrm{tr}\int_0^\infty\exp\{(\tilde{A}_{22}{}^i-L_2\tilde{A}_{12}{}^i)'t\}\exp\{(\tilde{A}_{22}{}^i-L^i\tilde{A}_{12}{}^i)t\}(-L^i,I)\Sigma^i(-L^i,I)'dt \quad (62)$$

where E_x : denote expectation on $\tilde{x}_0{}^i$

$$\Sigma^i \triangleq \begin{array}{cc}\begin{pmatrix} \Sigma_{11}{}^i & \Sigma_{12}{}^i \\ \Sigma_{21}{}^i & \Sigma_{22}{}^i \end{pmatrix} & \begin{array}{l} m_i \\ n_i-m_i+2 \end{array} \\ \begin{array}{cc} m_i & n_i-m_i+2 \end{array} & \end{array} : \text{variance of } \tilde{x}_0{}^i \quad (63)$$

The necessary condition to minimize $\bar{J}_z(L^i)$ is

$$\left.\frac{\partial\bar{J}_z(L^i)}{\partial L^i}\right|_{L^i=\overset{*}{L}{}^i}=0 \quad (64)$$

A Kleimann's lemma [7] is applied to derive the formula for $\partial\bar{J}_z/\partial L^i$ [see Appendix II], and then, Eq. (64) results in

$$L^i=(K^i\tilde{A}_{12}{}^i+\Sigma_{12}{}^{i\prime})\Sigma_{11}{}^{i-1} \quad (65)$$

$$K^i(\tilde{A}_{22}{}^i-L^i\tilde{A}_{12})'+(\tilde{A}_{22}{}^i-L^i\tilde{A}_{12}{}^i)K^i+(L^i\Sigma_{11}{}^iL^{i\prime}-\Sigma_{21}{}^iL^{i\prime}-L^i\Sigma_{12}{}^i+\Sigma_{22}{}^i)=0 \quad (66)$$

The iterative scheme for determining the optimal $\overset{*}{L}{}^i$ is summarized as follows:

i) Assume an initial $L^{i(k)}$ which stabilizes $(\tilde{A}_{22}{}^i-L^{i(k)}\tilde{A}_{12}{}^i)$. k (iteration count)$=0$

ii) Substitute $L^{i(k)}$ into Eq. (66), and solve it for $K^{i(k)}$, using any of the standard techniques to be found elsewhere.

iii) Substitute $K^{i(k)}$ into Eq.(65), and calculate $L^{i(k+1)}$. *If* $||L^{i(k+1)}-L^{i(k)}|| \leq$ error limit, use $L^{i(k+1)}$ as $\overset{*}{L}{}^i$. Otherwise go to ii).

With the L^{*i} thus determined, the decentralized observer on each area basis is optimally implemented by Eqs. (56) and (57).

4. Decentralized Estimator for Tie-line Power Flows

In Eq. (41), inputs to the decentralized controller are $x^i(=[y^{i\prime} \vdots x_u^{i\prime}]')$, p^i and x^0 (composed of y^{0i} and y^{0j} $(j \neq i)$).

It has been mentioned in the preceding sections that y^i and y^{0i} are accessible to the controller and x_u^i is estimated by the i-th observer, but information on y^{0j} (tie-line power flows between other areas) is assumed to be transmitted only at intervals to respective decentralized controller via the upper level station.

In such a case, paying attention to its relatively slow change, it is suggested that $y^{0j}(t)$ at time t between the sampling intervals is estimated by the following quadratic extapolation algorithm.

$$\hat{y}^{0j}(t)=y^{0j}(t_0)+\{y^{0j}(t_{-2})-4y^{0j}(t_{-1})+3y(t_0)\}\frac{t-t_0}{2\tau}+\{y^{0j}(t_{-2})-2y^{0j}(t_{-1})+y^{0j}(t_0)\}\frac{(t-t_0)}{2\tau^2} \tag{67}$$

where $\hat{y}^{0j}(t)$, $y^{0j}(t_0)$, $y^{0j}(t_{-1})$ and $y^{0j}(t_{-2})$: estimated values, immediate past, 1-step past and 2-step past sampled values of y^{0j}, respectively.

Consequently the structure of the decentralized load-frequency controller, observer and quadratic estimator in respect to area i is illustrated in Fig. 2.

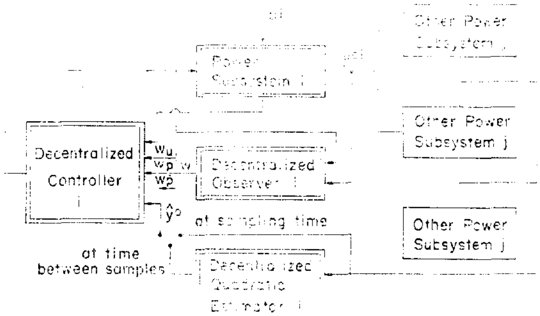

Fig. 2. Power subsystem with a decentralized load-frequency controller, observer and tie-line flow estimator.

5. Digital Simulations

The presented approach was tested on the two-area model system used by Elgerd [3] by means of computer system. In this model, the i-th area $(i=1,2)$ is associated with the following local measurable and unmeasurable states and interconnecting state:

$$x^i=[x_1{}^i, x_2{}^i, x_3{}^i]'=[y^i, x_{u1}{}^i, x_{u2}{}^i]'$$
$$=[\Delta f^i, \Delta g^i, \Delta p_g]'$$
$$x^{01}=y^{01}=-x^{02}=-y^{02}=\Delta p_{12}{}^1$$

where $\Delta f^i, \Delta g^i, \Delta p_g{}^i, \Delta p_{12}$: frequency, governor position, generator output deviation in area i and tie-line power flow deviation from area 1 to 2, respectively.

And the interlinked structures of the i-th subsystem $(i=1,2)$ are assumed as

$$A^i=\begin{pmatrix} -0.05 & 0 & 6 \\ -5.21 & -12.5 & 0 \\ 0 & 0.667 & -0.667 \end{pmatrix}, \quad B^i=\begin{pmatrix} 0 \\ 12.5 \\ 0 \end{pmatrix}, \quad D^i=\begin{pmatrix} -6 \\ 0 \\ 0 \end{pmatrix}$$

$$A^{01}=[\ 0.54 \quad 0 \quad 0\ ], \quad A^{02}=[-0.54 \quad 0 \quad 0\ ]$$
$$A^{10}=[\ -6 \quad 0 \quad 0\ ]', \quad A^{20}=[\ 6 \quad 0 \quad 0\ ]'$$

$$\Sigma^i=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad Q^i=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad R^i=\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$Q^0=1$$

For comparison purpose, four types of simulations were done: Type A) Optimal centralized control with the assumption that the entire system states and disturbances be measurable and so accessible to the controller. Type B) Near-optimal decentralized control with the assumption that the corresponding local states and inter-area tie-line states be measurable and so accessible to the controller. Type C) Near-optimal decentralized control with the use of the decentralized observer and inter-area tie-line states known and Type D) Near-optimal decentralized control with the use of both the decentralized observer and quadratic estimator for tie-line information.

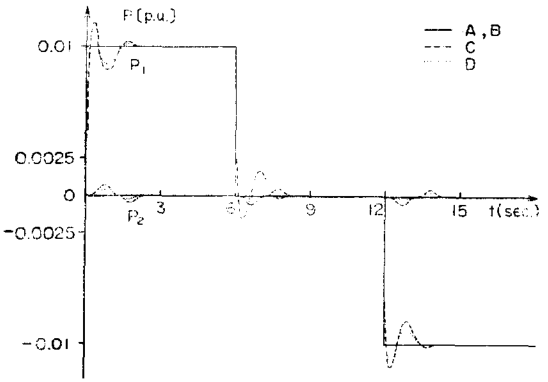

Fig. 3. Comqarison of actual disturbances (p^1, p^2) and identified disturbances (w_p^1, w_p^2). in cases of Typs C and D.

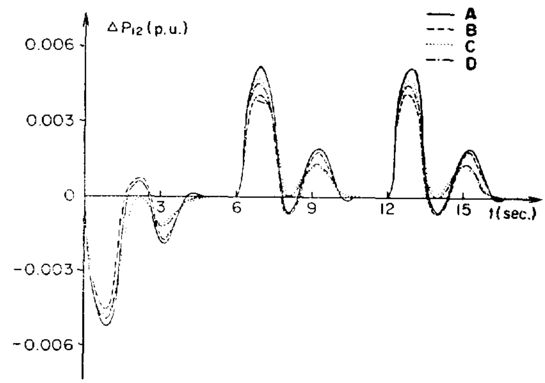

Fig. 6. Comparison of tie-line power flow deviation (Δp_{12}) in cases of Type A,B,C and D, respectively.

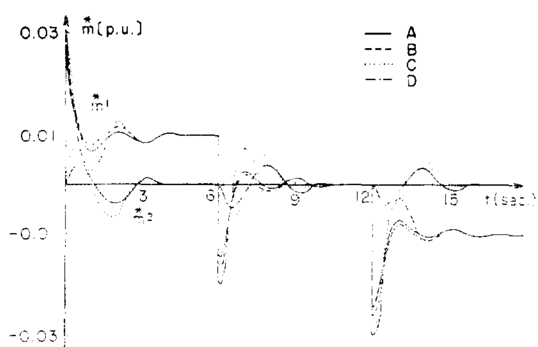

Fig. 4. Comparison of optimal and near-optimal controls (m^1, m^2) in cases of Type A,B,C and D, respectively.

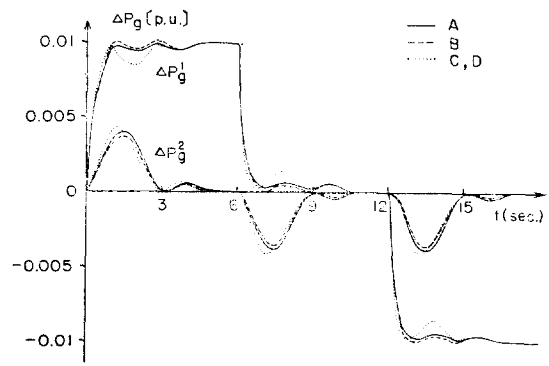

Fig. 7. Comparison of generator output deviations ($\Delta p_g^1, \Delta p_g^2$) in cases of Type A,B,C and D, respectively.

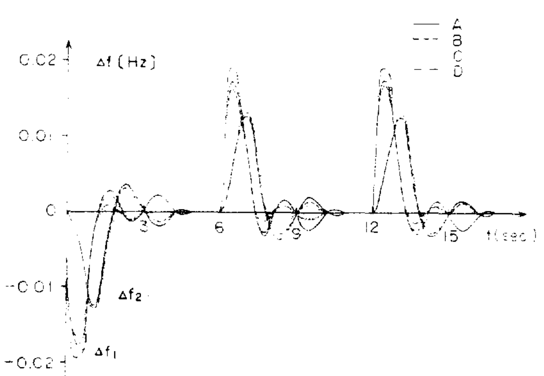

Fig. 5. Comparison of optimally and near-optimally controlled frequency deviations ($\Delta f^1, \Delta f^2$) in cases of Type A,B,C and D, respectively.

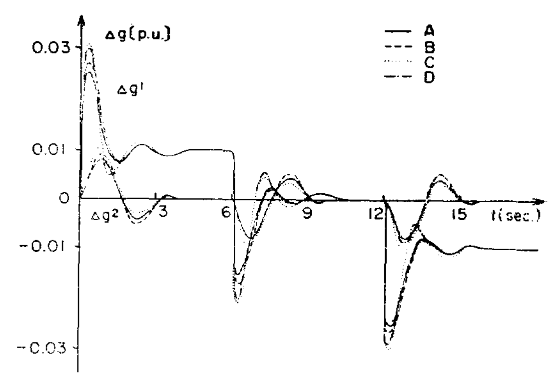

Fig. 8. Comparison of governor-position deviations ($\Delta g^1, \Delta g^2$) in cases of Type A,B,C and D, respectively.

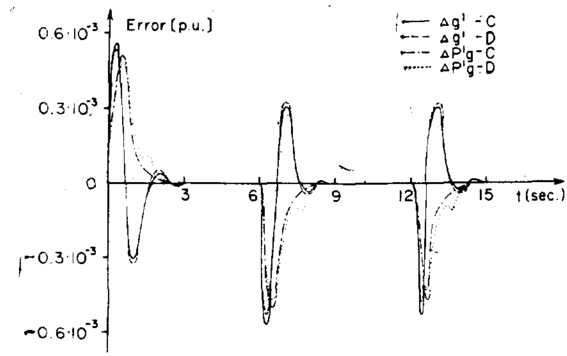

Fig. 9. Identification errors for generator output and governor position associated with area 1.

The following figures show representative digital simulations with computing step size $\Delta t=0.05$ [sec.] and tie-line flow sampling period $\tau=0.25$ [sec.] for the case of successive step changes of the load disturbance p shown in Fig.3.

As shown in Figs. 3~9, the near-optimal decentralized approach scheme (Type D) demonstrates a satisfactory approximation to the optimal centralized one.

If a performance deterioration η for a respective control type is defined as,

$$\eta_\kappa \triangleq \frac{\sqrt{J_\kappa}}{\sqrt{J_A}}\times 100 \ [\%]$$

where J_κ : J of Eq.(8) in case of Type $\kappa, \kappa=A, B, C$ and D.

The above simulations result in

$\eta_A=100[\%]$, $\eta_B=101[\%]$, $\eta_C=114[\%]$,
$\eta_D=(\tau/\Delta t=5)=114[\%]$, $\eta_D(\tau/\Delta t=10)$
$=117[\%]$

Conseqently, The decentralized controller and quadratic estimator are shown to deteriorate little performance measure, while the decentralized observer causes performance deterioration of 14% due to the initial transient phenomena, but after about 1 second of transient period, there are little differences between the centralized and decentralized control.

Conclusion

The near-optimal decentralized control structure with the decentralized optimal load-frequency controller, observer and tie-line estimator presented herein, possesses the desirable features mentioned in the introduction.

Moreover, the practical usefulness of the scheme will become more clear if some smoothing or filtering algorithm for excessive control actions are devised to prevent governor damages. It is hoped to report such a scheme in the future.

Acknowledgment

The author wishes to express his sincere thanks to the Korea Science and Engineering Foundation for the financial assistance in pursuance of this work.

Appendix I

In the final equilibrium point ($t\to\infty$), Eq.(1) or (45) can be rewritten, with $x_f{}^i=0$ and $y_f{}^{0i}=y_f{}^i=0$ ($\because$ target states are requlated to zero), as

$$\begin{pmatrix} A_{11}{}^i & A_{12}{}^i \\ A_{21}{}^i & A_{22}{}^i \end{pmatrix}\begin{pmatrix} 0 \\ x_{uf} \end{pmatrix}+\begin{pmatrix} B_1{}^i \\ B_2{}^i \end{pmatrix}m_f{}^i+D^i p_f{}^i=0 \qquad \text{a)}$$

and Eq. (a) is rearranged as

$$T^{i\prime}T^i\begin{pmatrix} x_{uf}{}^i \\ m_f{}^i \end{pmatrix}=-T^{i\prime}D^i p_f{}^i \qquad \text{(b)}$$

where $T^i\triangleq\begin{pmatrix} A_{12}{}^i & B_1{}^i \\ A_{22}{}^i & B_2{}^i \end{pmatrix}$, $x_{uf}{}^i$: final values of non-target states

In the case of assigning one regulating power plant to each area, since $T^{i\prime}T^i$ is nonsingular,

$$\begin{pmatrix} x_{uf}{}^i \\ m_f{}^i \end{pmatrix}=-(T^{i\prime}T^i)^{-1}T^{i\prime}D^i p_f{}^i=\begin{pmatrix} G^i \\ S^i \end{pmatrix}p_f{}^i \qquad \text{(c)}$$

On the other hand, in the case of assigning more than one regulating plants to each area, there exists, mostly, a certain proportionality among the inputs to the respective plants on some economic or capability basis. In this case, the dimension of $m_f{}^i$ can be reduced, using such prescribed proportionality relations, so that we might find out the equation corresponding to Eq.(c).

Therefore, x_f and m_f result, repectively in linear combinations of p_f as

$$x_f{}^i=\begin{pmatrix}0\\G^i\end{pmatrix}p_f=E^ip_f{}^i \quad \text{(d)} \qquad m_f{}^i=S^ip_f{}^i \quad \text{(e)}$$

Consequently,

$$x_f=Ep_f \quad \text{(f)} \qquad m_f=SP_f \quad \text{(g)}$$

Appendix II

Eq. (62) can be rewritten as

$$J=\mathrm{tr}\int_0^\infty \exp(F^{i\prime}t)\exp(F^it)[L^i\Sigma_{11}{}^iL^{i\prime}-\Sigma_{21}{}^iL^{i\prime}-L^i\Sigma_{12}{}^i+\Sigma_{22}{}^i]dt \quad \text{(a)}$$

where $F^i\triangleq\tilde{A}_{22}{}^i-L^i\tilde{A}_{12}{}^i$

$\frac{\partial J}{\partial L^i}$ can be obtained from the following Lemma [7].

Lemma

Let $f(x)$ be a trace function.

If $f(x+\varepsilon\Delta x)-f(x)=\varepsilon\mathrm{tr}[M(x)\Delta t]$ as $\varepsilon\to 0$,

where $M(x)$: $n\times r$ matrix

x : $r\times n$ matrix

then, $\frac{\partial f(x)}{\partial x}=M'(x)$

From Eq.(a),

$$\begin{aligned}J(L^i+\varepsilon\Delta L^i)&=\mathrm{tr}\int_0^\infty \exp[F^i-\varepsilon\Delta L^i\tilde{A}_{12}{}^i]'t\cdot\exp[F^i-\varepsilon\Delta L^i\tilde{A}_{12}{}^i]t\cdot[(L^i+\varepsilon\Delta L^i)\\&\quad\Sigma_{11}{}^i(L^i+\varepsilon\Delta L^i)'-\Sigma_{21}{}^i(L^i+\varepsilon\Delta L^i)'-(L^i+\varepsilon\Delta L^i)'\Sigma_{12}{}^i+\Sigma_{22}{}^i]dt\\&=\mathrm{r}\int_0^\infty\{-\int_0^t\exp[F^i(t-\sigma)]\Delta L^i\tilde{A}_{12}{}^i\cdot\exp[F^i\sigma]d\sigma+\exp[F^it]\}'\cdot\\&\quad\{-\varepsilon\int_0^t\exp[F^i(t-\sigma)]\Delta L^i\tilde{A}_{12}{}^i\exp[F^i\sigma]d\sigma+\exp[F^it]\}\cdot\{(L^i+\varepsilon\Delta L^i)\Sigma_{11}{}^i(L^i+\varepsilon\Delta L^i)'\\&\quad-\Sigma_{21}{}^i(L^i+\varepsilon\Delta L^i)'-(L^i+\varepsilon\Delta L^i)'\Sigma_{12}{}^i+\Sigma_{22}{}^i\}dt\end{aligned} \quad \text{(b)}$$

Expanding Eq. (b) to the 1-st order in ε,

$$\begin{aligned}J(L^i+\varepsilon\Delta L^i)&=\mathrm{tr}\int_0^\infty\{\exp(F^{i\prime}t)\exp(F^it)(L^i\Sigma_{11}{}^iL^{i\prime}-\Sigma_{21}{}^iL^{i\prime}-L^i\Sigma_{12}{}^i+\Sigma_{22}{}^i)\\&\quad+\varepsilon\exp(F^{i\prime}t)\exp(F^it)[L^i\Sigma_{11}{}^i\Delta L^{i\prime}+\Delta L^i\Sigma_{11}L^{i\prime}-\Sigma_{21}{}^i\Delta L^{i\prime}-\Delta L^i\Sigma_{12}{}^i]\\&\quad-\varepsilon\exp(F^{i\prime}t)\int_0^t\exp[F^i(t-\sigma)]\Delta L^i\tilde{A}_{12}{}^i\exp(F^i\sigma)d\sigma[L^i\Sigma_{11}{}^iL^{i\prime}-\Sigma_{21}{}^iL^{i\prime}-L^i\Sigma_{12}{}^i\\&\quad+\Sigma_{22}{}^i]-\varepsilon[\int_0^t\exp[F^{i\prime}(t-\sigma)]\tilde{A}_{12}{}^{i\prime}\Delta L^{i\prime}\exp(F^{i}\sigma)d\sigma\exp(F^it)[L^i\Sigma_{11}{}^iL^{i\prime}\\&\quad-\Sigma_{21}{}^iL^{i\prime}-L^i\Sigma_{12}{}^i+\Sigma_{22}{}^i]\}dt\end{aligned} \quad \text{(c)}$$

From the trace properties of matrix,

$$\begin{aligned}J(L^i+\varepsilon\Delta L^i)-J(L^i)&=2\varepsilon\mathrm{tr}\int_0^\infty\{(\Sigma_{11}{}^iL^{i\prime}-\Sigma_{12}{}^i)\exp(F^{i\prime}t)\exp(F^it)-\int_0^t\tilde{A}_{12}{}^i\exp(F^i\sigma)\\&\quad[L^i\Sigma_{11}{}^iL^{i\prime}-\Sigma_{21}{}^iL^{i\prime}-L^i\Sigma_{12}{}^i+\Sigma_{22}{}^i]\exp(F^{i\prime}t)\exp[F^i(t-\sigma)]d\sigma\}\cdot\Delta L^idt\end{aligned} \quad \text{(d)}$$

Using the Lemma,

$$\begin{aligned}\partial J/\partial L^i&=2\int_0^\infty\exp(F^{i\prime}t)\exp(F^it)[L^i\Sigma_{11}{}^i-\Sigma_{12}{}^{i\prime}]dt-\int_0^\infty\int_0^t\exp[F^{i\prime}(t-\sigma)]\cdot\exp(F^it)[L^i\Sigma_{11}{}^iL^{i\prime}\\&\quad-L^i\Sigma_{21}{}^{i\prime}-\Sigma_{12}{}^{i\prime}L^{i\prime}+\Sigma_{22}{}^i]\exp(F^{i\prime}\sigma)\tilde{A}_{12}{}^id\sigma\cdot dt\end{aligned} \quad \text{(e)}$$

Let $x\triangleq\int_0^\infty\int_0^t\exp[F^{i\prime}(t-\sigma)]\exp(F^it)T^i\Sigma^iT^{i\prime}\cdot\exp(F^{i\prime}\sigma)\tilde{A}_{12}{}^id\sigma dt$ (f)

and interchanging the order of integral,

$$x=\int_0^\infty\int_6^\infty\exp[F^{i\prime}(t-\sigma)]\exp(F^it)T^i\Sigma T^{i\prime}\exp(F^{i\prime}\sigma)\tilde{A}_{12}{}^{i\prime}dtd\sigma \quad \text{(g)}$$

By letting $\tau\triangleq t-\sigma$, above equation is rewritten as,

$$\begin{aligned}x&=\int_0^\infty\int_0^\infty\exp(F^{i\prime}\tau)\exp(F^i\tau)\exp(F^i\sigma)T^i\Sigma^iT^{i\prime}\exp(F^{i\prime}\sigma)\tilde{A}_{12}{}^{i\prime}d\tau d\sigma=\int_0^\infty\exp(F^{i\prime}\tau)\exp(F^i\tau)d\tau\\&\quad\int_0^\infty\exp(F^i\sigma)T^i\Sigma^iT^{i\prime}\exp(F^{i\prime}\sigma)\tilde{A}_{12}{}^{i\prime}d\sigma\end{aligned} \quad \text{(h)}$$

Substituting Eq.(h) into Eq. (e) and applying $\frac{\partial J}{\partial L^i}\Big|_{\overset{*}{L^i}}=0$,

$$\begin{aligned}&\int_0^\infty\exp(\overset{*}{F}{}^{i\prime}\tau)\exp(\overset{*}{F}{}^i\tau)d\tau\cdot[\overset{*}{L}{}^i\Sigma_{11}{}^i-\Sigma_{12}{}^{i\prime}]\\&=\int_0^\infty\exp(\overset{*}{F}{}^{i\prime}\tau)\exp(\overset{*}{F}{}^i\tau)d\tau\cdot\int_0^\infty\exp(\overset{*}{F}{}^i\sigma)\overset{*}{T}{}^i\Sigma^i\overset{*}{T}{}^{i\prime}\exp(\overset{*}{F}{}^{i\prime}\sigma)d\sigma\tilde{A}_{12}{}^{i\prime}\end{aligned} \quad \text{(i)}$$

So the optimal $\overset{*}{L}{}^i$ is

$$\overset{*}{L}{}^i=[\overset{*}{K}{}^i\tilde{A}_{12}{}^{i\prime}+\Sigma_{12}{}^{i\prime}]\Sigma_{11}{}^{i-1} \quad \text{(j)}$$

where

$$\overset{*}{K}{}^i=\int_0^\infty\exp(\overset{*}{F}{}^i\sigma)T^i\Sigma^iT^{i\prime}\exp(\overset{*}{F}{}^{i\prime}\sigma)d\sigma \quad \text{(k)}$$

Assuming there exist $\overset{*}{K}{}^i$ and $\overset{*}{L}{}^i$ such that $\overset{*}{F}(=\tilde{A}_{22}{}^i-\overset{*}{L}{}^i\tilde{A}_{12}{}^i)$ is stable and $\overset{*}{K}{}^i$ is solution of Eq (k), then, $\overset{*}{K}{}^i$ and $\overset{*}{L}{}^i$ are also solution of the following algebraic equation.

$$O=\overset{*}{K}^i[\tilde{A}_{22}{}^i-L^i\tilde{A}_{12}{}^i]'+[\tilde{A}_{22}{}^i-L^i\tilde{A}_{12}{}^i]\overset{*}{K}^i$$
$$+T^i\Sigma^i T^{i\prime} \qquad \text{(l)}$$
$$\overset{*}{L}^i=[\overset{*}{K}^i\tilde{A}_{12}{}^{i\prime}+\Sigma_{12}{}^{i\prime}]\Sigma_{11}{}^{i-1} \qquad \text{(m)}$$

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