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〈**Original**〉

Damping and Shear Distribution in Adhesive Bonded Lap Joint with Viscoelastic Adhesive

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粘彈性接着結合의 減衰 및 剪斷分布

洪　昌　善

抄　錄

正弦的 應力을 받고 있는 粘彈性 結合部의 에너지 손실을 여러가지 因子를 고려하여 구하였다. 彈性接着結合部의 解를 구하고 彈性解의 剪斷係數를 複素剪斷係數로 대치하여 粘彈性 接着結合部의 剪斷變形分布를 구하였다. 주어진 접착제의 최대減衰현상은 접착제의 剛性과 피접착물의 比를 증가시킴으로서 얻을 수 있다. 構造的 結合部에서 파괴가 일어나지 않도록 피접착부와의 剛性比를 결정함에 있어 주의를 요한다.

Nomenclature

θ	Fiber orientation from x-axis to 1-axis
ϕ	Phase angle
ω	Frequency of oscillation
ν	Poisson's ratio
η	Loss coeffcient
Δ	$\lvert G^*_c\rvert L^2/\lvert E^*_{xx}\rvert_u h_2 h_c$
N	$\lvert E^*_{xx}\rvert_U h_2/\lvert E_{xx}^*\rvert_L h_1$
$E^*_{xx}, E^*_{yy}, G^*_{xy}$	Complex moduli with x-y axis
$E^*_{11}, E^*_{22}, G^*_{12}$	Complex moduli with 1-2 axis
G_{12}', G_{12}''	Shear storage and loss moduli
u	Energy dissipated per cycle per unit volume
U	Total energy dissipated
L	Length of the beam
h_1	Thickness of lower adherend
h_2	Thickness of upper adherend
h_c	Thickness of adhesive
G^*_c	Complex shear modulus of the adhesive
U_0	Dimensionless function of the total energy dissipated

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1. Introduction

Adhesive bonding has been employed for centuries, but its use has been restricted by the availability of only animal and vegetable glues, so that the structural applications have been confined to timber, and even then only to supplement dowel and pin fasteners. All these early applications suffered from the susceptibilities of adhe sives to moisture, fungi and bacteria. The development of synthetic resin adhesives which were resistent to these conditions made possible the adhesive bonded joint of composite strructures on a much more ambitious scale. Recently, the biggest drawback to the structural application of composite materials in the aerospace vehicles is the predictable and reliable joining of one component to another. To date considerable work has been done to study the elastic analysis of adhesive joints* [1-9]. Among

* Numbers in brackets refer to the reference.

the foremost contributions in the field of bonded lap joints, the first paper is generally accredited to Volkersen[1]. He derived the load distribution of multi-row rivetted lap joints in tension, where he idealized the rivets to be replaced by a continuous medium of given flexibility, and considered the case of an adhesive bonding. By neglecting the bending of the adherends and stresses normal to the adhesive layer, his analysis of of a simple lap joint yielded a significant parameter relating the extensional stiffness of the adherends to the shear stiffness of the adheive layer.

de Bruyne [2] found good agreement with Volkersen using simple metal lap joints, and demonstrated the advantage of bevelled lap, for which it can be seen intuitively that the stress concentrations would be relieved, since this is elastically equivalent to a scarf joint while retaining the offset. At about the same time as de Bruyne's report, a rigorous analysis was published by Goland and Reissner [3] examining the stress distributions for adhesive joints with identical adherends under the action of tensile forces at the ends of the adherends. Hahn and Fouser[4] analyzed the bending of the adherends (ignoring the adhesive) in a lap joint assuming they act as cylindrically bent plates (analogous to the Goland and Reissner solution). The second part of the article considered the differential straining in a double lap joint (bending neglected) and determined the shear stress distribution in the adhesive. Cornell [5] considered a simplification of the Goland-Reisser method in which the bonding layer was idealized to a continuum of tension and shear springs. He used a brittle lacquer and photoelastic stress analysis to experimentally verify his analytical formulation of a single lap brazed joint in terms of a differential equation. Wah [6] reported the stress distribution of a bonded anisotropic lap joint in elastic range according to a strengt hof materials theory. The case of anisotropic adherends was investigated by the finite element mothod[7] or finie diffe rences [8]. Stress distribution in a stepped joint has been analytically discussed by Erdogan and Ratwani [9]. All the above analyses assume that the adhesive and the adherends are elastic. But it should be noted that the adhesive is a viscoelastic polymer and the material is characterized by complex modulus. The main objective of this paper is to investigate the effects of shear distribution and damping in the viscoelastic adhesive of the bonded lap joint.

2, Theoretical Analysis

The basic approach to analyzing adhesive joints has been to idealize the joint in terms of mathematical model whereby the material properties and joint geometry are related to the applied loads resulting in a differential equation.

The problem, originally solved by Goland and Reissner [3] for a completly isotropic material system, is extended as a problem with damping of a viscoelastic adhesive. In order to simplify assumptions that may be made, the lap considered in the theoretical analysis is a double lap joint which consists of two identical outer laminated adherends boned through two viscoelastic adhesive to a center adherend. For the practical application of composite material, we have supposed that the laminated adherends are symmetrical so that, with the middle plane as reference plane, the bending and stretching terms are uncoupled; therefore, the shear coupling compliances are zero [10].

For stress analyses the assumption must be made that there are no bending moments applied to the lap, neglect the shear deformation of the adherends, the shear stress does not vary across the bonding line, and the shear stress-strain relationship of the adhesive is constant. A double lap joint typifies the condition of no bending moments in the adherends (see Fig.1). Since a double lap presents a symmetrical configuration, only one bond may be considered to analyze the stress distribution.

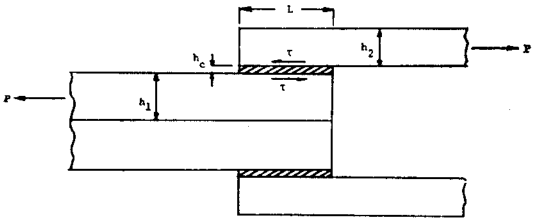

Fig. 1. Double lap joint and notations for analysis.

Let the coordinate x, measured from one end of lower adherend, locate the points along the axis of the lap. Lower adherend is described by $|E_{11}^*|_L$ and h_1, upper adherend by $|E^*_{11}|_u$ and h_2. The width of the lap is b, and its length is L. It is assumed that the adhesive material obeys the linear viscoelatic constitutive relations. The viscoelastic adhesive is described by G^*_c, ϕ, and h_c.

The loads P are transmitted between the laminate so that the force carried by the upper laminate at a section x—units from the origin is

$$P_2=\int_0^x \tau\, b_2\, dx \tag{1}$$

where b_2 is the joint width.

The force remaining in the lower laminate is then

$$p_1=p-p_2=p-\int_0^x \tau\, b_2\, dx \tag{2}$$

The strain in the upper laminate is

$$\varepsilon_2(x)=\frac{\partial u_2}{\partial x}=\frac{p_2(x)}{|E_{11}^*|_u h_2 b_2}=\frac{b_2}{|E_{11}^*|_u h_2 b_2}\int_0^x \tau(x)\, dx \tag{3}$$

From equation (2), $\varepsilon_2(x)$ may be written:

$$\varepsilon_2(x)=\frac{G_c}{|E^*_{11}|_u h_2}\int_0^x \gamma(x)\, dx \tag{4}$$

where G_c is the shear modulus of the adhesive cement.

The strain in lower laminate is

$$\varepsilon_1(x)=\frac{\partial u_1}{\partial x}=\frac{p-p_2(x)}{|E_{11}^*|_L b_1 h_1}=\frac{1}{|E_{11}^*|_L b_1 h_1}\left[p-b_2 G_c\int_0^x \gamma(x)\,dx\right] \tag{5}$$

The shear strain in the adhesive cement is

$$\gamma=u/h_c \text{ or } h_c\gamma=u \tag{6}$$

where h_c is the thickness of the adhesive cement and u is the displacement.

The total shear strain of the adhesive at any point along the glue line is composed of the difference in tensile strain between the adjacent adherends plus the shear strain of the adhesive at $x=0$.

Differentiating this expression twice with respect to x from equations (4) and (5)

$$\frac{\partial^2 u}{\partial x^2}=\frac{G_c}{|E_{11}^*|_u h_2}\gamma(x)+\frac{b_2 G_c}{|E_{11}^*|_L b_1 h_1}\gamma(x) \tag{7}$$

From equation (6), it is apparent that

$$h_c\frac{\partial^2\gamma}{\partial x^2}=\frac{\partial^2 u}{\partial x^2} \tag{8}$$

Equating equations (7) and (8)

$$h_c\frac{\partial^2\gamma(x)}{\partial x^2}=\frac{G_c}{|E_{11}^*|_u h_2}\gamma(x)+\frac{b_2 G_c}{|E_{11}^*|_L b_1 h_1}\gamma(x) \tag{9}$$

If the width is the same: $b_1=b_2=b$, equation (9) may be written

$$h_c \frac{\partial_2 \gamma(x)}{\partial x^2} = \frac{G_c}{|E_{11}^*|_u h_2} \gamma(x) + \frac{G_c}{|E_{11}^*|_L h_1} \gamma(x) \tag{10}$$

So equation (10) may be rewritten

$$\frac{\partial^2 \gamma(x)}{\partial x^2} - k^2 \gamma(x=0 \tag{11}$$

where

$$k^2 = \frac{L^2 G_c [|E_{11}^*|_L h_1 + |E_{11}^*|_u h_2]}{h_c |E_{11}^*|_L h_1 |E_{11}^*|_u h_2} \tag{12}$$

Solving the equation (11), the expression for the shear strain in the adhesive cement is

$$\gamma(x) = \frac{p}{bLG_c} \sqrt{\frac{L^2 G_c |E_{11}^*|_u h_2}{h_c |E_{11}^*|_L h_1 [_L h_1 + |E_{11}^*|_u h_2]}} \times \left(\frac{\frac{|E_{11}^*|_L h_1 + |E_{11}^*|_u h_2}{|E_{11}^*|_u h_2}}{} \right.$$

$$\left. \frac{\cosh k \frac{x}{L} + \cosh k \left(1 - \frac{x}{L}\right) - \cosh k \frac{x}{L}}{\sinh k} \right) \tag{13}$$

In the steady state, for a sinusoidally varying axial force with time, equation (13) may be adapted to an adhesive lap with a viscoelastic cement if $p_0 e^{i\omega t}$ is substituted for p, and G_c^* is substituted for G_c. Complex modulus G_c^* [11] is defined by

$$G_c^* = G_c' + iG_c''$$

or

$$G_c^* = |G_c^*| \ (\cos\phi + i \sin\phi) \tag{14}$$

where

$$|G_c^*| = \sqrt{(G_c')^2 + (G''_c)^2} \tag{15}$$

and ϕ=phase angle.

By substitituting these, equation (13) becomes the expression for shear strain in a viscoelastic lap at any point x and at time t, written by

$$\gamma(x, t) = \frac{p_0 e^{i\omega t} \sqrt{\Delta / W} (\cos\phi - i \sin\phi) (\cos\phi/2 + i \mathrm{isn}\phi/2)}{bL |G_c^*| (M_1^2 + M_2^2)}$$

$$\times (M_1 - iM_2) \Big\{ (W-1) \cosh \alpha \frac{x}{L} \cos \beta \frac{x}{L} + \cos \alpha \left(1 - \frac{x}{L}\right) \cos \beta \left(1 - \frac{x}{L}\right)$$

$$+ i [W(-1) \sinh \alpha \frac{x}{L} \sin \beta \frac{x}{L} + \sinh \alpha \left(1 - \frac{x}{L}\right) \sin \beta \left(1 - \frac{x}{L}\right) \Big\} \tag{16}$$

where

$$\alpha = \sqrt{W\Delta} \cos\phi/2, \qquad \beta = \sqrt{W\Delta} \sin\phi/2$$

$$\Delta = \frac{|G_c^*| L^2}{|E_{xx}^*|_u h_2 h_c}$$

$$W = 1 + \frac{|E_{xx}^*|_u h_2}{|E_{xx}^*|_L h_1}$$

$$M_1 = \sinh \alpha \cos \beta$$

$$M_2 = \cosh \alpha \sin \beta \tag{17}$$

In calculating the damping of a configuration, it is necessary to obtain the amplitude of sinusoidal shear strain that each element of the adhesive is subjected to. Equation (16) may be written in the form

$$\gamma(x, t) = \gamma_0(x) \sin(\omega t - \phi) \tag{18}$$

where $\gamma_0(x)$ is obtained by taking the square root of the sum of the squares of the real and imaginary parts of $\gamma(x, t)$ in equation (16). This is obtained to be

$$\gamma_0(x) = \frac{p_0 \sqrt{\Delta / W}}{bL |G^*_c|} \left\{ \frac{Q_1^2 + Q_2^2 + 2(W-1)(Q_1 R_1 + Q_2 R_2)}{M_{12} M_2^2} \right.$$

$$\left. \frac{+ (W-1)^2 (R_1^2 + R_2^2)}{} \right\}^{1/2} \tag{19}$$

where

$$Q_1 = \cosh\left[\alpha \left(1 - \frac{x}{L}\right)\right] \cos\left[\beta \left(1 - \frac{x}{L}\right)\right]$$

$$Q_2 = \sinh\left[\alpha \left(1 - \frac{x}{L}\right)\right] \sin\left[\beta \left(1 - \frac{x}{L}\right)\right]$$

$$R_1 = \cosh \alpha \frac{x}{L} \cos \beta \frac{x}{L}$$

$$R_2 = \sinh \alpha \frac{x}{L} \sin \beta \frac{x}{L} \tag{20}$$

It will be convenient to express the shear strain distribution in a dimensionless from given by

$$T(W, x, \Delta, \phi) = \frac{\gamma_0(x) bLG^*_c}{p_0}$$

$$= \left\{ \frac{\Delta}{W} \frac{Q_1^2 + Q_2^2 + 2(W-1)(Q_1 R_1 + Q_2 R_2)}{M_1^2 + M_2^2} \right.$$

$$+(W-1)^2(R_1^2+R_2^2)\Big\}^{1/2} \tag{21}$$

The energy dissipated per cycle by the unit volume is

$$\underline{u}=\int \tau\dot{\gamma}\,dt \tag{22}$$

where

$$\tau=\gamma_0(x)\,(G_c'\sin\omega t+G_c''\cos\omega t)$$
$$\dot{\gamma}=\omega\gamma_0(x)\cos\omega t \tag{23}$$

Integrating equation (22) over one period results in

$$\underline{u}=\pi G''_c\gamma_0^2\,(x) \tag{24}$$

Thus, the total energy dissipated per cycle in the viscoelastic adhesive of the lap joint is obtained as in the following:

$$U=\int_0^V \underline{u}\,dV=\pi\, bh_c\,|G_c^*|\,\sin\phi\int_0^L\gamma_0^2(x)\,dx \tag{25}$$

By substituting $\gamma_0(x)$ as given by equation (19) into equation (25), we get the total energy dissipated as follows

$$U=\frac{p_0^2\,h_c\,\Delta\sin\phi}{2bLW\,|G^*_c|}\left\{\frac{\dfrac{\sinh\alpha\cosh\alpha}{\alpha}+\dfrac{\sin\beta\cos\beta}{\beta}}{\sinh^2\alpha+\sin^2\beta}\right.$$
$$[2(W-1)\cosh\alpha\cos\beta+1+(W-1)^2] \tag{26}$$
$$\left.+2(W-1)\left(\frac{\dfrac{\cosh\alpha\sin^3\beta}{\beta}-\dfrac{\sinh^3\alpha\cos\beta}{\alpha}}{\sinh^2\alpha+\sin^2\beta}\right)\right\}$$

With $N=|E^*_{xx}|_u h_2/|E^*_{xx}|_L h_1$, the expression of W as given by equation (17) becomes

$$W=1+N \tag{27}$$

Substituting equation (27) into equation (26) results in

$$U=\frac{\pi p_0^2\,h_0\,\Delta\sin\phi}{2bL(1+N)\,|G^*_c|}\left\{\frac{\dfrac{\sinh\alpha\cosh\alpha}{\alpha}+\dfrac{\sin\beta\cos\beta}{\beta}}{\sinh^2\alpha+\sin^2\beta}\right.$$
$$[2N\cosh\alpha\cos\beta+1+N^2]$$
$$\left.+2N\left(\frac{\dfrac{\cosh\alpha\sin^3\beta}{\beta}-\dfrac{\sinh^3\alpha\cos\beta}{\alpha}}{\sinh^2\alpha+\sin^2\beta}\right)\right\} \tag{28}$$

For the purpose of discussion, it will be convenient to define a dimensionless function for equation (28) given by

$$U_0=\frac{2bL\,|G_c^*|\,U}{\pi\,p_0^2\,h_c}$$
$$=\frac{\Delta\sin\phi}{1+N}\left\{\frac{\dfrac{\sinh\alpha\cosh\alpha}{\alpha}+\dfrac{\sin\beta\cos\beta}{\beta}}{\sinh^2\alpha+\sin^2\beta}\right.$$
$$[2N\cosh\alpha\cos\beta+1+N^2]$$
$$\left.+2N\left(\frac{\dfrac{\cosh\alpha\sin^3\beta}{\beta}-\dfrac{\sinh^3\alpha\cos\beta}{\alpha}}{\sinh^2\alpha+\sin^2\beta}\right)\right\} \tag{29}$$

It is interesting to observe that the special case of joint which consists of identical materials of lower and upper adherend with the same thickness. For this special case, $N=1$, equation (29) becomes

$$U_0=\Delta\sin\phi\left\{\frac{\dfrac{\sinh\alpha\cosh\alpha}{\alpha}+\dfrac{\sin\beta\cos\beta}{\beta}}{\sinh^2\alpha+\sin^2\beta}\right.$$
$$[\cosh\alpha\cos\beta+1]$$
$$\left.+\left(\frac{\dfrac{\cosh\alpha\sin^3\beta}{\beta}-\dfrac{\sinh^3\alpha\cos\beta}{\alpha}}{\sinh^2\alpha+\sin^2\beta}\right)\right\} \tag{30}$$

3. Disussion of Results

The energy dissipated representing damping capacity in a viscoelastic adhesive lap joint are given in equation (29). The behavior and analysis of a lap joint depend on a number of factors including the material properties ot the adhesive and adherends, geometries and loading conditions. In order

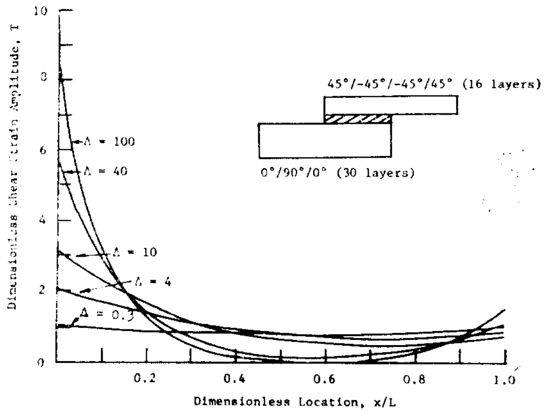

Fig. 2. Dimensionless shear strain amplitude vs. x/L for various Δ.

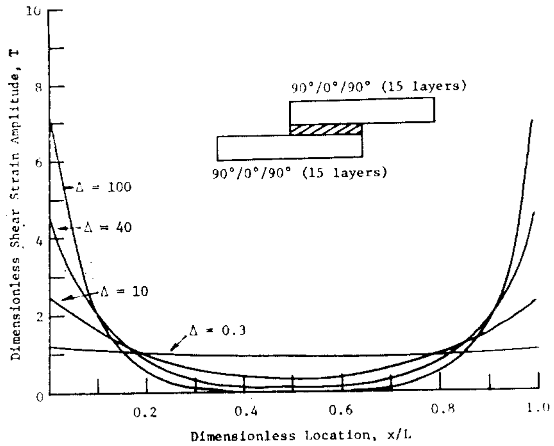

Fig. 3. Dimensionless shear strain amplitude vs. x/L for various values of Δ.

to derive the energy dissipated in the visco-elastic adhesive the amplitude of shear strain under sinusoidal loading conditions was obtained. The adherend material properties were taken from the reference [12].

For the purpose of discussion, dimensionless shear strain amplitude of two cases, namely unequal adherends of different properties and thickness and identical adherends of same properties and thickess are plotted in Figures 2 and 3. The upper laminate consists of 16 layers with fiber oreitnation and sequence 45°/−45°/−45°/45°, each layer being 0.01-in. thick, and the lower laminate consists of 30 layers with orientation and sequence 0°/90°/0°, each layer being 0.01-in. thick. The identical laminate case consists of 15 layers with orientations 90°/0°/90°. From examining Figures 2 and 3, the symmetry of shear strain amplitude distribution about the plane $\frac{x}{L}=0.5$ is influenced by varying the parameter N. It can also be seen that the parameter Δ determines the initial steepness of amplitude decay along the length of the adhesive lap for a given value N. The parameters N and Δ contain the quantities, $|E_{xx}^*|_U h_2$, $|E_{xx}^*|_L h_1$, and $G^*_c L^2 h_c$. These may be interpreted as measures of the rigidities of upper adherend, lower adherend, and the adhesive, respectively. Therefore, the parameter N is the ratio of the rigidity of upper adherend to that of lower adherend. On the other hand, the parameter Δmay be interpreted as the ratio of the rigidity of the adhesive to that of upper adhereend.

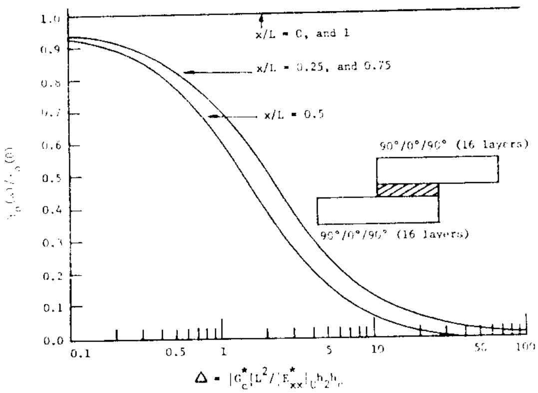

Fig.4. Dimensionless shear strain distribution, $\gamma_0(x)/\gamma_0(0)$ vs. Δ for various values of x/L, $\gamma_r=0.175$

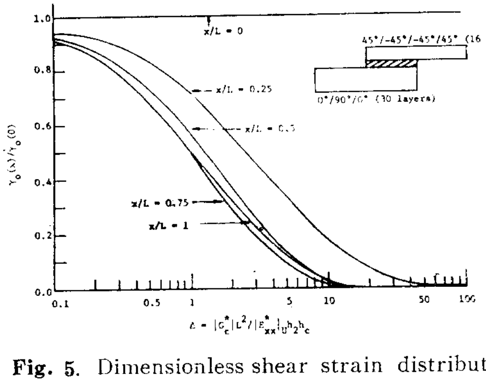

Fig. 5. Dimensionless shear strain distribution, $\gamma(x)/\gamma_0(0)$ vs. Δ for varihus values of x/L, $\gamma_r=0.175$

As shown, the energy dissipated per cycle in equation (24), the energy dissipated is a function of the square of shear strain amp-

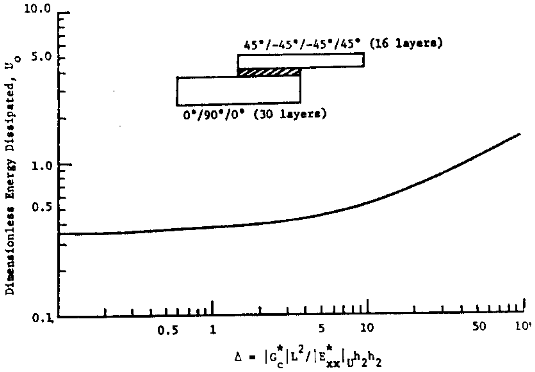

Fig. 6. Dimensionless energy dissipated vs. Δ for $\eta=0.175$

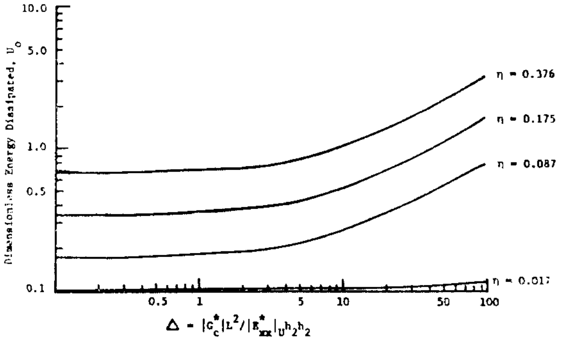

Fig. 7. Dimensionless enengy dissipated vs. Δ for various values of loss coefficient η, $N=0.1$

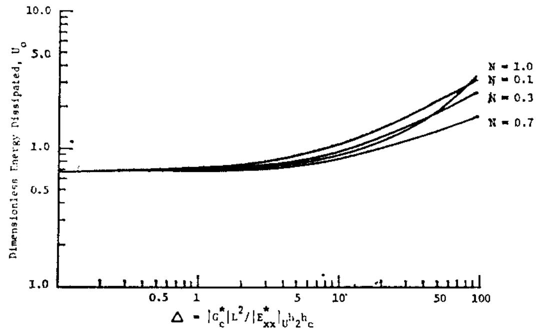

Fig. 8. Dimensionless energy dissipated vs. Δ for various values of $N=|E^*_{xx}|_U h_2/|^*E_{xx}|_L h_1$, $\eta=0.376$

litude. Therefore, the increased energy dissipated occurs when the effective shear strain distributed throughout the viscoelastic adhesive joint. The distribution of shear strain amplitude function, $\gamma_0(x)/\gamma_0(0)$ versus Δ for various values of $\frac{x}{L}$ is show in Figures 4 and 5. It can be seen from the figures that the value of Δ determines the slope of amplitude approaches to zero about the plane $\frac{x}{L}=0.5$ with increasing the value of Δ. It is interesting to note that the case of unequal laminates approaches to zero from the value of $\Delta=10$. On the other hand, the case of identical laminates approaches to zero from the value of $\Delta=30$. The energy dissipated versus Δ for unequal adherend case is shown in Figure 6. It can be seen that the energy dissipated is greatly influenced by the value of Δ. The energy dissipated is relatively unchanged in the range of the low value, $\Delta<1$, but is sharply increased in the range of the high value, $\Delta>5$. Figure 7 is a plot of the energy dissipated versus Δ for various values of the loss coefficient of the viscoelastic adhesive. It can be seen from the figure that the energy dissipated is increased by increasing the loss coefficient throughout all values of Δ. However, the energy dissipated is not greatly influenced by the value of for the low value of the loss coefficient, $\eta=0.017$. As expected, the loss coefficient is a significant parameter to increase the energy dissipated in the viscoelastic adhesive lap joint. Figure 8 shows the energy dissipated versus for various values of N. The energy dissipated is tabulated in Tables I and II for various values of N and Δ.

It can be seen from tables that the optimum value of Δ is the largest one that is available. Since the upper limit of Δ is determined by the minimum value of $|E_{xx}^*|_u h_2$, the practical application of lap should be done cautiously to be able to sustain without failure in the structural joint.

Table Ⅰ. Dimensionless energy dissipated vs. N and Δ for $\eta=0.174532$

Δ \ N	0.1	0.3	0.5	0.7	0.9	1
0.1	0.34736	0.34734	0.34732	0.34731	0.34731	0.34731
0.3	0.34784	0.34766	0.34754	0.34747	0.34745	0.34476
1	0.35262	0.35085	0.34968	0.34906	0.34894	0.34905
5	0.42063	0.39580	0.38205	0.376649	0.37704	0.37611
10	0.52150	0.46521	0.43765	0.42900	0.43301	0.43842
50	1.08118	0.90903	0.84020	0.82794	0.84900	0.87756
100	1.52547	1.28161	1.17008	1.11722	1.17812	0.98355

Table Ⅱ. Dimensionless energy dissipated vs. N and $\Delta$for $\eta=0.349066$

Δ \ N	0.1	0.3	0.5	0.7	0.9	1
0.1	0.68416	0.68412	0.68409	0.68408	0.68408	0.63408
0.3	0.68512	0.68477	0.68452	0.68439	0.68435	0.68437
1	0.69461	0.69110	0.68878	0.68754	0.68730	0.68752
5	0.83123	0.78130	0.75359	0.74235	0.74313	0.74759
10	1.03428	0.92082	0.86529	0.84790	0.85600	0.86693
50	2.15304	1.81013	0.67402	1.65268	1.69927	1.73682
100	3.04210	2.54999	2.40483	2.35768	3.16125	3.34004

4. Conclusions

The energy dissipated in the viscoelastic adhesive lap joint under sinusoidal stresses has been studied with various factors. The elastic solution for the shear distribution in the elastic adhesive lap joint has been utilized to find the shear distribution in the viscoelastic adhesive lap joint by replacing the shear modulus of the elastic solution by complex shear modulus.

The total energy dissipated in the viscoelastic adhesive lap may be increased by controlling the relative rigidities of the adherends and adhesive of a lap joint. For a given adhesive, maximum damping can be obtained by increasing the ratio of rigidity of the adhesive to that of upper adherend. Some caution must be exercised in determining the upper limit without failure in the structural joint.

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