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Optimal Resource Allocation for Fleet Availability Management in Closed Queueing Network*

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ABSTRACT

Interactions of major activities participating in fleet operations are investigated in the framework of a closed queueing network system with finite aircrafts assigned to it. An implementable algorithm is developed, which is useful for computing the distributions needed to evaluate the effects of the interactions on the fleet operations. The availability management program is focused on seeking an optimal resource allocation to multiple repair-shops to maximize the fleet availability subject to the budget constraint.

I. Introduction

This paper is concerned with a military activity operating a fleet of N identical aircrafts in a flying-base and repair-depot combination. Fleet availability is an important part of operational

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effectiveness and can be measured as the average fraction of aircraft available for use at a random instant (1). The purpose of this paper is to provide i) an effective algorithm to evaluate the fleet availability in the framework of a closed queueing network system and ii) a management program for improving the fleet availability with a limited amount of budget available. Furthermore, the model presented in this paper is intended to cover the more practical situations of the fleet operations compared to the previous works in the literature (1,2).

II. Description of the fleet operations

The following is the simplified description of the fleet operations to be modeled and analyzed in this paper.

Missions:

Two types of missions are assigned to the flying-base, i.e., one is alert missions against emergency or abnormal situations occuring unexpectedly and the other is routine missions such as crew training, coordinated operational exercise, aircraft performance checking, reconnaissance, and so on. The flying-base must have a required number of aircrafts, N_A, stand ready for alert missions if possible. However, to reduce the undue wear and tear on the aircrafts and to increase the probability of having at least N_A aircrafts when necessary, there is an upper limit, N_R, on the number of aircrafts authorized to operate simultaneously for the routine missions. If the total number of available aircrafts are greater than $N_A + N_R$, the remaining aircrafts stand by.

Maintenance:

Ordinarily, scheduled maintenance policy has direct influence on fleet availability. However, our concern is limited to corrective maintenance closely supporting the fleet operations. In this paper, analysis begins with a given value of N resulting from a specific scheduled maintenance policy. Corrective maintenance is particularly important when a specific fleet is temporarily deployed at forward area of conflict. In this situation, the prevailing practice is that repair-depot supports only corrective maintenance for the fleet operations.

Failure and repair:

The failure time distribution of aircraft in flight is assumed to be exponential with a rate λ per one hour of flight. However, the average flying hours of aircraft per unit time (in the fleet operations scale time) are different by the type of mission, i.e., f_A for alert missions and f_R for routine missions. Thus the converted failure rate of aircrafts in terms of the fleet operations scale time, u_M and u'_M , are;

$u_M = \lambda f_A$ for alert missions

and

$u'_M = \lambda f_R$ for routine missions.

The failed aircrafts arrive at the repair-depot to recover their malfunctions. For complex systems such as aircrafts, the repair-depot consists of multiple repair-shops each of which has a specialized repair function. Each single-channel repair-shop services the failed aircrafts arrived independently on a first-come-first-service basis. The repair time distribution is also assumed to be exponential with a rate u_i (i= 1,2,..., M-1) in the fleet operations scale time. The failed aircrafts return to the flying-base in operable state after passing through some of the repair-shops depending on their failure characteristics.

Closed network:

The are (M-1) repar-shops and the M^{th} "station" is the flying-base. Let i^{th} "station" (i=1,2,..., M-1) be used to represent the repair-shop in the i^{th} position when all repair-shops are placed in arbitrary sequence. Then M stations participating in the fleet operations form a closed network with N "customers" (=aircrafts) circulating within at all times as shown in Figure 1 in the last part of this paper. The customers transit in the network stochastically by a specific probability matrix $\| p_{ij} \|$, where p_{ij} is the probability that a customer will proceed to j^{th} station after leaving i^{th} station. The transition times of customers between stations are assumed to be zero.

Notations:

N total number of aircrafts assigned to the fleet

N_A required number of aircrafts to stand ready for alert missions

N_R max. number of aircrafts authorized to operate simultaneously for routine missions

M total number of stations, i.e., M-1 repair-shops and a flying-base

u_i service rate of i^{th} station (=repair-shops) for $i=1,2,\ldots, M-1$

u_M, u'_M converted failure rate of individual aircraft assigned to a alert mission and a routine mission in the fleet operations scale time, respectively

p_{ij} probability that a customer will proceed to j^{th} station after leaving i^{th} station

n_i, m_i number of customers present at i^{th} station

$\underline{n} = (n_1, n_2, \ldots, n_M)$, $\underline{m} = (m_1, m_2, \ldots, m_M)$ state of the network

$Pr(\underline{n})$, $Pr(\underline{m})$ state distribution at equilibrium

$q(\underline{n}, \underline{m})$ transition rate from state $\underline{n}$ to $\underline{m}$

$\mathscr{E}$ state space

$G(N)$ normalizing constant such that $\sum_{\underline{n} \in \mathscr{E}} Pr(\underline{n})=1$

c_i amount of budget allocated to i^{th} repair-shop

$\underline{C} = (c_1, c_2, \ldots, c_{M-1})$ vector indicating budget allocation

III. Mathematical model

State distribution for closed network:

There are M stations and N customers in the closed network. Let the state of the network be denoted by $\underline{n} = (n_1, n_2, \ldots, n_M)$ whose components n_i are the numbers of customers present at the i^{th} station Then the state $\underline{n}$ is a Markov process with q(n,m) representing the transition rate from state $\underline{n}$ to state $\underline{m}$ (4,5,9). The state space defined as

$$\mathcal{E} = \{ \underline{n} = (n_1, n_2, \ldots, n_M) \mid \sum_{i=1}^{M} n_i = N \text{ and } n_i \geq 0 \quad \text{for all} \quad i \}$$

is finite and has an irreducible class since each customer once returns to M^{th} station and then transits to other stations again.

Remark: If $\mathcal{E}$ is an irreducible class, the equilibrium state distribution on $\mathcal{E}$ is a set of positive numbers $Pr(\underline{n})$ satisfying:

(1) $$Pr(\underline{n}) \sum_{\underline{m} \in \mathcal{E}} q(\underline{n}, \underline{m}) = \sum_{\underline{m} \in \mathcal{E}} q(\underline{m}, \underline{n}) \quad Pr(\underline{m}), \quad \underline{n} \in \mathcal{E}$$

and

(2) $$\sum_{\underline{n} \in \mathcal{E}} Pr(\underline{n}) = 1.$$

When $\mathcal{E}$ is finite, (1) and (2) have unique solution, necessarily positive (3,6).

In (1) the only values of $\underline{m}$ for which $q(\underline{n},\underline{m})$ is nonzero are those with

$$m_i = n_i - 1, \quad m_j = n_j + 1, \text{ and } m_k = n_k \text{ for all } k \neq i, j,$$

since the internal transitions of customers between stations are permitted only. Thus

$$q(\underline{n},\underline{m}) = q(\underline{n}, \underline{n}-e_i + e_j)$$

where $\underline{e_i}$ is the vector with all components zero except for 1 in the i^{th} component. The process with the above property is a kind of Markov population processes in (4). Furthermore,

$$q(\underline{n}, \underline{n}-\underline{e}_i+\underline{e}_j) = u_i\, \beta_i(n_i)\, p_{ij} \qquad , i,j = 1,2,\ldots,M$$

where

$$\beta_i(k) = \min\{1,k\} \qquad \text{for } i=1,2,\ldots,M-1 \text{ and } k=0,1,2,\ldots,N$$

and

$$\beta_M(k) = \begin{cases} k & , k \le N_A \\ N_A + \dfrac{u'_M}{u_M}(k-N_A) & , N_A < k \le N_A+N_R \\ N_A + \dfrac{u'_M}{u_M} N_R & , k \ge N_A + N_R \end{cases}$$

Lemma : The equilibrium state distribution over $\mathcal{E}$, the solution of (1) and (2), is of the form

(4) $$\Pr(n_1,n_2,\ldots,n_M) = \frac{1}{G(N)} \prod_{i=1}^{M} A_i(n_i)$$

where (G(N) is a normalizing constant defined so that (2) is satisfied and

(5)
$$A_i(k) = \begin{cases} 1 & , k=0 \\ \prod_{\ell=1}^{k} \dfrac{\alpha_i}{\beta_i(\ell)} & , k=1,2,\dots,N \end{cases}$$

where $\alpha_1, \alpha_2, \dots, \alpha_M > 0$ are multiplicative constants such that

(6)
$$\alpha_i \mu_i = \Sigma_j \alpha_j \mu_j P_{ji} \qquad , i = 1,2,\dots,M.$$

For the proof of the lemma, see Appendix.

Computing algorithm:

The following procedures to compute $Pr(\underline{n})$ as well as marginal distributions, $Pr\{n_i=k\}$, are patterned after Buzen (7). Assuming that the ratios α_i are determined from (6), G(N) is defined as:

(7)
$$G(N) = \sum_{\underline{n} \in \mathcal{E}} \left[\prod_{i=1}^{M} A_i(n_i) \right]$$

Now let us introduce an auxiliary function to compute G(N), i.e.,

(8)
$$g(s, [i]) = \sum_{\underline{n}_{[i]} \in \mathcal{E}_{[i],s}} \left[\prod_{k=[1]}^{[i]} A_k(n_k) \right]$$

where

$$\underline{n}_{[i]} = (n_{[1]}, n_{[2]}, \cdots\cdots\cdots, n_{[i]})$$

and

$$\mathscr{E}_{[i],s} = \{\underline{n}_{[i]} = (n_{[1]}, n_{[2]}, \cdots\cdots, n_{[i]}) \mid \sum_{k=[1]}^{[i]} n_k = s \text{ and } n_k \geq 0 \text{ for } k = [1], \cdots, [i]\}$$

for [i]=[1],[2],...,[M] and s = 0,1,2,...,N. Note that [i] denotes the original number of the station in the i^{th} position when all stations are placed in arbitrary sequence regardless of their original numbers. From (7), we can observe that

(9) $$g(s,[1]) = \sum_{y=1}^{s} \frac{\alpha_{[1]}}{\beta_{[1]}(y)} \quad \text{for } s=0,1,\ldots,N,$$

(10) $$g(0, [i]) = 1 \quad \text{for } [i] = [1],[2],\ldots,[M]$$

and

(11) $$g(s,[i]) = \sum_{\underline{n}_{[i]} \in \mathscr{E}_{[i],s}} \left[\prod_{k=[1]}^{[i]} A_k(n_k) \right]$$

$$= \sum_{y=0}^{s} A_{[i]}(y) \left[\sum_{\underline{n}_{[i-1]} \in \mathscr{E}_{[i-1],s-y}} \{ \prod_{k=[1]}^{[i-1]} A_k(n_k) \} \right]$$

$$= \sum_{y=0}^{s} A_{[i]}(y) \; g\{s-y, [i-1]\}, [i] = [1],[2],\cdots\cdots,[M] \text{ and } s = 0, 1, \ldots, N.$$

The iterative relationship in (11), together with the initial conditions given in (9) and (10), completely defines the algorithm to compute G(N). Note that g(N,[M]) = G(N) in (11). The marginal distribution Pr$\{n_i=y\}$is derived from (4) by letting [M] = i as follows (i=1,2,...M and y=0,1,2,...,N) :

$$(12)\quad \Pr\{n_i = y\} = \sum_{\substack{\underline{n} \in \mathcal{E} \\ \& n_i = y}} \Pr(n_1, n_2, \ldots\ldots, n_M)$$

$$= \sum_{\substack{\underline{n} \in \mathcal{E} \\ \& n_i = y}} \frac{1}{G(N)} A_i(y) \prod_{k=[1]}^{[M-1]} A_k(n_k)$$

$$= A_i(y)\, g\{N-y, [M-1]\} / G(N).$$

The expected number of aircrafts available at the flying-base, $E\{n_M\}$, and the fleet availability, $A = E\{n_M\}/N$, can be immediately obtained from (12) by letting $[M] = M$.

IV. Fleet availability management

This section describes a management program for improving the fleet availability. The program is focused on allocating a limited amount of budget, B, to M-1 repair-shops competing with each other for the use of the budget available. Let the current repair-depot supportability be denoted by $\underline{u}^o = (u_1^o, u_2^o, \ldots, u_{M-1}^o)$. When a specific amount of budget, c_i, is allocated to i^{th} repair-shop, the improved repair rate is

$$u_i(c_i) = u_i^o + w_i(c_i) \qquad , \; c_i \geq 0 \text{ and } i = 1, 2, \ldots, M-1$$

where $w_i(c_i)$ is assumed to be continuous and differentiable at $c_i \geq 0$. Then, the expected number of aircrafts present at the flying-base is a function of $\underline{C} = (c_1, c_2, \ldots, c_{M-1})$, i.e.,

$$E\{n_M\} = h\{u_1(c_1), u_2(c_2), \ldots, u_{M-1}(c_{M-1})\}$$

$$= w(c_1, c_2, \ldots, c_{M-1}).$$

Therefore, our optimization problem is :

$$\text{maximize } w(c_1, c_2, \ldots, c_{M-1})$$

subject to a linear inequality constraint

$$b\ (\underline{C}) = c_1 + c_2 + \ldots + c_{M+1} \le B.$$

In order to solve the problem rigorously, constrained nonlinear programming techniques such as the gradient projection method developed by Rosen [8] could be used. However, this procedure is very time consuming and it is cumbersome to compute the projection of the gradient vector on the constraint hyperplane.

Intuitively, $E\{n_M\}$ must not decrease as the repair-depot supportability improves by investing resources in any of the repair-shops, and achieves its maximum when all the repair rates of the repair-shops become ∞ . The surface of the objective function $w(\underline{C})$ is relatively regular. Therefore, our optimization problem can be effectively solved by a modified unconstrained optimization technique in which "hillclimbing steps" of certain magnitudes are taken repeatedly until the available budget is exhausted.

The method of steepest ascent with "binary apportionment" is used effectively in our optimization problem based on the ideas that i) the hillclimbing proceeds along a steepest ascent path evaluated at each starting point of hillclimbing step until the total amount of budget is exhausted, ii) initially it takes a small step on steep surfaces, but progressively bigger steps on relatively flat surfaces, and iii) each step exhausts twice the amount of budget apportioned to the previous step. The algorithm begins by choosing some point $\underline{C}^o$ within the feasible region and evaluating the gradient $\nabla W(\underline{C}^o) = \left(\frac{\partial w}{\partial c_1}, \frac{\partial w}{\partial c_2}, \ldots\ldots, \frac{\partial w}{\partial c_{M-1}}\right)_{\underline{C}=\underline{C}^o}$ A new point $\underline{C}^1$ is then obtained from

(13) $$\underline{C}^{k+1} = \underline{C}^k + \nabla W(\underline{C}^k) \quad 2^k \tau / \sum_{i=1}^{M-1} \frac{\partial W(\underline{C}^k)}{\partial C_i}, \quad k = 0, 1, 2, \cdots\cdots$$

where superscript k indicates each hillclimbing step number and is the initial step size, i.e., the amount of budget apportioned to the initial step (k=0). The gradient $\nabla w(\underline{C}^1)$ is then reevaluated at $\underline{C}^1$, and another point $\underline{C}^2$ is determined from (13), and so on. The procedure is continued until some step k is found where i) remaining budget is less than or equal to $2^k \tau$, ii) $\nabla w(\underline{C}^k)$ converges closely to $\nabla w(\underline{C}^{k-1})$, or iii) $\nabla w(\underline{C}^k)$ becomes sufficiently small. Note that in the cases of i) and ii), all remaining budget is apportioned to the step k. In the case of iii), it means that the excessive amount of budget over $b(\underline{C}^k)$ has negligible effect on the fleet availability. They major computational effort in this method lies in obtaining the gradient vector $\nabla w(\underline{C})$ whose components are the respective partial derivatives evaluated at $\underline{C}$. For the computation of partial derivatives $\frac{\partial}{\partial c_i} w(c_1, \ldots, c_i, \ldots, c_{M-1})$, see Appendix. In the numerical example given below, the results obtained by our method are sufficiently close to those by the gradient projection method.

Numerical example:

As in Figure 1, seven stations form a closed network with 20 aircrafts. The data are given as; N_A=4, N_R=12, u_M=1.0, and u'_M=3.0. The values of $\| p_{ij} \|$ and u_i are given in Table 1 and Table 2, respectively. To obtain the fleet availability, we follow the computing steps; i) the ratios α_i from (6), ii) G(N) and g {s,[M-1]} for (9) - (11) setting [M] = i for E {n_i} , and iii) E {n_i} after obtaining Pr {n_i = l } from (12). Table 2 shows the results of i) and iii) as well as u_i. Then the fleet availability is:

$$A = \frac{E\{n_M\}}{N} = \frac{E\{n_7\}}{20} = 0.6025.$$

Table 1

Transition probabilities of aircrafts between stations

$\| p_{ij} \|$	1	2	3	4	5	6	7
1	0	0.6	0.4	0	0	0	0
2	0	0	0	0.7	0.3	0	0
3	0	0	0	0.2	0.8	0	0
4	0	0	0	0	0.5	0	0.5
5	0	0	0	0	0	0.6	0.4
6	0	0	0	0	0	0	1
7	1	0	0	0	0	0	0

Table 2

Values of μ_i, α_i and $E\{n_i\}$

Station number	Repair - depot						Flying-base	Sum
	1	2	3	4	5	6	7	
u_i	50.000	20.400	25.600	28.000	25.000	30.800	1.000	
α_i	0.020	0.029	0.016	0.018	0.020	0.018	1.000	
$E\{n_i\}$	1.164	3.004	0.743	0.938	1.164	0.938	12.050	20.000

Now we are concerned with allocating a specific amount of budget, i.e., ₵450, to the six repair-shops for improving the fleet availability. The function $u_i(c_i)$ is assumed to be as

$$u_i(c_i) = u_i^o + a_i \{ (1+c_i)^{0.8} - 1 \} , \qquad i=1,2,\ldots,6$$

where the values of a_i $(i=1,2,\ldots,6)$ are 6.5, 3.0, 3.6, 5.0, 2.0, and 8.0, respectively. Table 3 shows the results obtained from the method of steepest ascent with binary apportionment. Note that the optimal allocation of the budget is presented in the last row of Table 3. The results for various input data are summarized in Figure 2 to represent graphically the relationship between the total amount of budget and the fleet availability achievable through optimal resources allocation.

Table 3

Optimal allocation of budget and fleet availability improved

$\tau = 2$

Hill-climbing step (k)	Amount of budget apportioned to the step k	Amount of budget apportioned at step k						Fleet availability (A)
		1	2	3	4	5	6	
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.6025
1	2.000	0.213	1.013	0.120	0.213	0.131	0.310	0.6232
2	4.000	0.555	1.523	0.327	0.539	0.366	0.690	0.6495
3	8.000	1.308	2.357	0.843	1.208	0.986	1.298	0.6859
4	16.000	2.785	3.898	2.038	2,436	2.554	2.293	0.7367
5	32.000	5.565	6.916	4.512	4.715	6.090	4.202	0.8009
6	64.000	11.045	13.032	9.382	9.311	13.050	8.179	0.8667
7	128.000	22.091	25.268	19.158	18.732	26.340	16.411	0.9185
8	196.000	33.838	37.972	29.740	28.880	40.257	25.313	0.9473
Sum	450.000	77.402	91.974	69.119	66.033	89.773	58.697	0.9473

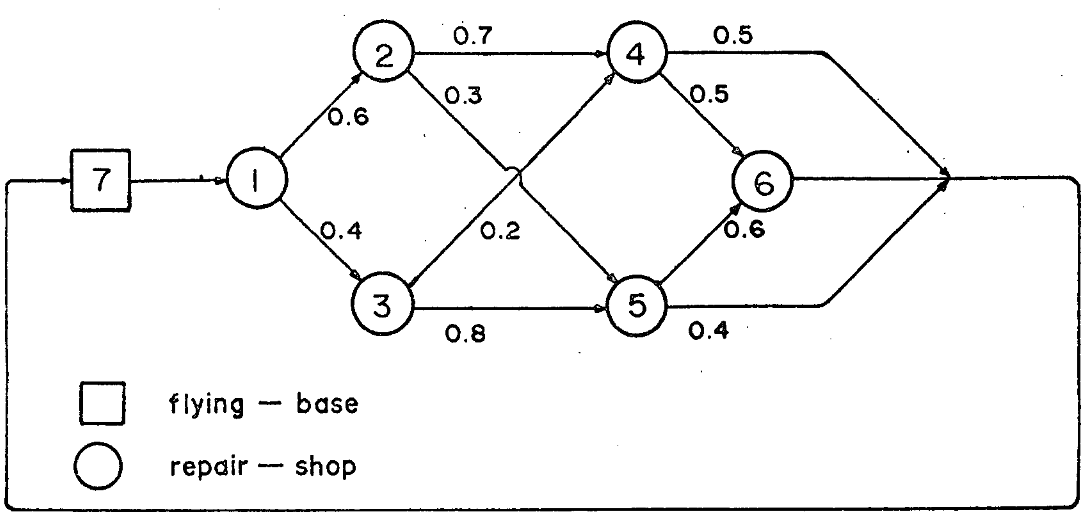

Figure 1. An example of closed network

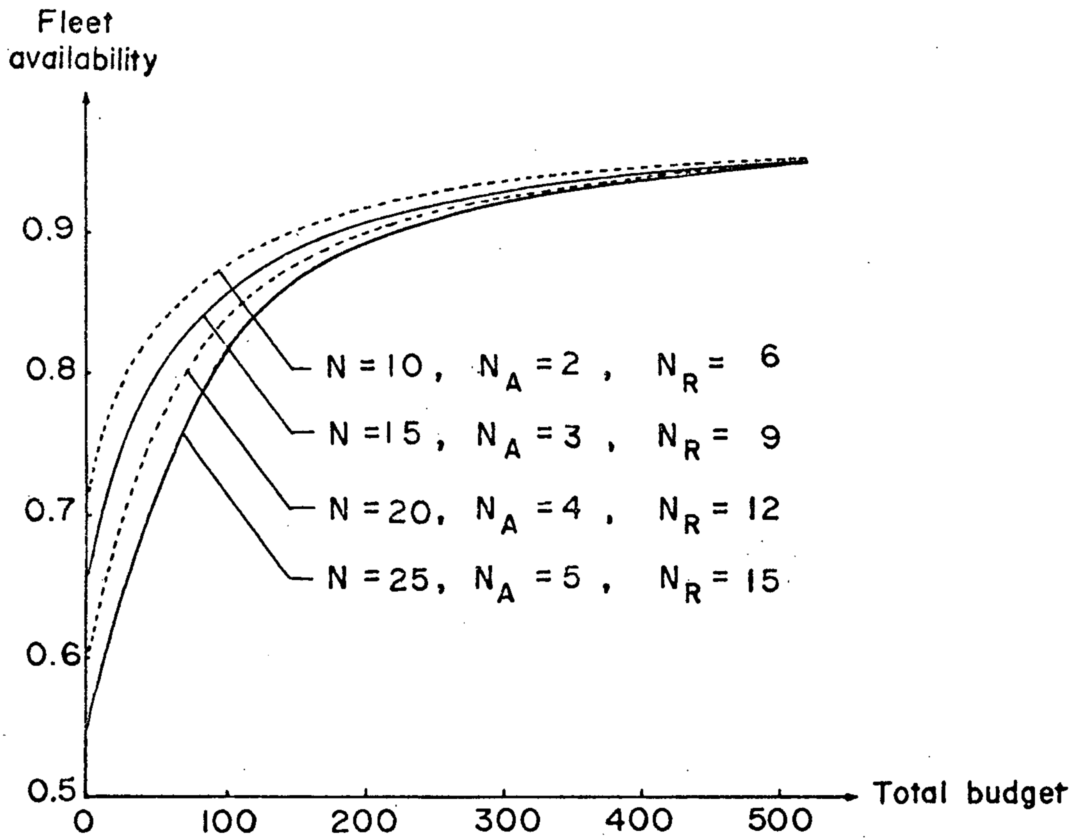

Figure 2. Relationship between total available budget and fleet availability achievable

V. Summary & conclusions

Interactions of major activities (in a flying-base with two typed missions and in repair-depot composed of multiple specialized repair-shops) participating in fleet operations are investigated in the framework of closed queueing network system with finite aircrafts assigned to it. An implementable algorithm is developed, which is useful for computing the distributions needed to evaluate the effect of the interactions on the fleet operations. The availability management program proposed in this paper is focused on seeking an optimal resource allocation to multiple repair-shops to maximize the fleet availability subject to the budget constraint. Furthermore, the methodology can be used as a powerful tool i) for the planners to design the optimal repair-depot supportability and ii) for the practioners to gain executive insights into the trade-offs between the available budget and the achievable fleet availability.

In this paper, air base/depot setting is used as a vehicle to convey the central theme. However, any piece of equipment or facility could repleace the aircraft. Application of the model and the solution methodology presented in this paper in many other industrial contexts is selfevident.

Appendix

Proof of Lemma:

The proof is patterned after Kelly [6] . Using (3), the balance equation (1) is written as

$$\text{(A 1)} \quad \Pr(\underline{n}) \Sigma_i \Sigma_j \mu_i \beta_i (n_i) p_{ij} = \Sigma_j \Sigma_i \mu_j \beta_j (n_j + 1) p_{ij} \Pr(\underline{n}+\underline{e}_j - \underline{e}_i).$$

Assume now that $\Pr(\underline{n})$ is the form stated in Lemma. Then from (4) and (5),

$$\frac{\Pr(\underline{n}+\underline{e}_j - \underline{e}_i)}{\Pr(\underline{n})} = \frac{\alpha_j \beta_i (n_i)}{\alpha_i \beta_j (n_j + 1)}$$

From the above result, (A 1) becomes

$$\text{(A 2)} \quad \Sigma_i \Sigma_j \mu_i \beta_i (n_i) p_{ij} = \Sigma_i \Sigma_j \alpha_j \mu_j p_{ji} \frac{\beta_i (n_i)}{\alpha_i}$$

For (A 2) to hold, the following equations must hold

$$\alpha_i \mu_i \Sigma_j p_{ij} = \Sigma_j \alpha_j \mu_j p_{ji} , \quad i = 1, 2, \cdots\cdots, M$$

and these are identical to (6) since $\sum_j p_{ij} = 1$ for all i. Note that (6), similar to (1), implies a balance equation in the case of a single customer while (1) implies the same in the case of multiple customers. Equations (6), a sufficient condition for (1), is an elementary result of Markov population processes as shown in [5,6] . Q.E.D.

Derivation of partial derivatives $\partial E[n_M]/\partial c_i$:

From (12), the expected number of aircrafts available is :

$$(A\ 3)\quad E\{n_M\} = \sum_{k=1}^{N} k \ \Pr\{n_M = k\}$$

$$= \sum_{k=1}^{N} \left[k \ A_M(k) \quad g\{N-k, [M-1]\} \frac{1}{G(N)} \right]$$

$$= \frac{\sum_{k=1}^{N} k \ A_M(k) \left[\sum_{y=0}^{N-k} A_{[M-1]}(y) \ g\{N-k-y, [M-2]\} \right]}{\sum_{k=0}^{N} A_M(k) \left[\sum_{y=0}^{N-k} A_{[M-1]}(y) \ g\{N-k-y, [M-2]\} \right]} \equiv \frac{A}{B}$$

In (6), we can see that the ratios $\alpha_i \mu_i$ are dependent only upon $\| p_{ij} \|$. This means that if μ_j is changed alone, all α_i are unchanged except for α_j corresponding to μ_j. This particular property of the closed network can be utilized for computing partial derivatives , $\partial E\{n_M\} / \partial \mu_i$, regardless of complex interdependencies between stations. Letting $[M] = M$ and $[M-1] = i$ $(i=1,2,\ldots,M-1)$, the term $g\{k, [M-2]\}$ in (A 3) is independent of μ_i. Thus

$$\frac{\partial E\{n_M\}}{\partial \mu_i} = \left\{ B \left[\sum_{k=1}^{N} k \ A_M(k) \left[\sum_{y=0}^{N-k} (-y) \lambda_i^{y} \mu_i^{-(y+1)} \ g\{N-k-y, [M-2]\} \right] \right] \right.$$

$$\left. - A \left[\sum_{k=0}^{N} A_M(k) \left[\sum_{y=0}^{N-k} (-y) \lambda_i^{y} \mu_i^{-(y+1)} \ g\{N-k-y, [M-2]\} \right] \right] \right\} / B^2$$

Note that from (5) letting $\alpha_i \mu_i = \lambda_i$,

$$A_i(k) = \alpha_i^k$$

$$= (\lambda_i / \mu_i)^k \quad , i = 1, 2, \cdots\cdots, M-1 \text{ and } k = 0, 1, \cdots\cdots, N.$$

Therefore, we obtain the partial derivatives, $\partial E\{n_M\}/\partial C_i$, as follows:

$$\frac{\partial E\{n_M\}}{\partial C_i} = \frac{\partial E\{n_M\}}{\partial \mu_i} \quad \frac{d\mu_i \; (c_i)}{d c_i} \quad , i = 1, 2, \cdots\cdots, M-1.$$

REFERENCES

1 V.V.S. Sarma, K. Ramchand, A.K. Rao, "Queueing models for estimating fleet availability", IEEE Trans. Reliability, vol. R-26, 1977 Oct., pp. 253-255.

2 M. Rose, "Determination of the optimal investment in end products and repair resources", Naval Res. Log. Quart., vol. 20, 1973 Mar., pp. 147-159.

3 D.G. Kendall, G.E.H. Reuter, "The calculation of the ergodic projection for Markov chains and processes with a countable infinity of states", Acta Mach., vol. 97. 1957, pp. 103-143.

4 J.F.C. Kingman, "Markov population processes", J. Appl. Prob., vol. 6, 1969, pp. 1-18.

5 M.S. Bartlett, "some evolutionary stochastic processes, J.R. Statist. Soc., B 11, 1949, pp. 211-229.

6 F.P. Kelly, "Networks of queues with customers of different type", J. Appl. Prob., vol. 12, 1975, pp. 542-553.

7 J.P. Buzen, "Computational algorithms for closed queueing networks with exponential servers," Communications of ACM, vol. 16, 1973, pp. 527-531.

8 J.B. Rosen, The gradient projection method for nonlinear programming. Part.I, linear constraints", Indust. Appl. Math., vol. 8, 1960, pp. 181-217

9 P. While, "Nonlinear migration processes," Proc. 36th Session of the International Statistical Institute, 1967.

10 K.S. Park and B.H. Ahn, "Fleet Availability in Closed Queueing Network with Inventory Stations," Journal of the Society of Logistics Engineers, vol. 14, no. 2, pp. 37-41 (1980)