<Research Paper>

Journal of the Korean Intistute of Industrial Engineers
Vol. 5 No. 1 June 1979

Development of a Stochastic Inventory System Model

Chang Sup Sung*

Abstract

The objective of this paper is to develop a stochastic inventory system model under the so-called continuous-review $\langle Q, r\rangle$ policy with a Poisson one-at-a-time demand process, iid customer inter-arrival times $\{Xi\}$, backorders allowed, and constant procurement lead time γ. The distributions of the so-called inventory position process $\{IP_{t-\gamma}\}$ and lead time demand process $\{D_{(t-\gamma,t)}\}$ are formulated in terms of cumulative demand by time t, $\{N_t\}$. Then, for the long-run expected average annual inventory cost expression, the "ensemble" average is estimated, where the cost variations for stock ordering, holding and backorders are considered stationary.

1. Introduction

Inventory systems are operated largely based on some operating policies concerning review systems and ordering rules. The so-called transactions-reporting (continuous) systems and periodic-review systems are commonly used for inventory system review.

Some examples of operating policies (doctrines) are the so-called $\langle Q, r\rangle$, $\langle R, r\rangle$, $\langle R, T\rangle$, $\langle nQ, r, T\rangle$ and $\langle R, r, T\rangle$ identified in [3], where Q is an order quantity, R and r are certain control limits on inventory level, and T is a review period. Among these five doctrines, the $\langle Q, r\rangle$ and $\langle R, r\rangle$ doctrines are associated with transactions reporting, and the other three are associated with periodic review.

In this study, the $\langle Q, r\rangle$ doctrine will be mainly dealt with. Under the assumptions that demands generate a Poisson process, the demands occurring when the system is out of stock are backordered, units are demanded one at a time, procurement lead time γ is constant, and the inventory system consisting of one stocking point with a single source for resupply. The doctrine is a continuous review model, under which an order is placed for the quantity Q to raise the inventory position (IP_t) at time t to the level $r+Q$ as soon as a demand drops the inventory position below the level $r+1$, where the inventory position is defined to be the amount on hand (OH_t) plus on order (OO_t) minus backorders (BO_t). Thus, the inventory position successively falls from $r+Q$ to $r+1$ during each procurement cycle, and instantaneously rises again up to $r+Q$.

One approach to the $\langle Q, r\rangle$ inventory system analysis is to optimize the parameters r and Q,

*Korea Institute of Science & Technology

in which the objective function for optimization is a suitable expected inventory cost, depending on Q and r as well as on a set of relevant unit costs. For this work, a model for the estimation of the "ensemble" average inventory cost (a long-run expected average annual cost) will be determined. In other words, the final objective is to determine the optimal value of Q and r which minimize the corresponding objective cost function, $\mathcal{L}(Q,r)$ involving the long-run expected terms of on-hand inventory $E[OH]$, backorders $E[BO]$, and average number of backorders incurred per year $E[\Delta BO]$. These long-run expected terms are to be computed in section 3 under the assumption of stationary cost variations. Let the family of random cumulative demands by time $t\{Nt;\ t\in T\}$ with the index set T generate a Poisson process, which denotes the cumulative demand by time $t\geq 0$. The assumptions concerning $\{N_t\}$ once made, one may in fer the relevant properties of the so-called inventory position process $\{IP_t\}$. Hence, the relevant properties of the so-called net inventory process $\{NIS_t\}$. from which the cost process is derived whose average we seek, where the net inventory is defined to be the amount on hand $\{OH_t\}$ minus backorders $\{BO_t\}$

It is shown in [2] and [3] that under the $\langle Q,r\rangle$ model the limiting distribution of $\{IP_t;\ t\geq 0\}$ is uniform on the set $\{r+1,\ r+2\cdots,r+Q\}$, when the inter-arrival times $\{X_i;\ i=1,2,\cdots\}$ between successive demands are independent and identically distributed (iid) random variables possessing negative exponential distribution and units are demanded one at a time.

Under the slightly modified replenishment policy $\langle nQ,r\rangle$, Simon [7] has also achieved the same result for the stationary demand process in which the demand quantity is random, lead times are arbitrarily distributed, and backorders are allowed.

Sivazian [8] has generalized the work done in [2] and [3].

It is shown in [5] that in the case of random demand quantity the limiting distribution is not uniform under the $\langle Q,r\rangle$ policy.

2. Model Development

Under the previous assumptions, $\{N_t\ ;\ t\geq 0\}$ is a discrete-valued continuous-parameter stochastic process (a renewal counting process) with sample paths increasing in unit steps. $\{N_t\}$ will be analyzed to describe probabilistically the inventory position $\{IP_t\ ;\ t\geq 0\}$.

It can be shown that $\{IP_t\}$ totally depends upon the demand process $\{N_t\}$. For example, if an inventory system is started with $IP_0=r+i$ $(i=1,2,\cdots,Q)$ at time $t=0$, than $IP_{t-\tau}=r+j$ $(j=1,2,\cdots,Q)$ at time $t-\tau>0$ can be reached after the $(i-j)^+$ or $\{i+(m-1)Q+(Q-j)\ ;\ m=1,2,\cdots\}$ demand materialization by time $t-\tau$, where τ is a constant procurement lead time, m denotes the total number of order placements by time $t-\tau$ and $(i-j)\equiv\max(0,\ i-j)$. In other words, $P\{IP_{t-\tau}=x\}$ is a function of $P\{N_{t-\tau}=y\}$, as $\{IP_{t-\tau}\}$ is determined by $\{N_{t-\tau}\}$. Moreover, under the assuption of the Poisson deman process and given $IP_0=r+i$, $\{IP_{t-\tau}\}$ and $\{D_{(t-\tau,t)}\}$ are mutually independent of each other, where $D_{(t-\tau,t)}$ is a lead time demand and so $D_{(t-\tau,t)}N_t-N_{t-\tau}$. That is,

$$P\{IP_{t-\tau}=r+j,\ D_{(t-\tau,t)}=k\}=P\{IP_{t-\tau}=r+j\}\cdot P\{D_{(t-\tau,t)}=k\}, \tag{1}$$

for $j=1,2,\cdots,Q$ and $k=0,1,2,\cdots$

Therefore, the analysis of $\{NIS_t\ ;\ t\geq 0\}$ becomes straightforward, from which the cost process can be immediately derived whose average we seek.

When demands arrive at time points $t_1, t_2, \cdots$ $(0<t_1<t_2<\cdots)$, the successive inter-arrival times $\{X_i\,;\, i>1\}$ are defined as $X_1=t_1$, $X_2=t_2-t_1, \cdots$, $X_n=t_n-t_{n-1}, \cdots$, where $\{X_i\}$ are assumed to be iid random variables with a common distribution F, $(F(0)=0)$. Thus, $\{IP_t\}$ also is a discrete-valued continuous-parameter stochastic process. Denote by S_n the renewal epoch of the n^{th} demand (the time of the n^{th} renewal or the waiting time to the n^{th} demand), so that $\{S_n\,;\, n=0,1,2,\cdots\}$ are the partial sums of the renewal process $\{X_i\}$, that is,

$$S_n=\sum_{i=1}^{n} X_i, \quad (S_0\equiv 0)$$

Then,

$$N_t=S_{up}\{n\,;\, S_n\leq t\} \tag{2}$$

so that for $t>0$ and $n=1,2,\cdots$,

$N_t\geq n$ iff $S_n\leq t$, and $N_t=n$ iff $S_n\leq t$ and $S_{n+1}>t$.

Therefore, $P\{N_t=n\}=F_n(t)-F_{n+1}(t)$, where $F_n(t)\equiv P\{S_n\leq t\}$ which denotes the n-fold convolution of F with itself, so that

$$F_{n+1}(t)=F_n{*}F(t)=\int_0^t F_n(t-x)dF(x)=\int_0^t F(t-x)dF_n(x), \text{ for } n=1,2,\cdots,$$

$$(F_0(t)\equiv 1 \text{ for } t\geq 0)$$

we shall first find the marginal distribution functions of $\{IP_t\}$, $\{D(t-\tau, t)\}$ and the residual waiting time (or excess waiting time) at epoch $t-\tau$ $\{Z_t-\tau\}$ for $t\in T$ which is the time from $t-\tau$ until the first demand subsequent to $t-\tau$, that is,

$$Z_{t-\tau}=S_{N_{t-\tau}+1}-(t-\tau),$$

$$\text{so that } S_{N_{t-\tau}}\leq t-\tau<S_{N_{t-\tau}+1} \tag{3}$$

For finding such distributions, the following so-called renewal-type equation plays an important role; For known functions $F(t)$ and $H(t)$ and an unknown function $g(t)$,

If $g(t)=H(t)+\int_0^t g(t-x)\; dF(x)$ $(t\geq 0)$, then

$$g(t)=H(t)+\int_0^t H(t-x)dm(x), \tag{4}$$

where $m(x)$ denotes the mean value function such that $m(t)=E\{N_t\}=\sum_{n=1}^{\infty} F_n(t)$. The proof of the equation appears in [1], [4] and [6]. The mean value function $m(t)$ can also be stated in the form of a renewal-type equation (see [4], [6]) ;

$$m(t)=F(t)+\int_0^t m(t-x)dF(x)$$

$$=F(t)+\int_0^t F(t-x)dm(x). \tag{5}$$

Suppose that we consider the sequence of events consisting of the times at which an order in the amount of Q is placed and received in the constant lead time τ. Defining Y_k to be the time elapsed between the $(k-1)$st and k^{th} orders, the sequence of random variables $\{Y_k\,;\, k=1,2,\cdots\}$ forms a modified renewal process in which the distribution functions are given by

$$P\{Y_1\leq y_1\}=P\{S_i\leq y_1\}\equiv F_i(y_1)=P\{N_{y_1}\geq i\}, \tag{6}$$

where i is the initial stock over the reorder point r, and likewise,

$$P\{Y_k\leq y_k\}=P\{S_Q\leq y_k\}=P\{N_{y_k}\geq Q\}=F_Q(y_k), \tag{7}$$

for $k=2,3,\cdots$,

since $\{Y_k\leq y_k\}\Longleftrightarrow\{(S_{i+(k-1)Q}-S_{i+(k-2)Q})\leq y_k\}$

$$\Longleftrightarrow \{S_Q \leq y_k\}.$$

Thus, another renewal process $\{W_m ; m=0,1,2,\cdots\}$ follows such that

$$W_m=\sum_{k=1}^{m} Y_k = S_{i+(m-1)Q} \text{ for } m=1,2,\cdots, \tag{8}$$
$$(W_0=Y_0=0),$$

where '$m=0$', means that no order is placed yet.

Let $(t-\tau-\theta)$ and m be, respectively, particular values of the time T and the serial M of the last order (say m^{th} order) placed no later than $t-\tau$. If we assume that $IP_{t-\tau}=r+j$ $(j=1,2,\cdots,Q)$ at time $t-\tau$, then we see that $(Q-j)$ demands are further needed in the time interval $(t-\tau-\theta, t-\tau]$ for $\theta \geq 0$, since $IP_{t-\tau-\theta}$ is $r+Q$ immediately after the m^{th} order is placed at time $t-\tau-\theta$.

Theorem 1

For the $\langle Q, r\rangle$ inventory system with backorders allowed, constant lead time $\tau \geq 0$, iid customer inter-arrival times with finite mean, units demanded one at a time, and with $IP_0=r+i$ $(i=1,2,\cdots Q)$,

$$P\{IP_{t-\tau}=r+j\}=P\{N_{t-\tau}=(i-j)\}^{+}+\sum_{m=1}^{\infty}\int_{\theta=0}^{\theta=t-\tau} P\{N_\theta=Q-j\}dP\{W_m \leq t-\tau-\theta\},$$

for $j=1,2,\cdots,Q$, where $P\{N_{t-\tau}=(i-j)\}^{+}=0$ if $i<j$.

PROOF: Denote by $\phi_m\{T\leq t-\tau-\theta\}$ the probability that $M=m$ and $T\leq t-\tau-\theta$ so that $\phi_m\{T\leq t-\tau-\theta\}=P\{W_m\leq t-\tau-\theta\}$.

Since the inventory position $IP_{t-\tau}=r+j$ $(j=1,2,\cdots,Q)$ can be reached after the demand materialization $D_{(0,t-\tau]}$ such that

$$D_{(0,t-\tau]}=N_{t-\tau}=\begin{cases}(i-j)^{+}, & \text{for } m=0\\ N_{t-\tau-\theta}+(N_{t-\tau}-N_{t-\tau-\theta})=\{i+(m-1)Q\}+(Q-j), & \text{for } m=1,2,\cdots\end{cases}$$

$$P\{IP_{t-\tau}=r+j\}=\sum_{m=0}^{\infty}\int_{\theta=0}^{\theta=t-\tau} P\{IP_{t-\tau}=r+j\,|\,M=m,\ T=t-\tau-\theta\}\cdot d\phi_m\{T\leq t-\tau-\theta\}$$
$$=P\{N_{t-\tau}=(i-j)\}^{+}+\sum_{m=1}^{\infty}\int_{\theta=0}^{\theta=t-\tau} P\{N_{t-\tau}-N_{t-\tau-\theta}=Q-j\,|\,M=m,\quad T=t-\tau-\theta\}\cdot dP\{W_m\leq t-\tau-\theta\}$$
$$=P\{N_{t-\tau}=(i-j)\}^{+}+\sum_{m=1}^{\infty}\int_{\theta=0}^{\theta=t-\tau} P\{N_\theta=Q-j\}dP\{W_m\leq t-\tau-\theta\} \quad \text{Q.E.D.}$$

Using the renewal equation for $m(t)$, the distribution function of $Z_{t-\tau}$ can be formulated as follows;

Theorem 2

For the inventory model of Theorem 1,

$$P\{Z_{t-\tau}\leq z\}=F(t-\tau+z)-\int_0^{t-\tau}[1-F(t-\tau+z-\xi)]dm(\xi)$$
$$=\int_{t-\tau}^{t-\tau+z}[1-F(t-\tau+z-\xi)]dm(\xi), \text{ for } z>0.$$

PROOF : Since

$$P\{Z_{t-\tau}\leq z\}=P\{0<S_{N_{t-\tau}+1}-(t-\tau)\leq z\}$$

$$=P\{t-\tau<S_{N_{t-\tau}+1}\leq t-\tau+z\}$$

$$=P\{t-\tau\leq X_1\leq t-\tau+z\}+\sum_{n=1}^{\infty}\int_0^{t-\tau}P\{t-\tau<S_{n+1}\leq t-\tau+z|S_n=\xi\}dP\{S_n=\xi\},$$

its proof is straightforward by use of Eq. (5).

The random variable $Z_{t-\tau}$ may have a different distribution from those of Xi's. Therefore, the distribution of $D_{(t-\tau,t)}$ can be determined in the next theorem by partitioning in accordance with the time $t-\tau+z$ at which the first demand occurs after time $t-\tau$ and the time interval $(t-\tau+z,\ t]$ during which $k-1$ demands occur.

Theorem 3

For the inventory model of Theorem 1,

$$P\{D_{(t-\tau,t)}=k\}=\begin{cases}\int_0^{\tau}P\{N_{\tau-z}=k-1\}dP\{Z_{t-\tau}\leq z\}, & \text{for } k=1,2,\cdots\\ P\{Z_{t-\tau}>\tau\}, & \text{for } k=0\end{cases}$$

PROOF : For $k=0$,

$$P\{D_{(t-\tau,t)}=0\}=P\{N_t-N_{t-\tau}=0\}$$
$$=P\{Z_{t-\tau}>\tau\},$$

For $k\geq 1$,

$$P\{D_{(t-\tau,t)}=k\}=P\{N_t-N_{t-\tau}=k\}$$
$$=\int_0^{\infty}P\{N_t-N_{t-\tau}=k|Z_{t-\tau}=z\}dP\{Z_{t-\tau}\leq z\}$$
$$=\int_0^{\tau}P\{N_{\tau-z}=k-1\}dP\{Z_{t-\tau}\leq z\}$$
$$=\int_0^{\tau}[F_{k-1}(\tau-z)-F_k(\tau-z)]dP\{Z_{t-\tau}\leq z\}.\ \text{Q.E.D.}$$

Recall that by definition,

$$\begin{aligned}NIS_t&=IP_{t-\tau}-D_{(t-\tau,t)} \text{ for } t\geq\tau\geq 0\\&=OH_t-BO_t \text{ and hence}\\NIS_t&=OH_t, \text{ is } NIS_t\geq 0\\&=BO_t, \text{ Otherwise.}\end{aligned}\qquad (9)$$

With the results of Theorems 1 & 3, we are about to find the distribution of $\{NIS_t\}$ which can also be used to determine $E[OH_t]$, $E[BO_t]$, $E[\Delta BO_t]$, and the probability $P_{os}(t)$ that the system is cut of stock, at time t.

Let's define that under the assumption of Poisson demand process,

$$\begin{aligned}P\{IP_{t-\tau}=r+j,\ D_{(t-\tau,t)}=j-s\}^+&=P\{IP_{t-\tau}=r+j\}\cdot P\{D_{(t-\tau,t)}=j-s\}^+\\&=P\{IP_{t-\tau}=r+j\}\cdot P\{D_{(t-\tau,t)}=j-s\}, \text{ if } j\geq s\\&=0, \qquad \text{otherwise}\end{aligned}\qquad (10)$$

Referring to Eqs. (9) and (10),

$$E[OH_t]=\sum_{x=0}^{\infty}x\cdot P\{OH_t=x\}$$

$$=\sum_{x=0}^{r+Q}x\cdot P\{OH_t=x\}$$

$$=\sum_{x=0}^{r+Q} x\cdot P\{NIS_t=x\}$$

$$=\sum_{x=0}^{r+Q} x\cdot\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\cdot P\{D_{(t-\tau,t)}=r+j-x\}^{+}$$

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\cdot\sum_{x=0}^{r+Q} x\cdot P\{D_{(t-\tau,t)}=r+j-x\}^{+}$$

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\cdot\sum_{x=0}^{r+j} x\cdot P\{D_{(t-\tau,t)}=r+j-x\}$$

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\sum_{n=0}^{r+j}(r+j-n)P\{D_{(t-\tau,t)}=n\}$$

replacing $r+j-x$ by n

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\Big[\left[(r+j)\sum_{n=0}^{r+j}P\{D_{(t-\tau,t)}=n\}-\sum_{n=0}^{r+j}P\{D_{(t-\tau,t)}=n\}\right], \tag{12}$$

$$E[BO_t]=\sum_{x=1}^{\infty} x\cdot P\{BO_t=x\}=\sum_{x=1}^{\infty} x\cdot P\{NIS_t=-x\}$$

$$=\sum_{x=1}^{\infty} x\cdot\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}P\{D_{(t-\tau,t)}=r+j+x\}$$

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\cdot\sum_{x=1}^{\infty} x\cdot P\{D_{(t-\tau,t)}=r+j+x\}$$

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\sum_{n=r+j+1}^{\infty}(n-r-j)P\{D_{(t-\tau,t)}=n\}$$

replacing $r+j+x$ by n

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\cdot\left[E[D_{(t-\tau,t)}]-(r+j)+\sum_{n=0}^{r+j}(r+j-n)P\{D_{(t-\tau,t)}=n\}\right], \tag{13}$$

$$P_{os}(t)=\sum_{x=1}^{\infty} P\{BO_t=x\}$$

$$=\sum_{x=1}^{\infty}\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\cdot P\{D_{(t-\tau,t)}=r+j+x\}$$

$$=\sum_{j=1}^{Q} P\{IP_{t-\tau}=r+j\}\left[1-\sum_{n=0}^{r+j} P\{D_{(t-\tau,t)}=n\}\right], \tag{14}$$

and

$$E[\Delta BO_t]=\lambda P_{os}(t), \tag{15}$$

Where λ denotes the mean rate of demand such that

$$\lambda=\lim_{\Delta t\to 0}\frac{E[D_{(t,t+\Delta t)}]}{\Delta t}$$

Now, we need to discuss the relevant cost variations associated with the $\langle Q, r\rangle$ inventory system.

The procurement cost is assumed to be composed of a fixed ordering cost \$A, which is approximately proportional to the number of orders placed, and of a variable cost \$C per unit associated with transportation costs, part of the receiving costs, and part of the inspection costs, Moreover, the unit cost \$C will be assumed independent of the quantity ordered.

For the inventory carrying (holding) costs, the instantaneous rate at which inventory carrying costs are incurred is assumed to be proportional to the investment in inventory at that point in time. The constant of the proportionality or just the carrying charge, denoted by "I", will be used to estimate the carrying costs. "I" has the dimension of "cost per unit time per monetary unit invested in inventory" (for example, dollars per year per dollar of inventory investment). Therefore, the instantaneous rate of incurring the carrying charges in the units of dollars per year is IC x, where x is the on-hand inventory level.

For the stockout costs, there are two cases such as backorder costs and lost-sales costs incurred by having demands occur when the system is out of stock. When units are demanded one at a time, the cost of each unit backordered can be estimated by $B(t)=B+\hat{B}t$, where B denotes the fixed cost per unit backordered and $\hat{B}$ represents the varying cost in proportion to the length of time. Denoting "units times years" by "unit years", B has the dimension of dollars per unit year of shortage in the case of which we want the cost for a year to come out in dollars. The lost sale costs won't be considered in this work.

The above expectations and relevant costs are put together to form the following expected inventory cost expression $\mathcal{L}(Q, r)_t$;

$$\mathcal{L}(Q, r)_t=\frac{\lambda}{Q}\cdot A+IC\cdot E[OH_t]+B\cdot E[\Delta BO_t]+\hat{B}\cdot E[BO_t], \tag{16}$$

where $\frac{\lambda}{Q}$ represents the number of orders placed per year which is obtained from the mean rate of demands per year λ and each order quantity Q.

2. Long-Run Expected Average Annual Cost Formulation

Taking limit of Eq. (16), as t tends to infinity, will lead to the formulation of the long-run expected average annual cost function, where its minimization is the criterion to determine the optimum Q and r.

We can define the following in the sense of "ensemble" average;

$$E[OH]=\lim_{t\to\infty}\frac{\int_0^t E[OH_t]dt}{t}=\lim_{t\to\infty} E[OH_t].$$

Then, $\mathcal{L}(Q, r)$, which denotes the limit of $\mathcal{L}(Q, r)_t$, is achieved from taking limits of $E[OH_t]$, $E[BO_t]$ and $E[\Delta BO_t]$ as to tends to infinity.

As pointed out in [8], it can be analytically proved by Laplace transform approach that

$$\lim_{t\to\infty}P\{IP_{t-\tau}=r+j\}=\frac{1}{Q}(j=1, 2, \cdots, R).$$

In order to determine the limit distribution of $D_{(t-\tau,t)}$, we need to introduce the following so-called Key Renewal Theorem (for its proof see [9] and [10]);

If the inter-arrival time X has finite mean μ and the distribution F is not arithmetic, and $H(t)$ is any function satisfying the conditions

a) $H(t)\geq 0$ for all $t\geq 0$

b) $\int_0^\infty H(t)dt<\infty$,

c) $H(t)$ is nonincreasing, then it is true that

$$\lim_{t\to\infty}\int_0^t H(t-x)dm(x)=\frac{1}{\mu}\int_0^\infty H(t)dt.$$

with this result and Theorem 2, the limit distribution will result in

$$\lim_{t\to\infty}P\{D_{(t-\tau,t)}=k\}=\begin{cases}\dfrac{\int_0^\tau F_{k-1}(y)dy-2\int_0^\tau F_k(y)dy+\int_0^\tau F_{k+1}(y)dy}{\mu}\text{ for } k=1, 2, \cdots,\\ 1-\dfrac{1}{\mu}\int_0^\tau[1-F(x)]dx, \text{ for } k=0\end{cases}$$

This result can be directly used for finding that

$\lim_{t\to\infty} E[D_{(t-\tau,t)}]=\frac{\tau}{\mu}$ for every $\tau>0$, which is necessary for computing $\lim_{t\to\infty} E[BO_t]$.

Thus, the above limiting estimations will end up with the following long-run expected average annual inventory cost $\pounds(Q,r)$;

$$\pounds(Q,r)=\frac{\lambda}{Q}\cdot A+IC\cdot E[OH]+B\cdot E[\Delta BO]+\hat{B}\cdot E[BO], \tag{17}$$

4. Conclusion

This study shows that the limit distributions of $D_{(t-\tau,t)}$, $IP_{t-\tau}$ and NIS_t are achieved by using the renewal theorems and the Laplace transform approach, so that the expected terms of the long-run expected average annual inventory cost expressions are analytically and explicitly described, and so it is more realistic.

This work can be extended to some continuous-review inventory systems which have demand processes such that the stochastic processes $\{IP_{t-\tau}\}$ and $\{D_{(t-\tau,t)}\}$ may be asymptotically independent under the assumption that demands occur one at a time. The non-stationary cost variations can also be considered without any significant difficulty.

The optimization problem of E. (17) can be solved by dynamic programming approach for the optima of Q and r.

REFERENCES

(1) Feller, William, "An Introduction to Probability Theory and its Applications," 2nd edition, Vol. II, John Wiley and Sons, New York, 1971.

(2) Galliher, H.P., Morse, P.M. and Simond, M., "Dynamics of Two Classes of Continuous-Review Inventory Systems," Operations Research, Vol. 7(1958), pp. 362-384.

(3) Hadley, G. and Whitin, T.M., "Analysis of Inventory Systems," Prentice-Hall, Englewood Cliffs, New Jersey, 1963.

(4) Prabhu, N.U., Stochastic Processes, Macmillan Company, New York, 1965.

(5) Richards, F.R., "Comments on the Distribution of Inventory Position in a Continuous-Review (s,S) Inventory System," Operations Research, Vol. 23, No. 2(1975), pp. 366-371.

(6) Ross, S.M., "Applied Probability Models with Optimization Applications," Holden-Day, San Francisco, California, 1970.

(7) Simon, R.M., "The Uniform Distribution of Inventory Position for Continuous Review (s,Q) Policies," The Rand Corporation P-3938, Santa Monica, California, 1968.

(8) Sivazlian, B.D., "A Continuous-Review (s,S) Inventory System with Arbitrary Interarrival Distribution between Unit Demand," Operations Research, Vol. 22(1974), pp. 65-71.

(9) Smith, W.L., "Renewal Theory and its Ramifications," Journal of the Royal Statistical Society, Series B, Vol. 20(1958), pp. 243-302.

(10) Takacs, L., "On a Probability Problem in the Theory of Counters," Annals of Mathematical Statistics, Vol. 29(1958), pp. 1257-1263.