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# Geodetic and Geophysical Analyses of Gravity Data in Korea

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**Abstract** : Geodetic and geophysical quantities related to gravity data are analyzed using three-dimensional sin x/x method for the southern part of the Korean peninsula and adjacent Japan Sea. The thickness of isostatic crust is found as 26 km. The average isostatic gravity anomaly in this area is appeared to be +24.8 mgal, of which result indicates that the surface features are under-compensation or the thickness of the crust is thinner than normal. It is noteworthy that the general trend of the deflections of the vertical in direction is nearly perpendicular to the geological structure having a direction of NNE-SSW in the southern part of Korea.

## Introduction

The Korean Peninsula, the present study area, is situated at a transition area between an island arc and a continent. Therefore a study of the structure of underlying crust is interesting from geophysical viewpoints. However, systematic studies for this area have not been done so far.

The main object of the present study is to analyze a crustal structure using gravity data and to make a stepping point for future studies.

Observed gravity values at the earth's surface provide important informations concerning the shape of the earth and the state of the interior. If the earth were in equilibrium, its figure would be close to an ellipsoid of revolution and the gravity field would be regular. There are, however, deviations from irregularities in the shape of the geoid and gravity anomalies. From gravity interpretations, we can derive the interesting quantities in geophysics and geodesy such as crustal thickness and deviations from an ellipsoid.

There are several methods in gravity interpretations. The starting point of gravity interpretations was the geodetic observation that mass deficiency under the mountain range of the Andes was reported by the French geodetic expedition in the year of 1740. They found the observed deflections of the vertical was much smaller than the value computed theoretically from the known topography of the Andes.

Similar results were found near the Himalayas in the 19th century and it was known to be a general phenomenon. For both the cases of Andes and Himalayas, the underlying mass deficiencies needed to explain the observed deflections of the vertical were approximately equal to the surface loads represented by the mountain ranges. Until 1855, however, a reasonable explanation was not given for the phenomenon.

In that year Airy postulated to explain the phenomenon that the earth had a floating crust of variable thickness supported on a denser substratum. According to the Airy's hypothesis, the floating body displaces its own mass in accordance with the Archimedes' principle, and there should be a downward projection of the root of the crustal material beneath mountain ranges into the underlying denser substratum.

On the other hand, Pratt formulated a different hypothesis that all crustal columns of unit cross section had equal mass above some level at depth, and changes in surface elevation were compensated by changes in mean crustal density down to the depth where equal mass was obtained.

Years later, Dutton introduced the word "isostasy" to describe the condition of compensation and the term has since been used everywhere.

Since that time, investigations into the hypothesis of isostasy began to be made with measurements of gravity and the deflections of the vertical. For many years the emphasis was on the testing of one or other hypothesis, Airy's or Pratt's and the attempt to find the best parameters such as density and crustal thickness, in each case.

To test the hypothesis of isostasy, isostatic gravity anomalies were calculated by a suitable adjustment of parameters.

In order to calculate the effect of masses compensating the topography on gravity at a station, Hayford and Bowie(1912) divided the region surrounding the station into compartments having the form of a cylindrical segment. The attraction of mass in this form, for a point on the axis of the cylinder, can readily be obtained. Then the density of each segment having the mean elevation $\bar{H}$ will be

$$\rho=\rho_n\cdot\frac{H}{H+\bar{H}} \quad\cdots\cdots(1)$$

where H is the constant depth of compensation and $\rho_n$ is the normal density of the crust. The anomalous density is then

$$\Delta\rho=\rho-\rho_n=-\frac{h\rho_n}{H} \quad\cdots\cdots(2)$$

With this value of $\Delta\rho$ the gravity effects of each compartments are obtained and the sum of the effects is added to the observed anomaly to give the Pratt-Hayford isostatic anomaly. From many observations in the United States, they found that a value H of 113.7 km gave the smallest sum of squares of anomalies, and suggested that this was the best value of the depth of compensation on Pratt's hypothesis. Although Pratt-Hayford system is useful in reduction of isostatic anomaly, we have no evidence that some discontinuity exists in the depth of 113.7km.

Heiskanen (1924) first showed that the same of compartments used by Hayford and Bowie(1912) could be used to test the Airy's hypothesis. For a segment of mean elevation $\bar{H}$ the downward projection of the root R is obtained from the relation

$$R=\bar{H}\cdot\frac{\rho_c}{\rho_s-\rho_c} \quad\cdots\cdots(3)$$

where $\rho_c$ and $\rho_s$ are the densities of the crust and subterranean mantle respectively. Then the Airy-Heiskanen isostatic anomaly can be obtained from the similar process to Hayford's.

It is well known that the depth of compensation H based on the Pratt-Hayford isostasy becomes twice as large as the thickness of the crust based on the Airy-Heiskanen one to give same gravity anomaly, in which the densities of the crust and subterranean mantle are assumed to be 2.67g/cm$^3$ and 3.27g/cm$^3$ respectively. And also the difference between these two sets of isostatic anomalies is small because two assumptions are nearly equivalent as far as the gravity effect of isostatic compensation is concerned (Heiskanen and Vening Meinesz, 1958).

Vening Meinesz (1941) assumed that a load placed on a floating crust would cause it to bend as an elastic plate, and that the compensation of the load would not be concentrated beneath it, but would be spread laterally. As in the Airy-Heiskanen system, the compensation is provided by the density contrast between the crust and subterranean mantle. He adopted a theoretical form for the downward deflection of the crust produced by a concentrated load, but left the distance at which the deflection became zero as a parameter to be determind. He prepared tables which allowed isostatic anomalies to be calculated for different crustal thickness and radii of regionality.

In the reduction of isostatic anomaly, both Heiskanen's and Hayford's method demand tedious labor to estimate the mean elevation of the segments.

Tsuboi (1938, 1940) developed more convenient method in interpreting several gravity problems such as the crustal structure and the shape of the geoid. He assumed harmonic variations in the gravity and elevations having constant interval on the plane surface, and that a variation in the gravity was caused by the anomalous mass distributions at the base of the crust. All the geophysical and geodetic quantities can be obtained in a form of Fourier syntheses in his method. Tsuboi's method was modified by Tomoda and Aki(1955). The modified one called sin x/x method is used in the present study. Tsuboi's method and modified one are given in the next section.

As the results of prevailed regional studies based on above isostatic system it could be recognized that most of the earth's major features were approximately in isostatic equilibrium because isostatic gravity anomalies showed relatively small values compared with the corresponding Bouguer or free-air anomalies. However, because of the ambiguity of gravity interpretations, it was not proved which was the more reasonable system of isostasy, and that no system could uniquely decide parameters such as the crustal thickness and the densities of the crust and subterranean mantle.

Tomoda (1960) developed an alternative method to remove ambiguities derived from the above isostatic systems, and the crustal thickness was determined by a slope of power spectrum of gravity anomalies without assumptions of densities and the hypothesis of isostasy. Although his method is a reasonable one in estimating the crustal thickness, the slope of the power spectrum is not always clear, and it is not easy to determine the crustal thickness.

On the other hand, the study of the crust laid on new situation by seismological studies originated from the discovery of Moho discontinuity. Heiskanen and Vening Meinesz(1958) showed that normal thickness of the crust could be determined from the relation between isostatic anomalies and elevations as summarized in below.

Bouguer reduction corresponds, in a way, to an isostatic reduction with infinite thickness and that the free-air reduction corresponds to an isostatic reduction with zero thickness of the crust. As the Bouguer anomalies are negative and free-air ones positive in mountains, it can be presumed that there must be some particular thickness for which isostatic anomalies become zero on the average. The particular thickness was adopted as a normal thickness of the crust They determined normal thickness of the crust in the regions of Europe and Africa, and the results were compared with the seismological ones. Some discrepancy was pronounced between the gravimetric results and seismological ones, when the depth of the Moho discontinuity was taken to be same as the normal thickness of the crust. They suggested that the discrepancy derived from the effect that only one discontinuity in the crust was assumed in the gravity-oriented structure, and that the discrepancy would become small, allowing for an adjustment of density contrast between the crust and subterranean mantle. In conclusion, they argued that many seismological evidences proved the hypothesis of isostasy to be true, and that the dependence of the thickness of the crust upon the elevations was practically valid.

Bott (1971) argued also that the oceans and continents are in broad isostatic equilibrium occurs principally according to variations in the crustal thickness predicted by the Airy's hypothesis but lateral variations in density within the crust or subterranean mantle must be considered locally as similar forms of the Pratt's hypothesis. He concluded no single universal hypothesis of isostasy could explain all the earth's major surface features, and that the crustal structure beneath any given feature needed to be investigated by combined gravity and seismic studies rather than to be assumed.

Above standpoints of Heiskanen and Bott were taken in many other studies (Worzel and Shurbet, 1955; Woolard, 1959; Kanamori, 1963; Talwani et al.. 1965).

Worzel and Shurbet (1955) made a standard crustal section from the careful study on the crustal structures determined by explosion seismic and gravity data. He adopted the density contrast 0.43g/cm (crust=2.84) for the standard crustal section of depth 33 km.

Woolard (1959) showed that the gravity-oriented structure could be in accordance with the seismic one, allowing for an effective adjustment of the density contrast between the crust and subterranean mantle.

According to the arguments described above, an approximate crustal structure would be derived from gravity data under the hypothesis of isostasy, if we choose reasonable density contrast.

## Method of analyses

Tsuboi's method, sin x/x method used in the present study and their applications are present in earlier papers (Tsuboi, 1938, 1940; Tomoda and Aki, 1955; Shimazu, 1961). Details of the numerical explanations of both Tsuboi's and sin x/x method are well summarized in Shimazu (1961). The descriptions given in this section follow his paper.

Tsuboi's method assumes that the surface of the earth is a plane and that any variation in gravity is caused by the mass distributions at the base of the crust or by undulations of the boundary between the crust and subterranean mantle.

Let the topographic heighds $H(x, y)$ and Bouguer gravity anomaly $\varDelta g\ (x, y)$ on a plane surface be expressed by the double Fouier series as

$$H(x,y)=\sum_m\sum_n H_{mn}\begin{matrix}\cos\\ \sin\end{matrix} mx\begin{matrix}\cos\\ \sin\end{matrix} ny \quad \cdots\cdots(4)$$

$$\varDelta g(x,y)=\sum_m\sum_n B_{mn}\begin{matrix}\cos\\ \sin\end{matrix} mx\begin{matrix}\cos\\ \sin\end{matrix} ny \quad \cdots\cdots(5)$$

The gravity potential U is the solution of Laplace equation

$$\nabla^2 U=O \quad \cdots\cdots(6)$$

subject to the boundary condition

$$-\left(\frac{\partial u}{\partial z}\right)_{z=d}=\varDelta g(x,y) \quad \cdots\cdots(7)$$

where the z-axis is taken to be positive upward in the vertical direction, z=0 at the base of the crust, and d is the thickness of the crust. The solution of equation (6) in the cartesian coordinate system has the form

$$U=\sum_m\sum_n A_{mn}\begin{matrix}\cos\\ \sin\end{matrix} mx\begin{matrix}\cos\\ \sin\end{matrix} ny\ e^{-\sqrt{m^2+n^2}\,z} \quad \cdots\cdots(8)$$

for the region z>o. Thus condition (7) indicates

$$U=\sum_m\sum_n \frac{B_{mn}}{\sqrt{m^2+n^2}}\begin{matrix}\cos\\ \sin\end{matrix} mx\begin{matrix}\cos\\ \sin\end{matrix} ny\ e^{-\sqrt{m^2+n^2}(z-d)} \quad (9)$$

For the practical applications it is assumed that the space mass distribution at the base of the crust caused by the undulations of the crust-mantle boundary is suitably expressed by a surface density distribution $\varDelta M(x, y)$. Then the following equation is obtained from Gauss' theorem.

$$-\left(\frac{\partial u}{\partial z}\right)_{z=o}=2\pi G\varDelta M(x,y) \quad \cdots\cdots(10)$$

where G is the universal contant of gravitation.

Equation (9) and (10) lead to the following for the subterraean mass distribution:

$$\varDelta M(x,y)=\frac{1}{2\pi G}\sum_m\sum_n B_{mn}\begin{matrix}\cos\\ \sin\end{matrix} mx\begin{matrix}\cos\\ \sin\end{matrix} ny\ e^{\sqrt{m^2+n^2}d} \quad \cdots\cdots(11)$$

The relief of the base of the crust $h(x, y)$ may be expressed by

$$h(x,y)=\frac{\varDelta M(x,y)}{\rho_s-\rho_c} \quad \cdots\cdots(12)$$

where $\rho_c$ and $\rho_s$ are densities of the crust and subterranean mantle respectively. Then $h(x, y)$ is positive downward.

On the other hand, the hypothesis of Airy's isostasy implies that surface elevations are exa-

-ctly compensated by the mass distributions $\Delta M'(x, y)$ at the bottom of the crust. These relationship may be expressed by

$$\Delta M'(x,y) = -\rho_c H(x,y) = -\rho_c \sum_m \sum_n H_{mn} \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} mx \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} ny \cdots\cdots (13)$$

The attraction at the earth due to the mass expressed in (13) is given by

$$\Delta g'(x,y) = -2\pi G \rho_c \sum_m \sum_n H_{mn} \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} mx \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} ny \; e^{-\sqrt{m^2+n^2}d} \cdots\cdots\cdots\cdots (14)$$

Therefore, the isosatic gravity anomaly defined by

$$\Delta g_{iso}(x,y) = \Delta g(x,y) - \Delta g'(x,y) \cdots\cdots\cdots\cdots (15)$$

Then, the crustal crustal thickness "d" which is unknown in equation (9) is determined under the following condition.

$$\sum (\Delta g_{iso})^2 \longrightarrow \text{minimum} \cdots\cdots\cdots\cdots (16)$$

It may be noted that the value "d" is the thickness of a isostatic crust having an average elevation H within the area. The depth of compensation "D" assumed in the Airy Heiskanen isostasy is given by the following.

$$d - D = \frac{\rho_c}{\rho_s - \rho_c} \bar{H} \cdots\cdots\cdots\cdots (17)$$

The anomaly in the vertical gradient of gravity $\Delta\left(\frac{\partial g}{\partial z}\right)$, the total deflections of the vertical $P(x, y)$ and the heights of the geoid $\rho(x, y)$ relative to an arbitary point $(x_o, y_o)$ can be shown as

$$\Delta\left(\frac{\partial g}{\partial z}\right)_{z=d} = \sum_m \sum_n \sqrt{m^2+n^2}\, B_{mn} \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} mx \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} ny \cdots\cdots\cdots\cdots (18)$$

$$\xi(x,y) = -\frac{1}{g} \sum_m \sum_n \frac{m}{\sqrt{m^2+n^2}} (\Delta g_{iso})_{mn} \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} mx \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} ny$$

$$\eta(x,y) = -\frac{1}{g} \sum_m \sum_n \frac{n}{\sqrt{m^2+n^2}} (\Delta g_{iso})_{mn} \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} mx \begin{smallmatrix}\sin\\ \cos\end{smallmatrix} ny$$

$$P(x,y) = \sqrt{\xi^2+\eta^2} \cdots\cdots\cdots (19)$$

$$\rho(x,y) = \frac{1}{g} \sum_m \sum_n \frac{(\Delta g_{iso})_{mn}}{\sqrt{m^2+n^2}} \left( \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} mx \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} ny - \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} mx_o \begin{smallmatrix}\cos\\ \sin\end{smallmatrix} ny_o \right) \cdots\cdots\cdots\cdots (20)$$

where $(\Delta g_{iso})_{mn}$ is the Fourier components of $\Delta g_{iso}$ and $\xi, \eta$ are the deflections of the vertical along eastward and northward directions respectively.

Though the Tsuboi's method is very useful in solving many gravity problems, it requires tedious and painstaking efforts to carry out the Fourier analyses and syntheses, if we wish to include the higher harmonics.

Tomoda and Aki (1955) developed a modified one called sin x/x method using Dirac delta function. All the quantities appeared above can be derived directly without computing the Fourier coefficients. The modified equations are shown as below:

Subterranean mass distribution

$$\Delta M(a,b) = \frac{1}{2\pi G} \sum_i \sum_j B_{ij}\, \phi^{(1)}_{a-i,b-j} \cdots\cdots (21)$$

Isostatic gravity anomaly

$$\Delta g'(a,b) = -2\pi G \rho_c \sum_i \sum_j H_{ij}\, \phi^{(2)}_{a-i,b-j} \cdots\cdots (22)$$

Anomaly in the vertical gradient of gravity

$$\Delta\left(\frac{\partial g}{\partial z}\right) = -\frac{\pi}{l} \sum_i \sum_j B_{ij}\, \phi^{(3)}_{a-i,b-j} \cdots\cdots\cdots (23)$$

Deflections of the vertical

$$\begin{cases} x\text{-direction};\ \xi(a,b) = -\frac{1}{g} \sum_i \sum_j (\Delta g_{iso})_{ij}\, \phi^{(4)}_{a-i,b-j,x} \\ y\text{-direction};\ \eta(a,b) = -\frac{1}{g} \sum_i \sum_j (\Delta g_{iso})_{ij}\, \phi^{(4)}_{a-i,b-j,y} \end{cases} \cdots\cdots\cdots (24)$$

Relative undulations of the isostatic geoid

$$\rho(a,b) = \frac{1}{g} \sum_i \sum_j (\Delta g_{iso})_{ij}\, \phi^{(5)}_{a-,b-j} \cdots\cdots\cdots\cdots (25)$$

with

$$\phi^{(1)}_{a,b} = \int_0^1 \int_0^1 \cos ma\pi \cos nb\pi\, e^{c\pi\sqrt{m^2+n^2}}\, dm dn \cdots (26)$$

$$\phi^{(2)}_{a,b} = \int_0^1 \int_0^1 \cos ma\pi \cos nb\pi\, e^{-c\pi\sqrt{m^2+n^2}}\, dm dn \cdots (27)$$

$$\phi^{(3)}_{a,b} = \int_0^1 \int_0^1 \sqrt{m^2+n^2} \cos ma\pi \cos nb\pi\, dm dn \quad (28)$$

$$\phi_{a,b,x}^{(4)}=\int_0^1\int_0^1 \frac{m}{\sqrt{m^2+n^2}} \sin ma\pi \cos nb\pi \, dmdn$$

$$\phi_{a,b,y}^{(4)}=\int_0^1\int_0^1 \frac{n}{\sqrt{m^2+n^2}} \cos ma\pi \sin nb\pi \, dmdn \quad \cdots\cdots (29)$$

$$\phi_{a,b}^{(5)}=\int_0^1\int_0^1 \frac{1}{\sqrt{m^2+n^2}} \cos ma\pi \cos nb\pi \, dmdn \quad (30)$$

where a and b are any integer defined by $x=a\pi$, $y=b\pi$. The ratio of the thickness of the crust "d" to the grid interval "l" is given by c=d/l along the $x$ and $y$ directions.

Several properties of the integrals $\phi_{a,b}^{(k)}$ may be noted asfollows:

Symmetry

$$\phi_{a,b}^{(1),(2),(3)}=\phi_{b,a}^{(1),(2),(3)}=\phi_{-ab.}^{(1),(2),(3)}=\phi_{a,-b}^{(1),(2),(3)}=\phi_{-a,-b}^{(1),(2),(3)}$$

$$\phi_{a,b,y}^{(4)} = \phi_{b,a,y}^{(4)} , \quad \phi_{b,a,y}^{(4)} = \phi_{a,b,x}^{(4)}$$

$$\phi_{a,-b,x}^{(4)} = \phi_{a,b,x}^{(4)} , \quad \phi_{a,b,y}^{(4)} = -\phi_{a,-b,y}^{(4)}$$

$$\phi_{-v,b,x}^{(4)} = -\phi_{a,b,x}^{(4)}, \quad \phi_{-a,b,y}^{(4)} = \phi_{a,b,y}^{(4)} \quad \cdots\cdots (31)$$

Convergency

$$\sum_{a=-\infty}^{+\infty}\sum_{b=-\infty}^{+\infty}\phi_{a,b}^{(1),(2)}=1, \quad \sum_{a=-\infty}^{+\infty}\sum_{b=-\infty}^{+\infty}\phi_{a,b}^{(3),(4)}=0 \cdots (32)$$

The numerical intergrations of $\phi_{a,b}^{(4)}$ and $\phi_{a,b}^{(5)}$ is not easy. But $\phi_{a,b}^{(4)}$ can easily be shown that

$$\phi_{a,b,x}^{(4)}=a\pi\ \phi_{a,b}^{(3)}, \quad \phi_{a,b,y}^{(4)}=-b\pi\phi_{a,b}^{(3)} \quad \cdots\cdots (33)$$

And the undulations of the isostatic geoid can be evaluated directly without computing $\phi_{a,b}^{(5)}$. In Fig. 1, let the deflections $(\xi, \eta)$ be known for centing grid point P. Then $\xi$ and $\eta$ can be defined by

$$\xi=\frac{1}{2l}(h_2-h_1+h_4-h_3)$$

$$\eta=\frac{1}{2l}(h_1-h_3+h_2-h_4) \quad \cdots\cdots (34)$$

where $h_1, h_2, h_3, h_4$ are the values of geoidal heights for the grid points in the sub-grid line with the interval $l$. Thus we obtain

$$h_2-h_3=l(\xi+\eta)$$

$$h_4-h_1=l(\xi-\eta) \quad \cdots\cdots (35)$$

Relative to a arbitarily selected grid point, a variation in geoidal heights can be determined step by step.

Several method are known for deducing the undulations of the geoid and the deflections of

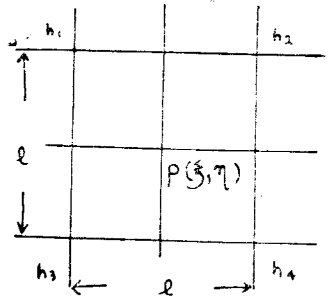


FIG. 1 Schematic diagram to illustrate the relation between relative geoidal heights and the deflections of the vertical.
$\xi$; eastward direction $\eta$; northward direction

the vertical from gravity anomalies. In the well-known method of Stokes, itis necessary that the gravity anomaly field over the whole surface of the earth is known with the sufficient density to permit a spherical harmonic expension of its distribution.

On the other hand, in the Tsuboi's method, only a rectangular portion of the earth's surface assumed a plane surface is considered. Nevertheless, Tsuboi (1940) indicated that the results obtained by his method in the United States of America were concordant with ones obtained by the other method.

On the other hand Talwani et al. (1959) evaluated gravity effct for the two-dimensional structure determined by the sin x/x method using their poligon method, and the calculated gravity values were compared with the observed ones. They indicated that the sin x/x method is very useful in making an approximate estimate of the crustal thickness.

Bott (1960) developed an alternative method to determine the boundary between two layers of known density contrast. The boundary is determined from iterative calculations for the evaluated gravity by the given structure to produce the same value as the obsered one. Bott's method is similar to Talwani's one in the process of calculation but simpler than Talwani's because Bott used simple blocks rather than poligons.

Tanner (1967) represented also that the result by the sin x/x method agreed with one by the Bott's method within on or two percents.

In conclusion, it may be stressed that all the quantities related to gravity can be evaluated very easily by the present method than by any others particulary in three-dimensional case.

This is the reason why the sin x/x method is selected in the present study.

## Data

First of all, gravity and elevation data at respective grid points having constant interval on the plane surface should be known.

Grid system is shown in Fig. 2 where the rectangular area is covered by a grid of 8×7 points with an interval of 60 km. Directions of $x$ and $y$ are parallel and orthogonal to lattitude respectively. Further, positive values of a and b indicate the eastward and northward directions respectively.

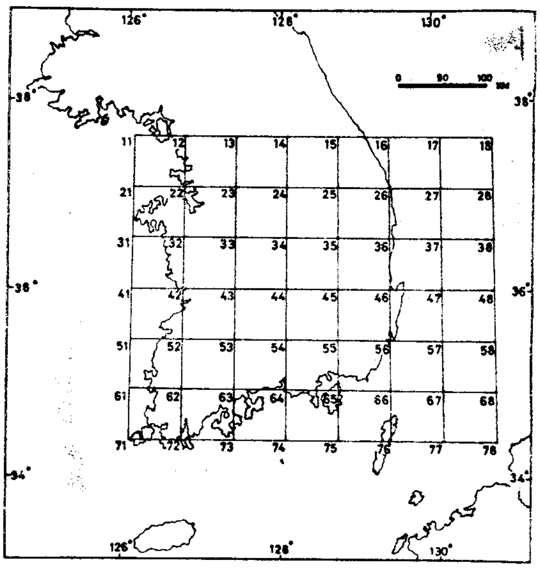


Fig. 2 Grid system used in the calculations.
To avoid the edge effects three grid points are added to the outside of the area. Consequently, total numbers of grid points become 14x13.

It is pointed out that this rectangular assumed to be a plane has not an exact orthogonal network on the actual surface of the earth but is distorted due to curvature. However, the effect of this distortion can be neglected in the present study.

To avoid the edge effects three grid points are added to the outside of the area, and consequently the total numbers of grid points become 14×13.

The elevation data for the ocean region are corrected by converting sea water of density 1.03g/cm$^3$ into the granitic layer of density 2.67 g/cm$^3$ and the revised values are in actual calculations.

Gravity data at the Korean Peninsula and adjacent Japan Sea are obtained from Bouguer Anomaly Map of Korea (1970) and free-air anomaly map of Japan and vicinity by Segawa and Tomoda (1976).

For a long time the density contrast between the crust and subterranean mantle was traditionally assumed to be 0.6g/cm$^3$ (crust=2.67).

However, the density contrast 0.43g/cm$^3$ (crust=2.84) in standard crustal section of Worzel and Shurbet (1955) is accepted as a reasonable one in recent studies (Kanamori, 1963; Talwani et al., 1959, 1965). Thus the value is used in the present study.

## Results and Discussions

Bouguer anomalies, topographic heights and isostatic gravity anomalies calculated for the crustal thickness of d=20. 24, 26, 28, 33 and 40 km at each grid points are shown in Tab. I

The thickness of the isostatic crust is found as 26 km from equation (16) and the depth of compensation based on Airy Heiskanen isostatic system becomes 26.3 km from equation (17).

Convergency of $\phi_{a,b}^{(k)}$ for c=26km/60km is shown in Table 2. From convergency test it is inferred that the present results have a reason-

**Tab. 1.** Bouguer anomalies, topographic heights and isostatic gravity anomalies for the thickness of the crust d=20, 24, 26, 28, 33 and 40 km. The thickness of isostatic crust is found as 26 km under the condition, $\Sigma(g_{iso})^2_{i,j} \rightarrow \min$.

| Grid No. | Elevation (m) | Bouguer Anomaly (mgal) | Isostatic gravity anomaly (mgal) d=20km | 24km | 26km | 28km | 33km | 40km |
|---|---|---|---|---|---|---|---|---|
| 11 | −10 | 20.5 | 23.9 | 24.4 | 24.7 | 24.9 | 25.3 | 25.8 |
| 12 | 50 | 22.5 | 34.9 | 35.4 | 35.6 | 35.7 | 36.0 | 36.2 |
| 13 | 330 | 0 | 29.3 | 28.0 | 25.4 | 26.8 | 25.4 | 23.6 |
| 14 | 310 | −30. | 5.9 | 4.5 | 3.6 | 2.8 | 0.6 | −2.6 |
| 15 | 920 | −12. | 38.4 | 31.0 | 27.6 | 24.5 | 17.5 | 9.5 |
| 16 | −800 | 75. | 34.0 | 35.6 | 36.3 | 37.0 | 38.3 | 39.8 |
| 17 | −1600 | 105. | 11.9 | 15.4 | 17.1 | 18.7 | 22.6 | 27.4 |
| 18 | −1500 | 123. | 22.2 | 24.0 | 24.9 | 25.8 | 28.2 | 31.6 |
| 21 | −30 | 22. | 21.8 | 22.2 | 22.4 | 22.6 | 23.0 | 23.6 |
| 22 | 0 | 15. | 20.7 | 21.4 | 21.7 | 22.0 | 22.6 | 23.4 |
| 23 | 200 | −7. | 13.2 | 12.7 | 12.4 | 12.2 | 11.6 | 10.8 |
| 24 | 290 | −10. | 21.1 | 20.0 | 19.4 | 18.8 | 17.2 | 15.0 |
| 25 | 780 | −15. | 36.4 | 30.5 | 27.9 | 25.3 | 19.5 | 12.6 |
| 26 | −20 | 32.5 | 22.4 | 21.2 | 20.5 | 20.0 | 18.7 | 17.1 |
| 27 | −1300 | 94.5 | 19.0 | 21.7 | 22.9 | 24.1 | 26.9 | 30.5 |
| 28 | −1700 | 112. | 6.4 | 9.4 | 11.0 | 12.5 | 16.2 | 21.2 |
| 31 | −20 | 21.5 | 21.6 | 21.9 | 22.0 | 22.1 | 22.4 | 22.8 |
| 32 | 70 | 15. | 23.2 | 23.3 | 23.4 | 23.4 | 23.6 | 23.8 |
| 33 | 120 | −2. | 15.9 | 15.9 | 15.9 | 15.9 | 15.6 | 15.2 |
| 34 | 240 | 0 | 24.9 | 24.1 | 23.7 | 23.3 | 22.2 | 20.7 |
| 35 | 110 | 12. | 29.7 | 29.3 | 29.0 | 28.7 | 27.7 | 26.0 |
| 36 | 10 | 32.5 | 25.9 | 24.8 | 24.3 | 23.8 | 22.7 | 21.4 |
| 37 | −800 | 65. | 7.3 | 8.1 | 8.6 | 9.2 | 10.6 | 12.8 |
| 38 | −2000 | 132.5 | 23.9 | 29.1 | 31.5 | 33.9 | 39.4 | 46.3 |
| 41 | −35 | 22.5 | 22.0 | 22.3 | 22.5 | 22.7 | 23.1 | 23.6 |
| 42 | 0 | 20. | 29.3 | 30.0 | 30.2 | 30.4 | 30.8 | 31.0 |
| 43 | 510 | −15. | 22.2 | 19.2 | 17.9 | 16.6 | 13.8 | 10.5 |
| 44 | 320 | −18. | 13.4 | 11.8 | 11.0 | 10.3 | 8.4 | 5.8 |
| 45 | 550 | 8. | 42.4 | 38.5 | 36.7 | 35.1 | 31.4 | 27.0 |
| 46 | 0 | 33. | 27.7 | 27.0 | 26.6 | 26.3 | 25.5 | 24.6 |
| 47 | −1300 | 109.5 | 48.2 | 52.3 | 54.1 | 55.8 | 59.6 | 64.1 |
| 48 | −1250 | 101. | 28.5 | 31.4 | 32.7 | 34.1 | 37.3 | 41.4 |
| 51 | −15 | 21. | 22.8 | 23.0 | 23.1 | 23.2 | 23.4 | 23.5 |
| 52 | 320 | −3. | 19.8 | 17.9 | 17.1 | 16.3 | 14.6 | 12.7 |
| 53 | 280 | −25. | 2.6 | 1.2 | 0.6 | 0.0 | −1.7 | −3.8 |
| 54 | 220 | 10. | 30.6 | 29.7 | 29.2 | 28.8 | 27.7 | 26.2 |
| 55 | 60 | 25. | 34.2 | 34.0 | 33.9 | 33.7 | 33.2 | 32.4 |
| 56 | −30 | 37. | 32.3 | 31.9 | 31.8 | 31.7 | 31.4 | 31.0 |
| 57 | −200 | 58.5 | 39.2 | 38.9 | 38.8 | 38.8 | 38.8 | 39.0 |
| 58 | −150 | 50.5 | 30.5 | 29.6 | 29.2 | 28.9 | 28.2 | 27.7 |
| 61 | 0 | 20. | 20.4 | 20.5 | 20.5 | 20.5 | 20.8 | 20.8 |
| 62 | 30 | 7. | 14.2 | 14.3 | 14.3 | 14.3 | 14.3 | 14.0 |
| 63 | 240 | 12. | 29.2 | 27.8 | 27.2 | 26.6 | 25.3 | 23.8 |
| 64 | −7 | 15. | 19.0 | 19.3 | 19.5 | 19.6 | 19.7 | 19.8 |
| 65 | −25 | 25. | 22.9 | 23.1 | 23.2 | 23.3 | 23.5 | 23.8 |
| 66 | −200 | 50.5 | 41.4 | 42.1 | 42.3 | 42.6 | 43.2 | 43.8 |

Table 1 (contiued)

| Grid No. | Elevation (m) | Bouguer Anomaly (mgal) | Isostatie d=20km | 24km | gravity 26km | 28km | anomaly (mgal) 33 | 40km |
|---|---|---|---|---|---|---|---|---|
| 67 | −150 | 50. 5 | 41. 1 | 41. 2 | 41. 3 | 41. 4 | 41. 5 | 41. 7 |
| 68 | −130 | 34. | 25. 8 | 25. 8 | 25. 9 | 25. 9 | 25. 8 | 25. 8 |
| 71 | −33 | 22. 5 | 20. 3 | 20. 4 | 20. 4 | 20. 5 | 20. 5 | 20. 7 |
| 72 | −12 | 21. | 20. 3 | 20. 4 | 20. 5 | 20. 5 | 20. 7 | 20. 9 |
| 73 | −15 | −21. | 21. 7 | 21. 8 | 21. 8 | 21. 9 | 22. 0 | 22. 0 |
| 74 | −50 | 33. 5 | 31. 2 | 31. 4 | 31. 5 | 31. 6 | 31. 9 | 32. 2 |
| 75 | −100 | 39. | 34. 0 | 34. 3 | 34. 4 | 34. 6 | 34. 9 | 35. 4 |
| 76 | −100 | 44. | 37. 8 | 38. 1 | 38. 2 | 38. 4 | 38. 7 | 39. 2 |
| 77 | −100 | 37. | 31. 6 | 32. 0 | 32. 2 | 32. 3 | 32. 7 | 33. 3 |
| 78 | −50 | 3. 5 | 2. 1 | 2. 4 | 2. 5 | 2. 7 | 2. 9 | 3. 2 |
| sum of square | | | 40282 | 39905 | 30267 | 39954 | 40456 | 41773 |

Tab. 2 Convergency of $\psi_{a,b}^{(k)}$

| k \ a, b | −1∼+1 | −3∼+3 | −5∼+5 | −7∼+7 |
|---|---|---|---|---|
| 1 | 0. 884448 | 1. 027993 | 1 ·036003 | 1. 033045 |
| 2 | 0. 781479 | 0. 895530 | 0. 931754 | 0. 946955 |
| 3 | 0. 111000 | 0. 065521 | 0. 045385 | 0. 034590 |

able accuracy for all grid points in the area.

The results are represented in forms of contour maps as Fig. 3, 4, 6, 7 and 8.

Fig. 3 is a relief map of the base of the crust. The variations of the undulations are ranging from −8% to −42% for the thickness of 26 km at the Japan Sea area, while they are relatively small at the land area. Along the east-west profile across the Peninsula the crust-mantle boundary is concave downward as being expected by the surface features. It is noted that variations at the Yellow Sea area are extremely small. It is remarkable that the crustal thickness is relatively thin in the region of the Kyungsang Sedimentary Basin located in the south-eastern part of the peninsula and shows little change toward the vicinity of Japan island arc.

Isostatic gravity anomalies for d=26 km are represented in a contour map as Fig. 4. which are ranging from 0 mgal to +54 mgal. The average value of +24. 8 mgal implies that the surface features are under-compensation or the crust is thin compared with that in a state of isostatic equilibrium.

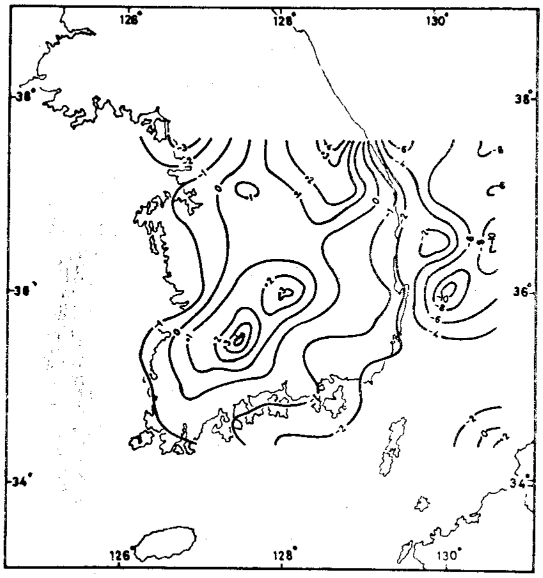


Fig. 3 Calculated relief at the base of the crust in km, which is the deviations from the depth of 26 km. A positive value means that the crust is thicker than the depth of 26 km. The contour intervals are 1 km in land and 2 km in ocean respectively.

For detailed discussions, the whole area is divided into four regions as shown in Fig. 5. Characteristics of respective regions is described as table 2. It is remarked that the isostatic

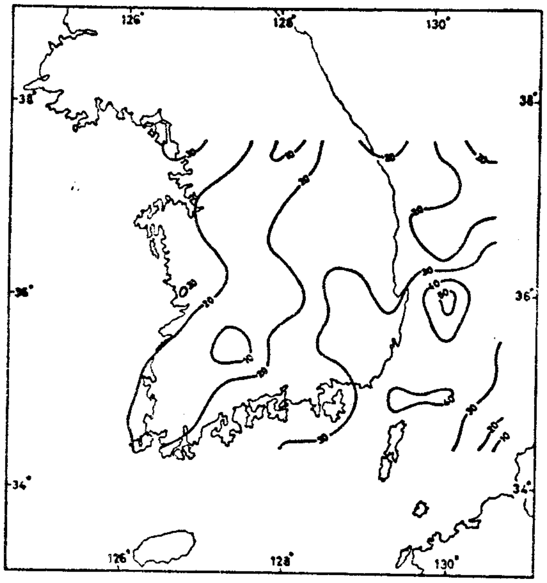


**Fig. 4** Calculated isostatic gravity anomalies in mgal. The contour interval is 10 mgal.

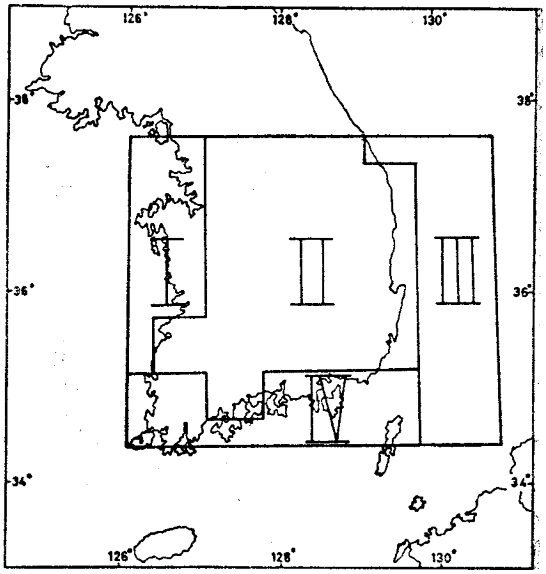


**Fig. 5** Divided regions. Isosatic gravity anomality · anomalies are reduced with increasing of the depth of compensation in region Ⅱ, while the opposite tendency is seen in other regions.

gravity anomalies are reduced with increasing of the depth of compensation in region Ⅱ while the opposite tendency is seen in other regions.

The average Bouguer anomalies, topographic heights and isostatic gravity anomalies in each region are present in table 3.

If we assume that the crust of each region in Fig. 5 has a vertical movement as an independent block to compensate the surface features, apparent directions of the vertical movement can easily be supposed.

Block Ⅱ is appeared as if it moves upward about the other blocks because the average isos tatic gravity anomaly is relatively small in region Ⅱ as shown in taple 3.

**Tab. 3** The average Bouguer anomalies, topographic heights and isostatic gravity anomalies for divided regions.

| Region | Topography (m) | Bouguer anomaly(m gal) | Isostatic anomaly(m gal) |
|---|---|---|---|
| Ⅰ | 6 | 20.0 | 25.1 |
| Ⅱ | 275 | 3.1 | 22.1 |
| Ⅲ | −537 | 76.8 | 27.3 |
| Ⅳ | −27 | 27.1 | 26.1 |

Features of the coastline at the Korean Peninsula are complex in the south-western part, whereas opposite trends are seen in the eastern part.

It is assumed that the features of coastline is caused by the vertical movement of upward direction in the eastern part and downward in the south-western part. Although these features of coastline is well explained by the vertical movements of divided blocks, many questions arise from the above assumptions because the vertical movement of the crust crust can never be explained by the hypothesis of isostasy alone.

In the above, it was concluded that some deviations from isostatic equilibrium could be recognized in the whole area. However many gravity observations showed that the regional effect of positive or negative isostatic anomalies must be balanced over the extended area by the opposite effects. Therefore, the actual state is not unambiguous in the present study using gravity data alone.

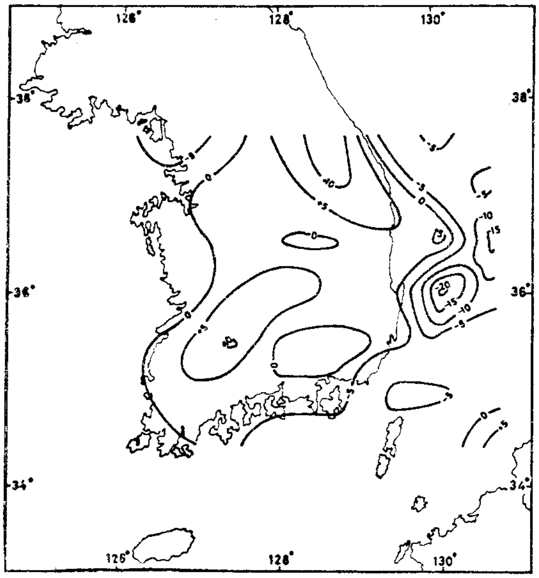


Fig. 6 Calculated values of anomalous vertical gradients of gravity. The contour interval is $5\times10^{-4}$ mgal/m. Normal value is $-3086\times10^{-4}$ mgal/m.

In Fig. 6 the vertical gradient of gravity is shown to be smaller or larger than normal value of $-3086\times10^{-4}$ mgal/m. In general these values are positive in land and negative in ocean respectively. It can be neglected in the free-air reduction of gravity since the fluctuations are everywhere than 0.1%.

The deflections of the vertical are illustrated vectorially in Fig. 7 and these not exceed 6″. It may be noted that the general trend of the deflections of the vertical in direction is nearly perpendicular to the geological structure having a direction of NNE-SSW in the southern part of Korea. The computed deflections can not be compared directly with the astro- geodetic deflections because the latter refer to the reference ellipsoid whereas the former are derived from isostatic gravity anomalies.

The relative undulations of geoidal heights are shown in Fig. 8 and these relative to the point which is marked by the cross.

It is interesting to note that the deflections of the vertical in direction show systematic changes corresponding to the undulations of geoidal heights and isostatic gravity anomalies.

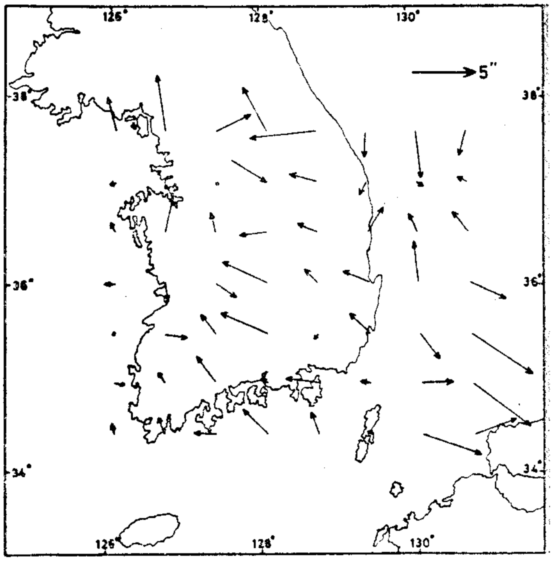


Fig. 7 Calculated deflections of the vertical.

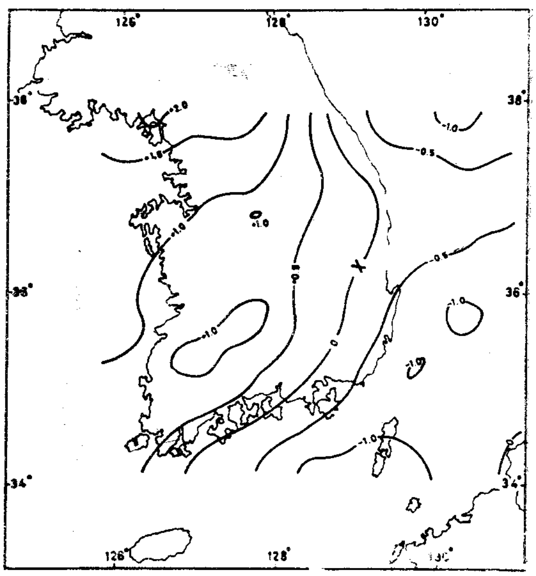


Fig. 8 Calculated undulations in geoidal heights relative to the point which is marked by the cross. The contour interval is 0.5m.

The present author wants to stress that all the results obtained here can never be unique and that they are proposed as an approximate model. As indicated by Shimazu (1961), alternative interpretations are possible such as the lateral changes of the density within the crust and subterranean mantle, support of surface features by the strength of the crust itself rather than by isostatic balance, or other mechanism.


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## 요 약

3차원의 sinx/x 방법을 이용하여 한반도 남부와 인접 일본해에서의 중력 資料로부터 측지학적 및 지구물리학적으로 정량적인 해석을 시도하였다.

한반도에서 均衡狀態에 있는 地殼의 두께는 26km로 밝혀졌으며 이 지역에서의 isostatic gravity이상치는 평균+24.8 mgal로 나타났다. 이 결과는 地表地形의 보정이 덜되어 있거나 또는 지각의 두께가 정상보다 얇음을 의미한다.

또한 일반적으로 중력치의 수직성분의 편차방향이 한반도 남부지역의 지구조방향인 북북동—남남서에 수직인것은 특기할만한 사실이다.


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