

河道에서의洪水追跡

—修正 Muskingum-Cunge方法—

Flood Routing on the River by Revised
Muskingum-Cunge Method

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Summary

The prediction of a design flood hydrograph at a particular site on a river may be based on the derivation of a discharge or stage hydrograph at an upstream section, together with a method to route this hydrograph along the rest of the river.

In order to limit this investigation to cases where the assumption like uniform rainfall may be reasonably valid, the derivation of unit hydrographs has been limited to catchment with an area less than 500 km². Consequently, flood routing methods provide a useful tool for the analysis of flooding in all but the smaller catchment, particularly where the shape of the hydrograph as well as the peak value is required. The author, therefore, will introduce here a flood routing method on the open channel with a peak discharge of the catchment area concerned. The importance of being able to route floods accurately is also reflected in the large number of flood routing method which have been developed since the year 1900.

There are the modified puls method, Steinberg method, Goodrich method, Ekdahl method, Tatum's mean continuously Equation, wisler-Brater method, Muskingum, cheng, and Muskingum-cunge (M-C) method and so on.

The author will try to introduce a flood routing method which is revised Muskingum-cunge method. In calculating flood routing by the M-C method, whole variable parameters on the river were assumed to almost uniform values from the upstream to the downstream.

In the results, the controlled flood rates at the 40km downstream on the river is appeared to decrease 22m³/sec or 12 percent of the peak flood 170m³/sec.

I. 序 言

人類社會의 生活構造가 現代化 乃至 超科學化 됨에 따라 自然의 惠澤 못지 않게 科學理論과 應用으로 人爲의인 手段으로서 自然攝理를 變造하여 살고 있다.

그러나 自然現象中에도 물資源을 利用하는 河川(江)과에 相關性은 東西古今을 通해 人類文明發達過程을 보면 河川을 中心으로 文明이 發祥되었고 生存의 기틀이 마련되었다. 가까운 亞細亞文明의 發

祥地인 中國의 黃河, 印度의 Indas江, Mesopotamia의 Euphrates江 等の 大河川을 中心으로 固有의 文明이 發達하였다. 現代科學이 發達하면 할수록 河川水流를 利用하는 일들이 많아진다. 메마른 耕地에 農工學의 手段으로 農業用水를 給水하고 河口를 締切하여 淡水湖를 建設하고, 舟運, 各種 產業用水, 發電, Recreation等이 모두가 水理水文學의 理論을 適用 開發함으로서 人類生活을 潤澤하도록 도움을 주게 되는 것이다. 自然河川水流가 人間에 베푸는 例를 江을 中心으로 들어보면, 서울市의 大動脈인 漢江水가 서울市民에게 주는 도움, 黃褐色의

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Rhine江물이 獨逸人에게, 鈍濁하게만 보이는 Thames江물이 London市民에게, Seine江물이 Paris의 시민에게, Tebere江이 Rome文化에, Danube江이 Vienna市民에게 주는 도음 등 現代生活과의 函數關係를 수없이 찾을 수 있다. 그러나 이런 現象들은 災難이 없을 境遇이고 조금이라도 水流와 河川과에 函數關係를 소홀히 할 경우엔 말할 수 없는 水魔를 가져다 준다. 우리는 온갖 技術을 動員하여 洪水氾濫을 막기 爲해 河川에 防水堤를 建設하고 河川改修 調節 Dam建設을 하게 된다. 建設事業施行에 앞서 任意의 洪水가 誘發할시 最適通水斷面, 適正 洪水位 및 流量 等を 如何히 決定하면 經濟性이고 安全性이 있겠느냐 하는 것이 重要な 일일 것이다. 주어진 條件으로서 가장 經濟的인 建設을 爲해서는 適正設計와 施工後에 維持管理일 것이며 이들은 洪水追跡의 過程을 거쳐 얻게 된다. 本稿에서는 諸般 水理工學의 變數를 考慮하고 計劃地點 上流部에서 水文曲線圖를 알고 있을 때 河道에서 洪水追跡을 施行하는 修正 Muskingum-Cunge方法을 筆者가 英國에서 水文分野 技術訓練時에 Lecture note를 中心으로 하여 記述하고자 한다. 大體的으로 洪水追跡은 Dam內에서의 洪水追跡과 河川에서의 洪水追跡으로 區分하겠는데 Dam內에서 洪水追跡法은 本分野의 先輩諸位께서 누누히 記述한 바 있어 省略하기로 하며 實際計算例와 應用理論을 簡單히 記述하고자 하며 農工分野事業에 從事하는 여러분들께 多少나마 도움이 되었으면 하는 마음 간절하나.

II. 一般概要

河川에 流入하는 洪水는 一般的으로 下流로 流去함에 따라 그 量의으로 減少變化가 생긴다. 流下途中 河床의 遊水地(凹地)나, 貯水池等을 지나면 그 變化는 더욱 크게 된다. 任河意川에서 計劃洪水量의 變化狀況을 알고 短期 洪水豫報를 한다는 것은 重要的 일이다. 이와같이 變化過程을 上流地點에 洪水量圖를 利用하여 下流部 計劃地點에 洪水量圖를 求하는 것을 洪水追跡이라고 한다. 洪水追跡은 水文 및 水理工學의 理論을 根據로 하여 不等流의 運動方程式 및 連續方程式을 樹立하고 流路를 數個로 區分한 後 其區間의 洪水波(Flood waves)로 因한 水文量의 移行(Translation), 變形(Deformation), 減殺(Attenuation)를 찾는 것이다.

洪水追跡 方法엔 貯溜方程式①(Channel Storage Equation)과 水流連續方程式 및 運動量 方程式에서 逐次計算하는 精密法② 등이 利用되나 後者의 方法

은 初期條件으로서 下端의 水位曲線 및 流量曲線을 알고 上流側으로 計算하는 方法으로서 그 複雜性이 많아 前者 ①에 方法을 많이 쓰고 있다.

貯溜方程式을 適用함에 있어 河道斷面 A,B의 上下地點을 取하고 Q_1, Q_2 의 流入 및 流出量, 河道貯溜量(兩斷面 사이에 있는 流量)을 S로 하여 다음 一般式을 使用한다.

$$\frac{ds}{dt} = Q_1 - Q_2$$

$$S = k[eQ_1 + (1-e)Q_2]$$

여기서 S를 어떻게 假定하는가에 따라 Puls法, Steinberg, Goodirch, Ekdahl, Muskingum法等에 많은 方法이 開發되어 各己 特徵을 가지고 쓰여진다. 筆者經驗에 依하면 Dam內에 調節貯溜量追跡엔 修正 Puls法이, 一般河川에서는 Muskingum法이 先進諸國에도 主로 利用되고 있는 것으로 알고 있다. Muskingum方法은 美國工兵團(U.S. Army Corps of Engineers)의 G.T. McCarthy (1938)가 Muskingum洪水調節事業을 研究하면서 考察된 方法인데 基本假定은 貯溜量은 단지 流達區間 下端의 流出量에 따른 函數이고 貯溜量을 本質的인 角柱型(prismatic shape)으로 增減한다고 假定한 것이다.

여기서 記述하고자 하는 方法은 Cunge(1969)氏에 依해서 修正된 Muskingum-Cunge方法으로 逐次計算하는 方法을 記述하고자 한다.

III. 洪水追跡의 基本資料

1. 資料

가. 水位 및 流量曲線圖(stage and discharge hydrograph)

나. (水位 및 流量에 對한 幾何學的 感應度)Sensitivity of stage and discharge to local Geometry

다. 洪水波(flood waves)—Translation
—Deformation
—Attenuation

라. 洪水波의 影響을 주는
河川幅(Channel width)
河床勾配(Bottom slope)
粗度係數(Roughness coefficient)
幾何學的인 不均衡性(Geometric Irregularities)
堤防(Channel Embankments)
流量制御施設(Control structures)
潮流(Tide)

支流入量(Inflow from the lateral tributary)等等

2. 洪水波(Flood Waves)

洪水波는 洪水時 河川 上下流에 있어 여러 地點의 水位曲線을 求하고 여기서 同時刻의 水位를 取하여 出水前부터 上昇 graph를 表示하면 水位는 河川에 따라 어떤 波狀을 이룬다. 이것을 洪水波라고 하는데, 卽 河川에 따라서 거리를 橫軸으로 各地點의 同水位를 縱軸으로 한 水面曲線이다.

가. 洪水波의 傳播速度(waves speed)

傳播速度를 ω 로 表示하면

$\omega = \frac{\text{流量變化}}{\text{河積變化}}$ 로 表示할 수 있다.

$A = b \cdot H$ 이므로 b 가 變하지 않는다던

$$\omega = V + \frac{A}{b} \cdot \frac{\frac{\partial V}{\partial x}}{\frac{\partial H}{\partial x}} = V + \frac{A}{b} \frac{\partial V}{\partial H} \dots\dots(1)$$

斷面이 短形이라 假定하고

Chezy式의 $V = CH S^{\frac{1}{2}}$ 을 使用하여

$$\frac{\partial V}{\partial H} = \frac{C}{2} S^{\frac{1}{2}} H^{-\frac{1}{2}}$$

$$\omega = V + \frac{C}{2} H \frac{S^{\frac{1}{2}}}{H^{\frac{1}{2}}} = V + \frac{1}{2} CH S^{\frac{1}{2}} = \frac{3}{2} V \dots(3)$$

여기서 $A = \text{河川水理斷面(流積), } m^2$

$H = \text{水位, } m$

$b = \text{河幅, } m$

$V = \text{流速, } m/sec$

$C = \text{Chezy의 流速係數}$

$S = \text{河川勾配}$

$\omega = \text{洪水波傳播速度, } m/sec$

上記와 같이 ω 를 平均流速法에 依해서 算出하거나, 다음 (4)式에 依하거나 하면 된다.

$$\omega = \frac{L}{T_p} - \frac{2d}{L^2} Q^* \dots\dots(4)$$

$$\alpha = \frac{1}{2W_c} \left\{ \frac{1}{L} \sum_{m=1}^M \frac{L_m}{S_m^2} \right\}^{-3} \sum_{m=1}^M \left(\frac{L_m}{S_m^2} \right) \dots(5)$$

或은 $\alpha \approx 1.0$

$$Q^* = \left(\frac{\alpha_p}{T_p} \right)^3 Q_p \left\{ \frac{d^2 Q_p}{d t^2} \right\} \dots\dots(6)$$

或은 $\frac{\partial Q}{\partial T} \approx 0 \quad Q_p \approx Q^*$

여기서 $\omega = \text{Flood waves speed(洪水波傳達速度)}$

$L = \text{河川流路延長}$

$T_p = \text{流路長(L)에 最大洪水到達時間}$

$\alpha = \text{流量減殺係數(Attenuation parameter)}$

$Q^* = \text{attenuation of peak discharge}$

$W_c = \text{平均河幅}$

$L_m = m\text{番째 區間距離}$

$S_m = \text{河床勾配}$

$M = \text{總區間數}(\alpha\text{계 산하는데 區間數})$

$m = m\text{番째 區間的 變數計算하는데 番號}$

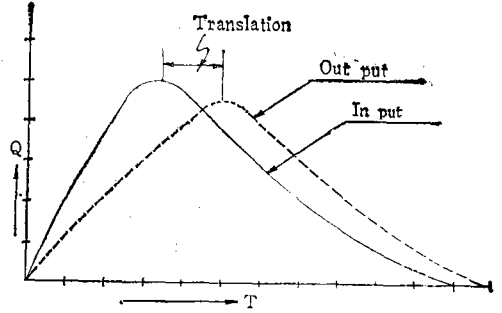


그림. 1. 洪水調節後 peak移行圖

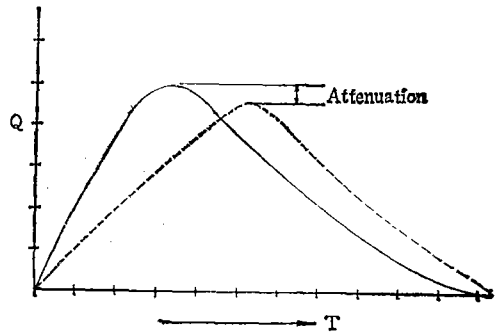


그림. 2. 洪水調節後 peak變形圖

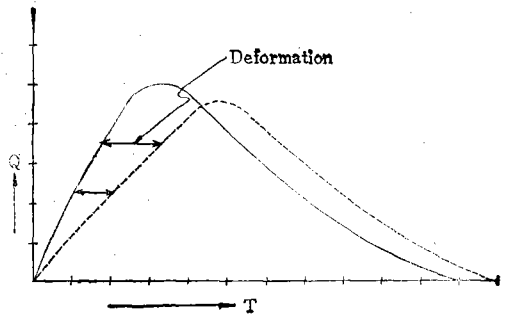


그림. 3. 洪水調節後 流出量 上昇勾配減殺圖

IV. Muskingum法의 基本理論

本 洪水追跡方法은 G.T. McCarthy(1938)氏에 依해서 提議된 方法으로 流入量(Q_{in})과 流出量(Q_{out})

파의 河道貯溜方程式을 다음 式과 같이 表示하였다.

$$\frac{ds}{dt} = Q_{in} - Q_{out} \dots\dots\dots(7)$$

$$S = k[eQ_{in} + (1-e)Q_{out}] \dots\dots\dots(8)$$

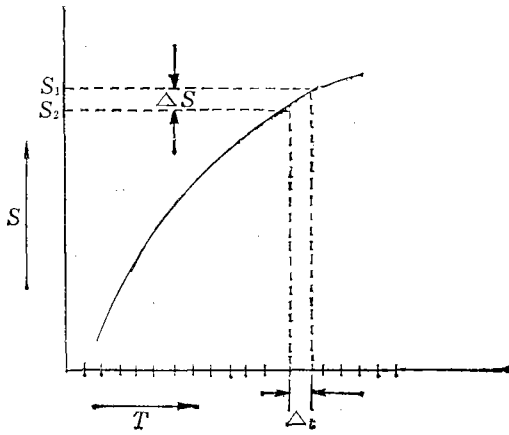


그림. 4. 貯溜關係圖

方程式 (7)을 微分方程式으로 表示하면

$$\begin{aligned} \frac{ds}{dt} &\approx \frac{\Delta s}{\Delta t} \\ &= \frac{s^{n+1} - s^n}{\Delta t} \\ &= \frac{1}{2} [Q_{in}^{n+1} + Q_{in}^n] - \frac{1}{2} [Q_{out}^{n+1} + Q_{out}^n] \dots\dots(9) \end{aligned}$$

여기서 s^n : n 時間間 Δt 時間增加時에 貯溜量
式 (8)과 (9)를 聯立하면

$$\begin{aligned} \frac{k}{\Delta t} [e(Q_{in}^{n+1} - Q_{in}^n) + (1-e)(Q_{out}^{n+1} - Q_{out}^n)] &= \frac{1}{2} \\ [Q_{in}^{n+1} + Q_{in}^n] - \frac{1}{2} [Q_{out}^{n+1} + Q_{out}^n] &\dots\dots\dots(10) \end{aligned}$$

或은

$$\begin{aligned} Q_{out}^{n+1} \left[\frac{k(1-e)}{\Delta t} + \frac{1}{2} \right] &= Q_{in}^n \left[\frac{ke}{\Delta t} + \frac{1}{2} \right] + Q_{in}^{n+1} \left[\frac{1}{2} \right. \\ &\left. - \frac{ke}{\Delta t} \right] + Q_{out}^n \left[\frac{k(1-e)}{\Delta t} - \frac{1}{2} \right] \dots\dots\dots(11) \end{aligned}$$

或은

$$Q_{out}^{n+1} = C_1 Q_{in}^n + C_2 Q_{in}^{n+1} + C_3 Q_{out}^n \dots\dots\dots(12)$$

$$\left. \begin{aligned} \text{여기서 } C_1 &= \frac{Ke + \frac{1}{2}\Delta t}{K(1-e) + \frac{1}{2}\Delta t} \\ C_2 &= \frac{\frac{1}{2}\Delta t - Ke}{K(1-e) + \frac{1}{2}\Delta t} \\ C_3 &= \frac{K(1-e) - \frac{1}{2}\Delta t}{K(1-e) + \frac{1}{2}\Delta t} \end{aligned} \right\} \dots\dots\dots(13)$$

$C = C_1 + C_2 + C_3 = 1$ 이 되도록 안분 修正함.

$$K = \frac{T \left(\frac{1}{2} (Q_{in}^n + Q_{in}^{n+1}) - \frac{1}{2} (Q_{out}^n + Q_{out}^{n+1}) \right)}{X(Q_{in}^{n+1} - Q_{in}^n) + (1-X)(Q_{out}^{n+1} - Q_{out}^n)} \dots\dots(14)$$

K = 流量增加에 따른 貯溜量 增加比

X = 流入量 및 流出量에 의해 決定되는 變數

T = 時間增加

Q_{in}^n = n 時間初의 流入量

Q_{in}^{n+1} = n 時間後의 流入量

Q_{out}^{n+1} = n 時間初의 流出量

Q_{out}^n = n 時間後에 流出量

C = 常數

以上 記述한 바와 같이 一定區間에 對한 K, X 값을 決定하기 爲하여 流入 및 流出量平均値를 그에 該當하는 溪谷貯溜量과 比較하면서 方眼紙에 點描하여 X 값을 假定하는 등 複雜性이 뒤따른다.

V. Muskingum-Cunge理論 및 實際

Cunge(1969)는 Muskingum方法中에 一部 函數計算하는 方式을 修正 發表하였다. 河道貯溜量은 流入 및 流出量과 Δt 에만 關係되는 것이 아니라 附隨의 所以 河川에 水理學的 變數들 卽 河幅, 勾配, 洪水波傳播速度 等에 函數로 本 것이다.

Muskingum公式에서 上流區間 斷面에서 流入量

$$Q_j, \text{ 流出量 } Q_{j+1} \text{로 하면, } K \frac{d}{dt} \{eQ_j + (1-e)Q_{j+1}\} = Q_j - Q_{j+1} \dots\dots\dots(15)$$

$$\frac{K}{\Delta t} \{eQ_j^{n+1} + (1-e)Q_{j+1}^{n+1} - eQ_j^n - (1-e)Q_{j+1}^n\} = \frac{1}{2} \{Q_j^{n+1} - Q_{j+1}^{n+1} + Q_j^n - Q_{j+1}^n\} \dots\dots\dots(16)$$

과 같이 表示되는데

여기서 k 값은 다음과 같이 計算된다.

$$k = \frac{\Delta x}{\omega} \dots\dots\dots(17)$$

ω : 最大洪水量의 平均速度

Δx : 區間延長(河道地點間距離)

式 (17)을 運動量 微分方程式으로 表示하면

$$\frac{\partial Q}{\partial t} + \omega \frac{\partial Q}{\partial x} = 0 \dots\dots\dots(18)$$

Hayami (1951), Thomas & Wormleaton (19)

70. 1971)式에서 쓰이는

“Linear Diffusion method”方程式인

$$\frac{\partial Q}{\partial t} + \omega \frac{\partial Q}{\partial x} = \mu \frac{\partial^2 Q}{\partial x^2} + \omega q \dots\dots\dots(19)$$

(19)式을 導入하면

ω, μ 을 常數(constant parameters)로서 取扱되여 여기서 $\mu = \frac{\alpha(\bar{Q}_p)}{L}$ 로서 定義한다.

$$\omega = \frac{L}{T_p} - \frac{2\alpha}{L^2} Q^*$$

Cunge는 Taylor Expansions方程式 (10)形, $\{Q_j^n\}$ 形으로 表示하여

$$\frac{\partial Q}{\partial \pi} + \omega \frac{\partial Q}{\partial x} = \mu \frac{\sigma^2 Q}{\sigma x^2} \dots\dots\dots(20)$$

$$\left. \begin{aligned} \mu &= \left(\frac{1}{2} - e\right) \omega \cdot \Delta x \\ \mu &= \frac{\alpha \bar{Q}_p}{L} \end{aligned} \right\} \text{式을 聯立하여}$$

$$e = \frac{1}{2} - \frac{\alpha \bar{Q}_p}{L \omega \Delta x} \dots\dots\dots(21)$$

以上과 같이 e 값을 計算할 수 있다.

$$\text{或은 } e = \frac{1}{2} - \frac{\bar{Q}_p}{2B \cdot S \cdot \omega \Delta x} \dots\dots\dots(22)$$

$$\text{단 } \bar{Q}_p = Q_p - \frac{1}{2} Q_{new}^* \dots\dots\dots(23)$$

$$Q_{new}^* = Q_p \left\{ 1 - \exp\left(-\frac{Q^*}{Q_p}\right) \right\} \dots\dots\dots(24)$$

$$S = \frac{Q^2 n^2}{A^2 R^3} = \frac{V^2 n^2}{R^3} \dots\dots\dots(25)$$

ω =waves speed, m/sec

Δx =區間延長, m

B =河幅, m

S =河床勾配

\bar{Q}_p =最大平均流量

n =manning's의 n 數

R =徑深, m

V =流速 m/sec

以上과 같이 計算된 常數로서 다음 式에 依하여 洪水追跡을 行하면 된다.

$$Q_j^{n+1} = C_1 Q_j^n + C_2 Q_j^{n+1} + C_3 Q_{j+1}^n + C_4 \dots\dots\dots(26)$$

$$\left. \begin{aligned} C_1 &= \frac{Ke + \frac{1}{2} \Delta t}{K(1-e) + \frac{1}{2} \Delta t} \\ C_2 &= \frac{\frac{1}{2} \Delta t - K \cdot e}{K(1-e) + \frac{1}{2} \Delta t} \\ C_3 &= \frac{K(1-e) - \frac{1}{2} \Delta t}{K(1-e) + \frac{1}{2} \Delta t} \end{aligned} \right\} \dots\dots\dots(27)$$

$$C_4 = \frac{q \cdot \Delta t \cdot \Delta x}{K(1-e) + \frac{1}{2} \Delta t}$$

단 q :單位길이當 支流 流入量

以上 修正 M-C方法에 關한 說明을 하였으며 式(26)에서 河道洪水追跡에 支流 流入量이 없으면 C_4 項은 無示한다.

Cunge의 依해 作成된 $\frac{\Delta x}{\omega \Delta t}$ 와 e 에 關係圖表 그림. 5로서 上記한 計算上에 複雜性을 減 수가 있다.

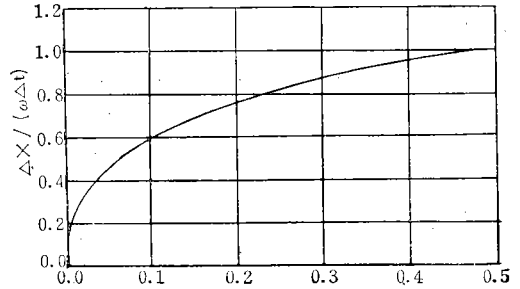


그림. 5. Curve for $\left(\frac{\Delta x}{\omega \Delta t}\right)$ versus (e) : Taken from Cunge (1969)

VI. 計算例

위 그림과 같이 河川길이가 40km되는 河道에서 upstream hydrograph와 같은 最大洪水量(170m³/sec)으로서 0km, 10km, 20km, 30km, 40km地點에 對한 洪水追跡을 하기로 한다.

條件: $Q_p = 170 \text{ m}^3/\text{sec}$

河幅 (B)=50m

河床勾配 (S)= 5×10^{-4}

傳播速度 (ω)=1.5m/sec

$\Delta x = 10 \text{ km}$

$\Delta t = 2 \text{ hrs}$

但 支流 流入量은 없는 것으로 假定함.

計算順序:

- ① Upstream (0地點)에 對한 洪水量圖를 求함.
- ② 全河川長을 任意距離로 區分함. 但 支流 流入量이 有할 時엔 當該區間末端에 有하도록 區分함이 좋음.

普通 $\Delta x = \frac{L}{10}$ 程度가 좋음. $\left(\frac{L}{10} = \Delta x < \frac{L}{10}\right)$

- ③ 區間別 平均河川幅을 計算함.

(比較的 河川幅이 平均인 境遇엔 全體를

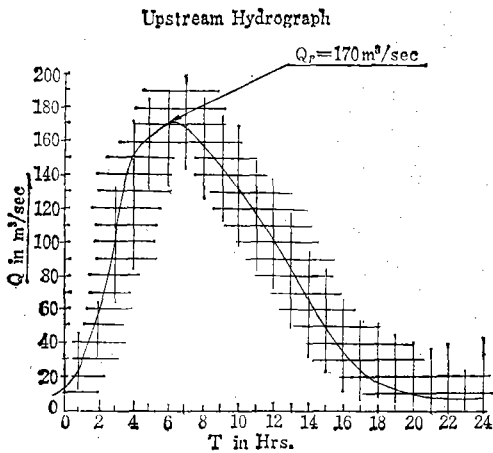


그림. 6. 任意流域에서의 홍수량圖

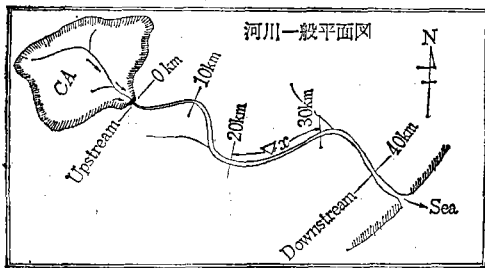


그림. 7. Upstream Hydrograph 流域과 河川區間 分割圖

個 斷面을 取하기도 함)

④ 區間別 河床勾配를 計算.

⑤ 洪水傳播速度(ω) 計算.

$$\omega = \frac{L}{T_p} - \frac{2\alpha}{L^2} Q^*$$

⑥ k 計算(河川形狀이 均一한 境遇엔 平均 k 計算)

⑦ e 計算(公式 또는 그림. 5로서)

⑧ C_1, C_2, C_3, C_4 의 計算(但支流流入量이 없을때엔 C_4 는 不要)

⑨ Δt 選定(普通 $\Delta t = \frac{T_p}{5}$ 이나, $T_p > 3hr$ 일 경

우엔 適正하게 定함)

修正 Muskingum-Cunge 方法에 依해 實際計算 함.

$$Q_i^{n+1} = C_1 Q_i^n + C_2 Q_i^{n+1} + C_3 Q_{i+1}^n + C_4$$

$$K = \frac{\Delta x}{\omega} = \frac{10,000}{1.5} = 6,666.2$$

(22)式에 依하여, $e = \frac{1}{2} - \frac{\bar{Q}_p}{2B.S.e\Delta x} = 0.273$

$$C_1 = \frac{Ke + \frac{1}{2}\Delta t}{K(1-e) + \frac{1}{2}\Delta t} =$$

$$\frac{6,666.7 \times 0.273 + \frac{1}{2} \times 7,200}{6,666.7(1-0.273) + \frac{1}{2} \times 7,200}$$

$$= 0.642 \approx 0.643$$

$$C_2 = 0.210$$

$$C_3 = 0.147$$

$$C \text{檢定: } C = 0.643 + 0.210 + 0.147 = 1 \text{ OK}$$

Predicted Hydrograph

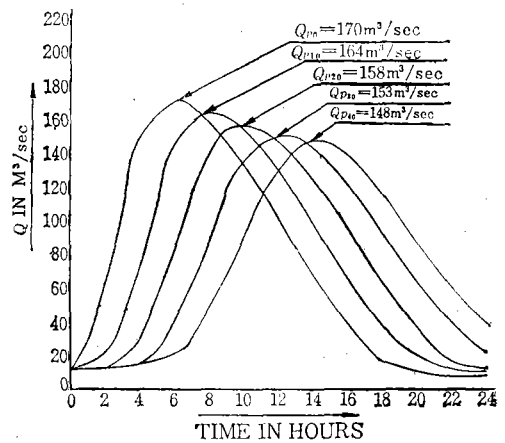


그림. 8. 洪水追跡後의 地點別 洪수량圖

計算方法:

— 計算表의 (B)欄(㉠……㉤)은 upstream hydrograph에서 읽은 時間別 流入量

— 橫으로 (B)(C)(D)(E)(F)는 地點距離

— $Q_i^{n+1} = C_1 Q_i^n + C_2 Q_i^{n+1} + C_3 Q_{i+1}^n$ 에서 番號 ①~④

까지의 計算은 다음과 같다.

①의 $Q_1 = C_1(㉠) + C_2(㉡) + C_3(H) = (0.643 \times 10) + (0.210 \times 65) + (0.147 \times 10) = 21.55$

② $Q_2 = C_1(H) + C_2(㉠) + C_3(I) = (0.643 \times 10) + (0.210 \times 21.55) + (0.147 \times 10) = 12.42$

③ $Q_3 = C_1(I) + C_2(2) + C_3(J) = (0.643 \times 10) + (0.210 \times 12.42) + (0.147 \times 10) = 10.51$

④ $Q_4 = C_1(J) + C_2(3) + C_3(K) = (0.643 \times 10) + (0.210 \times 10.51) + (0.147 \times 10) = 10.11$

⑤ $Q_5 = (0.643 \times 65) + (0.210 \times 155) + (0.147 \times 21.55) = 77.52$

⑥ $Q_6 = (0.643 \times 21.55) + (0.210 \times 77.52) + (0.147 \times 12.42) = 31.97$

表-1.

計算表

$\Delta x=10\text{km}$

(A) 時間(Δt)	(B) 0km 流入量(Q)	(C) 10km	(D) 20km	(E) 30km	(F) 40km	(G) 備 考
0	hr ㉑ m ³ /sec 10.00	(H) m ³ /sec 10.00	(I) m ³ /sec 10.00	(J) m ³ /sec 10.00	(K) m ³ /sec 10.00	
2	㉒ 65.00	㉓ 21.55	㉔ 12.47	㉕ 10.51	㉖ 10.11	
4	㉗ 155.00	㉘ 77.52	㉙ 31.97	㉚ 16.24	㉛ 11.66	
6	㉜ 170.00	㉝ 148.76	㉞ 85.79	㉟ 40.97	㊱ 20.75	
8	㊲ 155.00	㊳ 163.73	㊴ 142.64	㊵ 91.13	㊶ 48.53	
10	㊷ 130.00	㊸ 151.04	㊹ 157.97	㊺ 138.29	㊻ 94.77	
12	㊼ 98.00	㊽ 126.37	㊾ 146.88	㊿ 152.74	㋀ 134.93	
14	㋁ 65.00	㋂ 95.24	㋃ 122.85	㋄ 142.69	㋅ 148.00	
16	㋆ 36.00	㋇ 63.36	㋈ 92.61	㋉ 119.43	㋊ 138.59	
18	㋋ 17.00	㋌ 36.03	㋍ 61.92	㋎ 90.11	㋏ 116.08	
20	㋐ 10.00	㋑ 18.33	㋒ 36.12	㋓ 60.55	㋔ 87.74	
22	㋕ 10.00	㋖ 11.22	㋗ 19.46	㋘ 36.24	㋙ 59.50	
24	㋚ 10.00	㋛ 10.18	㋜ 12.21	㋝ 20.40	㋞ 36.33	

㉑ $Q_7 = (0.643 \times 12.42) + (0.210 \times 31.97) + (0.147 \times 10.51) = 16.24$

㉒ $Q_8 = (0.643 \times 10.51) + (0.210 \times 16.24) + (0.147 \times 10.11) = 11.66$

위와 같은 방법으로 逐次計算을 番號 ㉑~㊿까지 하면 各地點에 豫測洪水量圖를 求할 수 있으며 그 結果는 表. 1. 計算表에 보는 바와 같고 各地點에 對한 洪水量圖는 그림. 8. 에 圖示한 것과 같다.

VII. Computer program

Muskingum-Cunge 洪水追跡法을 쉽게 計算할 수 있도록 Computer program을 附隨的으로 紹介하여 設計에 더욱 便利하게 한다. 그 program list와 必要한 Data는 다음과 같다.

Muskingum-Cunge method

A. Prepare card data

The program listed below reads the data in the following order. Eight cards containing the heading for the output from the program. Cards containing 11 integers (free I-format):

- JX the number of space nodes
- LEND the number of time steps
- IQH the number of data points for the upstream hydrograph
- IQDNS the number of data points for the downstream hydrograph
- ITR1 the number of data points for the hydrograph from the first tributary
- ITR2 the number of data points for the hydrograph from the second tributary
- JT1 the space node for the confluence of the first tributary with the main channel
- JT2 the space node for the confluence of the

J1 second tributary with the main channel
the discharges along the reach are
written at J1 intervals apart

L1 the discharges along the reach are
written when the number of times
steps is equal to a multiple of L1

L2 only the downstream discharge is
written when the number of time steps
is equal to a multiple of L2

Cards containing 12 real numbers (free E-form-
at):

DXLR length of reach
DT time step
DTQHYDRO time interval between successive
data points for each specified hydrograph
QINIT initial discharge along the reach
QINB base flow for the lateral inflow
QINA amplitude of the variable part of the
lateral inflow
TQIN time when the peak lateral inflow
occurs

TSQIN timescale for the variable part of the
lateral inflow
WSP value of L/T_p
AP value of α
QCON value of \bar{Q}_p
TDEVN time after which the error parameters
are calculated.

Notes

i If there is no downstream hydrograph IQDNS
=0 and TDEVN is set to a value greater than
the real time for routing the flood. Similarly
ITR1, ITR2, JT1 and JT2 should be set equa-
to zero if there are no tributaries.

ii The computer version above of the Muskin,
gum-Cunge method can also be used in a desk
calculation if JX is not too large.

The computer program below is written in
FORTRAN IV. The program can be extended
to deal with and number of different reaches
in the same river.

B. Computer program list.

```

C  Q1 - ARRAY FOR THE DISCHARGE ALONG THE REACH AT THE NEW TIME LEVEL.
C  Q2 - ARRAY FOR THE DISCHARGE ALONG THE REACH AT THE OLD TIME LEVEL.
C  DX - SPACE INCREMENT.
C  DXLR - LENGTH OF THE REACH.
C  JX - NUMBER OF SPACE GRID POINTS.
C  X - ARRAY FOR THE DISTANCES OF THE GRID POINTS ALONG THE CHANNEL.
C  DT - TIME INCREMENT.
C  T - TIME IN SECONDS.
C  TH - TIME IN HOURS.
C  LPM - NUMBER OF TIME STEPS.
C  TITLE - ARRAY WHICH STORES DATA FOR THE HEADING OF THE OUTPUT.
C  J1, L1 AND L2 REFER TO WHEN AND HOW THE RESULTS ARE WRITTEN.

C  WSP - WAVE SPEED.
C  AP - ATTENUATION PARAMETER.
C  QCON - DISCHARGE CONSTANT.
C  C1, C2, C3 AND C4 ARE PARAMETERS IN THE MUSKINGUM METHOD.
C  DTQHYDRO - TIME INTERVAL IN HOURS BETWEEN THE DATA POINTS FOR THE INPUT AND
C  OTHER DISCHARGE HYDROGRAPHS.
C  QHYDRO - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C  AT THE UPSTREAM SECTION.
C  IQM - NUMBER OF DATA POINTS FOR THE UPSTREAM HYDROGRAPH.
C  QTR1B1 - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C  AT THE FIRST TRIBUTARY.
C  ITR1 - NUMBER OF DATA POINTS FOR THE HYDROGRAPH AT THE FIRST TRIBUTARY.
C  JT1 - POSITION OF THE INPUT DISCHARGE FROM THE FIRST TRIBUTARY.
C  QTR1B2 - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C  AT THE SECOND TRIBUTARY.
C  ITR2 - NUMBER OF DATA POINTS FOR THE HYDROGRAPH AT THE SECOND TRIBUTARY.
C  JT2 - POSITION OF THE INPUT DISCHARGE FROM THE SECOND TRIBUTARY.
C  QINIT - INITIAL DISCHARGE FOR THE FLOW THROUGHOUT THE REACH.
C  QINB - BASE VALUE FOR THE LATERAL INFLOW.
C  QINA - AMPLITUDE OF THE FUNCTION FOR THE LATERAL INFLOW.
C  TQIN - TIME WHEN THE LATERAL INFLOW TAKES ITS MAXIMUM VALUE.
C  TSCQIN - TIME SCALE FOR THE LATERAL INFLOW FUNCTION.
C  QDNS - ARRAY WHICH STORES THE DATA FOR THE RECORDED DISCHARGE HYDROGRAPH
C  AT THE DOWNSTREAM END OF THE REACH.
C  IQDNS - NUMBER OF DATA POINTS FOR THE DOWNSTREAM HYDROGRAPH.

C  TDEVN - TIME IN HOURS WHEN THE CALCULATIONS OF THE ERROR PARAMETERS ARE
C  BEGUN.
C  TOTDIS - AVERAGE RECORDED DISCHARGE AFTER THE TIME TDEVN.
C  TOTD - AVERAGE PREDICTED DISCHARGE AFTER THE TIME TDEVN.
C  DISDIF - DIFFERENCE BETWEEN THE AVERAGE RECORDED AND PREDICTED DISCHARGES
C  AS A PERCENTAGE OF THE RECORDED DISCHARGES.
C  DEVN - ULTIMATELY RECORDS THE STANDARD DEVIATION OF THE PREDICTED
C  DISCHARGE AS A PERCENTAGE OF THE AVERAGE RECORDED DISCHARGE.
    
```


河道에서의洪水追跡

```

MASTER FLOODS1
      REAL Q1(50),Q2(50),X(50)
      COMMON DX,DT,JX,DXLR,LEND,TH,J1,L1,L2,TITLE(60),
      1JT1,JT2,QTRIB1(200),QTRIB2(200),QHYDRO(200),DTQHYDRO,
      2QINIT,QINB,QINA,TWIN,TSCQIN,AP,WSP,QCON,
      4      QDNS(200),TDEVN

C FIRST READ IN THE APPROPRIATE DATA AND WRITE IT OUT FOR EASY REFERENCE.
      CALL DATIN

C NEXT CALCULATE VARIOUS CONSTANTS AND PARAMETERS FOR USE BELOW.
      DX=DXLR/FLOAT(JX-1)
      DK=DX/WSP
      EPSILON=0.5*(1.0-QCON+AP+2.0/(DX+WSP+DXLR))
      CC=DK*(1.0-EPSILON)+DT*0.5
      C1=(DK+EPSILON+DT*0.5)/CC
      C2=(DT*0.5-DK+EPSILON)/CC
      C3=(DK*(1.0-EPSILON)-DT*0.5)/CC
      C4=DT+DX/CC
      JXM1=JX-1
      T=0.0

C THE FOLLOWING VARIABLES ARE ONLY USED WHEN A RECORDED DOWNSTREAM
C HYDROGRAPH IS AVAILABLE FOR COMPARISON. ALL SUCCEEDING
C STATEMENTS BEGINNING IN COLUMN 13 REFER TO THIS CASE ONLY, AND
C SHOULD NOT BE INCLUDED OTHERWISE.
      DEVN=0.0
      TOTDIS =0.0
      TOTD=0.0
      QP1=QINIT
      QP2=QINIT
      QDNP2=QINIT
      QDNP1=QINIT
      JCOUNT=0
      ICOUNT=0

C DEFINE X AND THE INITIAL VALUES OF Q1 AND Q2
      DO 2 J=1,JX
      X(J)=DX*FLOAT(J-1)
      Q1(J), Q2(J)=QINIT
2      CONTINUE

C BEGIN THE MAIN TIME LOOP
      DO 14 L=1,LEND
      T=T+DT
      TH=T/3600.0

      QP2=QP1
      QP1=Q1(JX)
      QDNP2=QDNP1
      QDNP1=QUN
      QDN=FF(QDNS,TH,DTQHYDRO)

C UPDATE THE VALUES FOR THE DISCHARGE
      DO 4 J=1,JX
      Q2(J)=Q1(J)
4      CONTINUE

C FIND THE VALUE OF Q1(1) FROM THE INPUT HYDROGRAPH AND ADD THE
C CONTRIBUTION FROM THE TRIBUTARY, IF ANY
      QTOT=0.0
      IF(JT1.EQ.1) QTOT=FF(QTRIB1,TH,DTQHYDRO)
      Q1(1)=FF(QHYDRO,TH,DTQHYDRO)+QTOT

C THE MAIN CALCULATIONS FOR Q1 ARE PERFORMED IN THE FOLLOWING LOOP
      DO 6 J=2,JX
C QTOT REFERS TO THE INFLOW DISCHARGE FROM TRIBUTARIES AND LATERAL
C RUNOFF. UNDER SNOW-MELT CONDITIONS IT MAY BE PREFERABLE TO TAKE
C THE LATERAL INFLOW AS PROPORTIONAL TO THE DISCHARGE AT THE
C UPSTREAM BOUNDARY
      TX=(TH-TQIN)/TSCQIN
      QTOT=QINA*EXP(-TX+TX)+QINB
      QTOT=Q1(1)*CONST/DXLR
C
      IF(J.EQ.JT1) QTOT=QTOT+FF(QTRIB1,TH,DTQHYDRO)/DX
      IF(J.EQ.JT2) QTOT=QTOT+FF(QTRIB2,TH,DTQHYDRO)/DX
      Q1(J)=C1+Q2(J-1)+C2*Q1(J-1)+C3*Q2(J)+C4*QTOT
6      CONTINUE

```

C PERFORM VARIOUS SUMS ON THE ERROR PARAMETERS

```

      IF(TH.LT.TDEVN) GO TO 8
      DEVN=DEVN+(Q1(JX)-QDN)*(Q1(JX)-QDN)
      ICOUNT=ICOUNT+1
      JCOUNT=JCOUNT+1
      IF(JCOUNT.NE.2) GO TO 8
      TOTD=TOTD+2.0*DT*(Q1(JX)+Q1(JX+Q2))/3.0
      TOTDIS=TOTDIS+2.0*DT*(QDN+QDNP1+QDHP2)/3.0
      JCOUNT=0
  
```

C NOW WRITE OUT THE RESULTS - IF IT IS TIME

```

  8   IF(L.NE.L1+(L/L1)) GO TO 12
      WRITE(3,301) (TH,X(J),Q1(J),J=1,JXN1,J1)
301  FORMAT((20X,F6.2,9X,F8.1,8X,F7.2))
  10  WRITE(3,302) TH,X(JX),Q1(JX),QDN
302  FORMAT((20X,F6.2,9X,F8.1,2(AX,F7.2)))
      WRITE(3,303)
303  FORMAT(1H //)
      GO TO 14
  12  IF(L.EQ.L2+(L/L2)) GO TO 10
  14  CONTINUE
  
```

C FINALLY, WORK OUT THE ERROR PARAMETERS

```

      TOTDIS=TOTDIS/((TH-TDEVN)+3600.0)
      TOTD=TOTD/((TH-TDEVN)+3600.0)
      DISDIF=(1.0-TOTD/TOTDIS)*100.0
      DEVN=SQRT(DEVN/FLOAT(ICOUNT))+100.0/TOTDIS
304  WRITE(3,304) TOTDIS,DISDIF,DEVN
      FORMAT(34H AVERAGE RECORDED DISCHARGE = ,F7.2,
  1   9H CUMEC/S/
  2   56H DIFFERENCE BETWEEN RECORDED AND PREDICTED AVERAGE D.
  3   11H ISCHARGE = ,F6.2,1HX/
  4   26H STANDARD DEVIATION = ,F6.2,1HX//)
      STOP
      END
  
```

C
 SUBROUTINE DATIN

C THIS SUBROUTINE READS IN THE RELEVANT DATA, WRITES IT OUT FOR EASY
 C REFERENCE, AND PROVIDES A HEADING FOR THE RESULTS

```

      COMMON DX,DT,JX,DXLR,LEND,TH,J1,L1,L2,TITLE(60),
  1  JT1,JT2,QTR1B1(200),QTR1B2(200),QHYDRO(200),DTQHYDRO,
  2  QINIT,QINB,QINA,QIN,TSCQIN,AP,WSP,QCON,
  3  QDNS(200),TDEVN
  4
  100 READ(1,100) (TITLE(I),I=1,60)
      FORMAT(10A8/5A8)
  101 READ(1,101) JX,LEND,IQH,IQDNS,ITR1,ITR2,JT1,JT2,J1,L1,L2
      FORMAT(11I0)
  102 READ(1,102) DXLR,DT,DTQHYDRO,QINIT,QINB,QINA,QIN,TSCQIN,WSP,AP,
      1QCON
      2 ,TDEVN
  102 FORMAT(13E0.0)
  300 WRITE(3,300) (TITLE(I),I=1,60)
      FORMAT(1H ,(15A8))
  C300 WRITE(3,301) JX,LEND,IQH,IQDNS,ITR1,ITR2,JT1,JT2,J1,L1,L2
      FORMAT - FOR WRITE STATEMENT
  C301 WRITE(3,302) DXLR,DT,DTQHYDRO,QINIT,QINB,QINA,QIN,TSCQIN,WSP,AP,
      1QCON
      2 ,TDEVN
  C302 FORMAT - FOR WRITE STATEMENT
  
```

C READ IN THE DISCHARGE HYDROGRAPHS

```

      READ(2,200) (QHYDRO(I),I=1,IQH)
      IF(ITR1.NE.0) READ(2,200) (QTR1B1(I),I=1,ITR1)
      IF(ITR2.NE.0) READ(2,200) (QTR1B2(I),I=1,ITR2)
      READ(2,200) (QDNR(I),I=1,IQDNS)
  200 FORMAT(41X,F6.2)
      WRITE(3,306)
  306 FORMAT(1H ,21X,4HTIME,10X,8HDISTANCE,6X,10NCALCULATED,6X,9HPROTOTYP
  1PE/1H ,20X,5H(HRS),15X,3M(M),9X,9HDISCHARGE,6X,9HDISCHARGE/S)
      RETURN
      END
  
```

C
 FUNCTION FF(Q,T,DT)

C THIS FUNCTION INTERPOLATES USING A CUBIC POLYNOMIAL

```

REAL Q(200)
      L=INT(T/DY)+1
      IF(L.LE.1) L=2
      PRINT/DT=FLOAT(L-1)
      A=(Q(L-2)+3.0*Q(L-1)+Q(L))/6.0
      B=(Q(L-1)+4.0*Q(L)+Q(L+1))/6.0
      C=(Q(L)+6.0*Q(L+1)+Q(L+2))/6.0
      D=Q(L)
      FF=(A*FR1+D)*FR1+C+FR2
      RETURN
      END
  
```

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