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On the Numerical Evaluation of the Wave Pattern of a Havelock Source*

by

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Abstract

A method of evaluating Kelvin wave pattern is presented in this paper. The mathematical manipulation of x-derivative of the Green function of the Havelock source by the use of contour integration on the complex plane has resulted in the expressions that can be readily incorporated with computer program. The efficiency and accuracy that can be secured by the use of the present mathematical expressions seem to be excellent when suitable numerical quadratures are employed. The wave patterns for particular submergences of the singularity are presented.

Notation of Symbols in General Use

- g : Acceleration due to gravity
- U : Speed of the free stream
- K : Wave number (g/U^2)
- f : Submerged distance of the singularity
- $\boldsymbol{x}$: Position vector of a field point $(\boldsymbol{x}=(x, y, z))$
- $\boldsymbol{x}'$: Position vector of the singularity $(\boldsymbol{x}'=(x', y', z'))$
- ζ : Wave elevation
- i : Imaginary unit $(=\sqrt{-1})$

1. Introduction

When the flow problem around a ship is formulated with the assumption of ideal fluid, the field equation turns out to be a Laplace equation of the velocity potential. One way of solving this linear partial differential equation is the use of integral solution method, that is, the use of singularities based on the solution of Poisson's equation. If a floating body problem is approached with this method, the free surface demands distribution of singularities over itself to restrict the flow within the limited space out of the body and below the free surface, and at the same time to satisfy the required boundary condition. However this raises unwieldy difficulties originating from the fact that its configuration in association with the particular speed and geometry of the body is by no means available in advance.

These difficulties forced the early investigators to introduce the approximation about the position of application of Bernoulli's equation and thereafter the linearisation of the free surface boundary condition. If these simplifications are accepted, it is not absolutely necessary to distribute singularities on the free surface to restrict the flow domain. Instead, it would be more plausible to endow the property of generating free surface to the singularity itself. This is possible because the linearised free surface boundary condition resulted in the form of differential equation appears to be satisfiable by some function satisfying the field equation, if the suitable form of solution of the homogeneous Poisson equation (Laplace equation) is added to its particular

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integral to make up the function. This means that the free surface boundary condition is to be absorbed as the built-in nature of the singularity. Such a singularity has been customarily called a Havelock source (or a Kelvin source) and the Green function of this singularity has been constructed by a number of investigators [1], [2], [3].

No doubt the construction of such a function is a triumphant success in the field of the linearised ship wave-making theory. Indeed it is accepted as one of the fundamental solutions of the field equation. In contrast, however, the practical use of the function has been quite a different matter because of the complexity of its numerical evaluation. Although this situation has been greatly altered with the appearance of high speed computing machines, it is still of indispensable requirement from both economic and mathematical reasons to provide a reliable and efficient means of obtaining the numerical value.

Historically the evaluation of numerical values of the function in the form of wave pattern has never failed to arouse interests of naval architects and considerable amount of efforts has duly been poured into it. Wigley dealt with this problem but, because of prohibitive amount of calculation, showed wave profile along the centerline only, i.e. the path of the singularity. More recently Adee [4], Standing [5], Noblesse [6] and many others have considered the problem. These recent investigations reflected that the main attention has been shifted to the mathematical manipulation of the function so that the basis for the algorithm that could work with best efficiency and accuracy may be provided.

It is the purpose of this paper to introduce one method of such decompositions of the function, an algorithm based on, which has proved to work satisfactorily in both speed and accuracy.

A moving Cartesian coordinate system is employed in this paper with its x-axis directed opposite to the motion of the singularity (or equivalently coincident with the free stream), z-axis directed vertically upwards and y-axis directed so as to compose the mentioned coordinate system right-handed one, and with its origin located directly above the singularity, at the undisturbed water level.

2. The Wave Elevation

One of a number of alternative forms of the Green function which satisfies the linearised free surface boundary condition and the radiation condition is, by Wehausen [7],

$$G(\boldsymbol{x};\boldsymbol{x}')=\frac{1}{|\boldsymbol{x}-\boldsymbol{x}'|}-\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_1'|}-\frac{4K}{\pi}\int_0^{\pi/2}\sec^2\theta\int_0^{\infty}\frac{e^{k(z+z')}}{k-K\sec^2\theta}\cos(k(x-x')\cos\theta)\cos(k(y-y')\sin\theta)\,dk\,d\theta$$
$$+4K\int_0^{\pi/2}\sec^2\theta\; e^{K(z+z')\sec^2\theta}\sin(K(x-x')\sec\theta)\cos(K(y-y')\sec^2\theta\,\sin\theta)\,d\theta \quad (1)$$

where $K=g/U^2$

$\boldsymbol{x}=(x,y,z)$

$\boldsymbol{x}'=(x',y',z')$

$\boldsymbol{x}_1'=(x',y',-z')$

with $-\infty<x<\infty,\quad -\infty<x'<\infty$

$-\infty<y<\infty,\quad -\infty<y'<\infty$

$-\infty<z\leq 0,\quad -\infty<z'<0$

for the uniform onset velocity field with the magnitude U in the positive x-direction. $\boldsymbol{x}$ denotes the position vector of a field point and $\boldsymbol{x}'$ that of the source in the coordinate system fixed with respect to the source.

The wave elevation which is consistent with the approximation introduced for the derivation of the above Green function is given by the expression

$$\zeta(x,y)=\frac{U}{g}G_x[(x,y,0);(x',y',z')]$$

the subscript x denoting partial derivative. Without losing generality, the position of the source can be brought directly below the origin of the coordinate system, i.e. $(x',y',z')=(0,0,-f)$, f being the submerged distance of the source from the undisturbed waterplance. With this arrangement, the wave elevation will be given by

$$\zeta(x,y)=\frac{U}{g}G_x[(x,y,0);(0,0,-f)] \quad (2)$$

The straightforward differentiation of eq. (1) shows that

$$G_x[(x,y,0);(0,0,-f)]$$
$$=\frac{4K}{\pi}\int_0^{\pi/2}\sec\theta\int_0^\infty \frac{ke^{-kf}}{k-K\sec^2\theta}\sin\ (kx\cos\theta)\cos\ (ky\sin\theta)dk\ d\theta$$
$$+4K^2\int_0^{\pi/2}\sec^3\theta e^{-Kf\sec^2\theta}\cos\ (Kx\ \sec\theta)\ \cos\ (Ky\sec^2\theta\ \sin\theta)d\theta$$
$$=\frac{2K}{\pi}\int_{-\pi/2}^{\pi/2}\sec\theta\int_0^\infty \frac{ke^{-kf}}{k-K\sec^2\theta}\sin(k\overline{w})dkd\theta$$
$$+2K^2\int_{-\pi/2}^{\pi/2}\sec^3\theta e^{-Kf\sec^2\theta}\cos(K\overline{w}\sec^2\theta)d\theta$$
$$=\frac{2K}{\pi}\int_{-\pi/2}^{\pi/2}\sec\theta\left[Re\int_0^\infty \frac{-ik}{k-K\sec^2\theta}e^{-k(f-i\overline{w})}dk\right]d\theta$$
$$+2K^2\int_{-\pi/2}^{\pi/2}\sec^3\theta e^{-Kf\sec^2\theta}\cos(K\overline{w}\sec^2\theta)d\theta \qquad (3)$$

where $\overline{w}=x\cos\theta+y\sin\theta$

Because eq. (3) is an even function of y, it is sufficient to consider the region $y\geq 0$ only.

3. Contour Integration

In view of the pole appearing in the inner integral of the first term of eq. (3), use will be made of the residue theorem. Defining the integral as I, i.e.

$$I=\int_0^\infty \frac{-ik}{k-K\sec^2\theta}e^{-k(f-i\overline{w})}dk \qquad (4)$$

a suitable contour should be sought to manipulate it. Consider a complex plane h, where

$$h=k+ik'$$
$$=re^{i\varphi}$$

then the integral I will be, in general, of the form

$$\int \frac{-ih}{h-K\sec^2\theta}e^{-h(f-i\overline{w})}dh \qquad (5)$$

on this complex plane. If a contour is chosen so that the integrand is analytic inside it and tends to zero on the large circular path, then the integral I will be, from the Cauchy-Goursat theorem and the residue theorem,

$$I=\int_0^\infty \frac{-ik}{k-K\sec^2\theta}e^{-k(f-i\overline{w})}dk$$
$$=\int_0^\infty \frac{-ire^{i\varphi}}{re^{i\varphi}-K\sec^2\theta}e^{-re^{i\varphi}(f-i\overline{w})}e^{i\varphi}dr+\mathrm{Res}(K\sec^2\theta)$$
$$=\frac{-i}{\sqrt{f^2+\overline{w}^2}e^{-i\varphi}}\int_0^\infty \frac{re^{-re^{i(\varphi-\alpha)}}}{r-K\sec^2\theta\sqrt{f^2+\overline{w}^2}e^{-i\varphi}}dr$$
$$+\mathrm{Res}(K\sec^2\theta) \qquad (6)$$

where $\alpha=\tan^{-1}(\overline{w}/f)$ and φ is the angle between the real axis and a radial line. As is obvious from this expression, if φ is chosen identical to α, the exponent will become a purely real quantity. Then, since $\sqrt{f^2+\overline{w}^2}e^{-i\alpha}=f-i\overline{w}$,

$$I=\frac{-i}{f-i\overline{w}}\int_0^\infty \frac{re^{-r}}{r-K\sec^2\theta(f-i\overline{w})}dr$$
$$+\mathrm{Res}(K\ \sec^2\theta) \qquad (7)$$

Depending on the sign of $\overline{w}$, the contour is chosen according to the following two classifications.

A) When $\overline{w}>0$

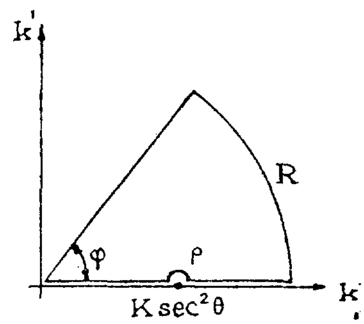

Fig. 1. Contour of integration when $\overline{w}>0$

The contour shown in Fig. 1 has been chosen to ensure that the real part of the exponent of eq. (5) is always negative on the large circular path R. The angle φ is given by

$$\varphi=\tan^{-1}(\overline{w}/f) \qquad (8)$$

which is positive. The residue on the small half circle ρ is

$$\mathrm{Res}(K\sec^2\theta)=\pi K\sec^2\theta e^{-K\sec^2\theta(f-i\overline{w})} \qquad (9)$$

B) When $\overline{w}<0$

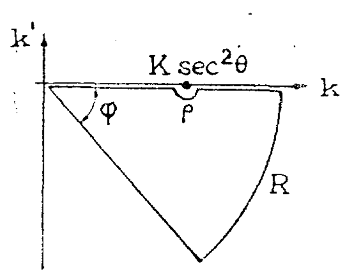

Fig. 2. Contour of integration when $\overline{w}<0$

The contour is chosen in the fourth quadrant to ensure that the integral eq. (5) will tend to zero on the circular path R. The angle φ is again given by eq. (8) and the residue is, in this case, because of the negative direction of integration on the half circle ρ,

$$\mathrm{Res}(K\sec^2\theta)=-\pi K\sec^2\theta e^{-K\sec^2\theta(f-i\varpi)} \tag{10}$$

As is clear from eq. (9) and eq. (10), the residue is given by the identical expression in both cases except for the sign and therefore the integral I may be uniquely expressed taking the sign into account as follows;

$$I=\frac{-i}{f-i\varpi}\int_0^\infty \frac{re^{-r}}{r-K\sec^2\theta(f-i\varpi)}dr+sgn(\varpi)\pi K\sec^2\theta e^{-K\sec^2\theta(f-i\varpi)} \tag{11}$$

where $sgn(\varpi)$ means

$sgn(\varpi)=\ 1$ if $\varpi>0$

$sgn(\varpi)=-1$ if $\varpi<0$

The real part of the integral I is

$$Re(I)=\frac{\varpi}{f^2+\varpi^2}\int_0^\infty \frac{r-2Kf\sec^2\theta}{(r-Kf\sec^2\theta)+K^2\varpi^2\sec^4\theta}re^{-r}dr+sgn(\varpi)\pi K\sec^2\theta e^{-Kf\sec^2\theta}\cos(K\varpi\sec^2\theta) \tag{12}$$

When ϖ is zero, $Re(I)$ must vanish as is obvious from eq. (3). This fact offers the limiting value of the first term of eq. (12) when ϖ approaches zero, i.e.

$$\left[\frac{\varpi}{f^2+\varpi^2}\int_0^\infty \frac{r-2Kf\sec^2\theta}{(r-Kf\sec^2\theta)^2+K^2\varpi^2\sec^4\theta}re^{-r}dr\right]_{\varpi=+0}=-\pi K\sec^2\theta e^{-K\sec^2\theta}$$

$$\left[\frac{\varpi}{f^2+\varpi^2}\int_0^\infty \frac{r-2Kf\sec^2\theta}{(r-Kf\sec^2\theta)^2+K^2\varpi^2\sec^4\theta}re^{-r}dr\right]_{\varpi=-0}=\pi K\sec^2\theta e^{-Kf\sec^2\theta} \tag{13}$$

This result will be very useful in programming.

4. Change of Integrating Parameters

The substitution of eq. (12) into eq. (3) yields the following expression for G_x

$$\begin{aligned}G_x=&\frac{2K}{\pi}\int_\Theta^{\pi/2}\sec\theta\frac{\varpi}{f^2+\varpi^2}\int_0^\infty \frac{r-2Kf\sec^2\theta}{(r-Kf\sec^2\theta)^2+K^2\varpi^2\sec^4\theta}re^{-r}drd\theta\\&+\frac{2K}{\pi}\int_{-\pi/2}^{\Theta}\sec\theta\frac{\varpi}{f^2+\varpi^2}\int_0^\infty \frac{r-2Kf\sec^2\theta}{(r-Kf\sec^2\theta)^2+K^2\varpi^2\sec^4\theta}re^{-r}drd\theta\\&+4K^2\int_\Theta^{\pi/2}\sec^3\theta e^{-Kf\sec^2\theta}\cos(K\varpi\sec^2\theta)d\theta\\=&\frac{2K}{\pi}\int_\Theta^{\pi/2}\cos\theta\frac{\varpi}{f^2+\varpi^2}\int_0^\infty \frac{r\cos^2\theta-2Kf}{(r\cos^2\theta-Kf)^2+K^2\varpi^2}re^{-r}drd\theta\\&+\frac{2K}{\pi}\int_{-\pi/2}^{\Theta}\cos\theta\frac{\varpi}{f^2+\varpi^2}\int_0^\infty \frac{r\cos^2\theta-2Kf}{(r\cos^2\theta-Kf)^2+K^2\varpi^2}re^{-r}drd\theta\\&+4K^2\int_\Theta^{\pi/2}\sec^3\theta e^{-Kf\sec^2\theta}\cos(K\varpi\sec^2\theta)d\theta\end{aligned}$$

where Θ is an angle which makes $\varpi=x\cos\theta+y\sin\theta$ zero, i.e.

$$\Theta=\tan^{-1}(-x/y).\left(-\frac{\pi}{2}\leq\Theta\leq\frac{\pi}{2}\right) \tag{15}$$

Because of the discontinuity of the integrand as shown by eq. (13), the range of θ-integration is divided into two parts in eq. (14).

To convert the integrals into more convenient forms, the following transformations are made:

A) for the first integral, let s be

$s=\sin\theta+\frac{x}{\sqrt{x^2+y^2}}$

$=\sin\theta+a$

where $a=x/\sqrt{x^2+y^2}$, then

$\varpi=\varpi_1=x\cos\theta+y\sin\theta$

$=x\sqrt{1-(s-a)^2}+y(s-a)$,

and $s=0$ and $(1+a)$ when $\theta=\Theta$ and $\pi/2$ repectively;

B) for the second integral, let q be

$q=-\sin\theta-\frac{x}{\sqrt{x^2+y^2}}$

$=-\sin\theta-a$

then $\varpi=\varpi_2=x\sqrt{1-(q+a)^2}-y(q+a)$, and $q=0$ and $(1-a)$ when $\theta=\Theta$ and $-\pi/2$ respectively;

C) for the last integral, let t be

$t=\tan\theta$

then $\sec^2\theta=1+t^2$

$\varpi\sec^2\theta=(x+yt)\sqrt{1+t^2}$

and $t=-x/y$ and ∞ when $\theta=\Theta$ and $\pi/2$ respectively.

Substituting these results into eq. (14), G_x becomes

$$\begin{aligned}G_x=&\frac{2K}{\pi}\int_0^{1+a}\frac{\varpi_1}{f^2+\varpi_1^2}\int_0^\infty \frac{r[1-(s-a)^2]-2Kf}{\{r[1-(s-a)^2]-Kf\}^2+K^2\varpi_1^2}re^{-r}drds\\&+\frac{2K}{\pi}\int_0^{1-a}\frac{\varpi_2}{f^2-\varpi_2^2}\int_0^\infty \frac{r[1-(q+a)^2]-2Kf}{\{r[1-(q+a)^2]-Kf\}^2+K^2\varpi_2^2}re^{-r}drdq\end{aligned}$$

$$+4K^2\int_{-x/y}^{\infty}\sqrt{1+t^2}\,e^{-Kf(1+t^2)}\cos[K\sqrt{1+t^2}\,(x+yt)]dt \quad (16)$$

5. Individual Cases

5.1 When $y>0$

A) when $x<0$

In this case $a=x/\sqrt{x^2+y^2}$ is negative and hence from eq. (16)

$$G_x=\frac{2K}{\pi}\int_0^{1+a}F_1(s)ds+\frac{2K}{\pi}\int_{1+a}^{1-a}\frac{\bar{w}_2}{f^2+\bar{w}_2^2}F_2(s)ds$$
$$+4K^2\int_{-x/y}^{\infty}\sqrt{1+t^2}e^{-Kf(1+t^2)}\cos[K\sqrt{1+t^2}\,(x+yt)]dt \quad (17)$$

where if $s\neq 0$

$$F_1(s)=\int_0^{\infty}\Big[\frac{\bar{w}_1}{f^2+\bar{w}_1^2}\,\frac{r[1-(s-a)^2]-2Kf}{\{r[1-(s-a)^2]-Kf\}^2+K^2\bar{w}_1^2}+\frac{\bar{w}_2}{f^2+\bar{w}_2^2}\,\frac{r[1-(s+a)^2]-2Kf}{\{r[1-(s+a)^2]-Kf\}^2+K^2\bar{w}_2^2}\Big]re^{-r}dr \quad (18a)$$

if $s=0$, $F_1(0)=0$ (18b)

and $$F_2(s)=\int_0^{\infty}\frac{r[1-(s+a)^2]-2Kf}{\{r[1-(s+a)^2]-Kf\}^2+K^2\bar{w}_2^2}re^{-r}dr \quad (19)$$

B) when $x=0$

Because $a=0$ and $\bar{w}_1=-\bar{w}_2$, the first two integrals in eq. (16) cancel each other and therefore

$$G_x=4K^2\int_0^{\infty}\sqrt{1+t^2}\,e^{-Kf(1+t^2)}\cos(Kyt\sqrt{1+t^2})dt. \quad (20)$$

C) when $x>0$

In this case, a is positive and so, from eq. (16)

$$G_x=\frac{2K}{\pi}\int_0^{1-a}F_1(s)ds+\frac{2K}{\pi}\int_{1-a}^{1+a}\frac{\bar{w}_1}{f^2+\bar{w}_1^2}F_3(s)ds$$
$$+4K^2\int_{-x/y}^{\infty}\sqrt{1+t^2}e^{-Kf(1+t^2)}\cos[K\sqrt{1+t^2}\,(x+yt)]dt \quad (21)$$

where $F_1(s)$ is given by eq. (18a) and eq. (18b), and

$$F_3(s)=\int_0^{\infty}\frac{r[1-(s-a)^2]-2Kf}{\{r[1-(s-a)^2]-Kf\}^2+K^2\bar{w}_1^2}re^{-r}dr. \quad (22)$$

It must be noted that the first two integrals of eq. (16) should be treated with care when $\bar{w}_1$ and $\bar{w}_2$ are noughts at the lower limits of the integrating parameters s and q respectively, because in such cases the two inner integrands have poles between the integration range of r. However this trouble can be overcome by suitable use of the known fact shown by eq. (13) —suitable, because the part corresponding to $Re(I)$, eq. (12), has been altered due to the series of manipulations stemming from the second part of eq. (14).

As a way of doing this, if the first two integrals of eq. (16) are combined on their overlapping integrating intervals, as shown by the first terms of eq. (17) and eq. (21) with the definition of $F_1(s)$ by eq. (18a), a careful study of eq. (13) reveals that the combined integrand vanishes at the lower integrating limit, i.e. $F_1(0)=0$, which is the source of trouble. Then the integrand $F_1(s)$ becomes a well-behaved function within the integrating interval. The unoverlapped part of the integral, the second terms of eq. (17) and eq. (21), pose no specific difficulty.

5.2 When $y=0$

A) when $x<0$

In this case, $\bar{w}$ is given by $\bar{w}=x\cos\theta$ and from eq. (15) Θ is $\pi/2$. Then the first and third integrals of eq. (14) vanish leaving the second integral only. Hence, considering that the integrand is an even function of θ,

$$G_x=\frac{4K}{\pi}\int_0^{\pi/2}\cos\theta\frac{\bar{w}}{f^2+\bar{w}^2}\int_0^{\infty}\frac{r\cos^2\theta-2Kf}{(r\cos^2\theta-Kf)^2+K^2\bar{w}^2}re^{-r}\,dr\,d\theta$$
$$=\frac{4K}{\pi}\int_0^1\frac{\bar{w}}{f^2+\bar{w}^2}\int_0^{\infty}\frac{r(1-s^2)-2Kf}{[r(1-s^2)-Kf]^2+K^2\bar{w}^2}re^{-r}\,dr\,ds \quad (23)$$

using the transformation $s=\sin\theta$ to obtain the latter expression, and then $\bar{w}=x\sqrt{1-s^2}$.

B) when $x=0$

It is simplest to consider this case from eq. (3). By the use of the transformation $t=\tan\theta$ with $\bar{w}=0$,

$$G_x=4K^2\int_0^{\infty}\sqrt{1+t^2}e^{-Kf(1+t^2)}dt. \quad (24)$$

C) when $x>0$

Because Θ is $-\pi/2$ from eq. (15), the second integral of eq. (14) disappears, and because integrands of the first and third integrals are even functions of θ

Table 1. Wave Configuration of a Havelock Source (Depth of Sonrce F=0.98cm, Wave Number K=0.102/cm)

(UNITS IN CM)

X \ Y	0.0	4.92	9.83	14.75	19.67	24.58	29.50	34.42	39.34	44.25	49.17	54.09	59.00	63.92	68.84	73.75
-24.59	0.0002	[illegible]	0.0002	0.0001	0.0001	0.0001										
-19.67	[illegible]	[illegible]	[illegible]	0.0002	0.0002	0.0001										
-14.75	[illegible]	[illegible]	[illegible]	0.0002	0.0002	0.0002										
-9.83	[illegible]	[illegible]	[illegible]	0.0003	0.0003	0.0002										
-4.92	0.0012	[illegible]	0.0007	0.0005	0.0003	0.0003										
0.0	0.0227	[illegible]	0.0011	0.0007	0.0004	0.0003	0.0002									
4.92	[illegible]	[illegible]	0.0017	0.0009	[illegible]	0.0004	0.0002									
9.83	[illegible]	[illegible]	0.0024	0.0012	0.0006	0.0004	0.0002	0.0002								
14.75	-.0101	[illegible]	0.0026	0.0012	0.0007	0.0004	0.0002	0.0001								
19.67	[illegible]	[illegible]	0.0011	0.0009	0.0005	0.0002	0.0001	0.0001	0.0000							
24.59	[illegible]	[illegible]	-.0027	-.0003	-.0001	-.0000	-.0001	-.0001	-.0001							
29.50	[illegible]	-.0115	-.0092	-.0025	-.0010	-.0005	-.0003	-.0002	-.0002	-.0001						
34.42	-.0011	[illegible]	-.0116	-.0051	-.0023	-.0012	-.0007	-.0005	-.0003	-.0002						
39.34	0.0012	0.0049	-.0075	-.0056	-.0036	-.0019	-.0011	-.0007	-.0005	-.0003	-.0002					
44.25	[illegible]	[illegible]	0.0049	-.0055	-.0041	-.0024	-.0015	-.0009	-.0006	-.0004	-.0003					
49.17	0.0039	[illegible]	0.0146	[illegible]	-.0030	-.0023	-.0015	-.0010	-.0007	-.0005	-.0003	-.0002				
54.09	0.0039	[illegible]	0.0092	0.0080	0.0003	-.0013	-.0012	-.0009	-.0006	-.0004	-.0004	-.0002				
59.00	0.0031	[illegible]	-.0042	0.0127	0.0050	0.0010	-.0002	-.0004	-.0004	-.0003	-.0002	-.0002	-.0001			
63.92	0.0016	0.0020	-.0037	0.0085	0.0088	0.0038	0.0014	0.0004	0.0001	-.0000	-.0001	-.0001	-.0001			
68.84	-.0002	[illegible]	0.0066	-.0130	0.0075	0.0061	0.0041	0.0015	0.0007	0.0004	0.0002	0.0001	0.0000	0.0000		
73.76	-.0015	[illegible]	0.0007	-.0104	0.0025	0.0061	0.0043	0.0025	0.0014	0.0008	0.0004	0.0003	0.0002	0.0001		
78.67	-.0029	-.0024	-.0071	-.0054	-.0065	0.0026	0.0039	0.0029	0.0019	0.0011	0.0007	0.0004	0.0003	0.0002	0.0001	
83.59	-.0033	[illegible]	-.0009	0.0037	-.0116	-.0036	0.0015	0.0023	0.0018	0.0013	0.0009	0.0006	0.0004	0.0002	0.0002	
88.51	-.0025	-.0029	-.0066	0.0013	-.0077	-.0092	-.0027	0.0004	0.0011	0.0010	0.0008	0.0006	0.0004	0.0003	0.0002	0.0001
93.42	-.0010	-.0018	-.0057	-.0080	0.0018	-.0101	-.0069	-.0025	-.0004	0.0003	0.0004	0.0004	0.0003	0.0002	0.0002	0.0001
98.34	[illegible]	[illegible]	0.0072	-.0047	0.0065	-.0043	-.0084	-.0053	-.0024	-.0009	-.0007	0.0000	0.0001	0.0001	0.0001	0.0001
103.26	0.0010	0.0019	-.0002	0.0055	0.0012	0.0044	-.0055	-.0063	-.0041	-.0023	-.0011	-.0005	-.0002	-.0001	-.0000	0.0000
108.17	0.0024	0.0021	0.0070	0.0037	-.0049	0.0090	0.0013	-.0044	-.0046	-.0032	-.0019	-.0011	-.0006	-.0003	-.0002	-.0001
113.09	0.0027	[illegible]	[illegible]	-.0017	-.0004	0.0053	0.0060	0.0004	-.0030	-.0031	-.0023	-.0015	-.0010	-.0006	-.0004	-.0002
118.01	0.0026	[illegible]	[illegible]	0.0047	0.0085	-.0020	0.0097	0.0061	0.0007	-.0017	-.0020	-.0014	-.0011	-.0008	-.0005	-.0003
122.93	0.0019	[illegible]	0.0026	0.0051	0.0061	-.0036	0.0047	0.0092	0.0049	0.0010	-.0007	-.0011	-.0010	-.0008	-.0005	-.0004
127.84	0.0008	[illegible]	[illegible]	-.0131	-.0039	0.0024	-.0028	0.0072	0.0077	0.0041	0.0014	0.0001	-.0003	-.0005	-.0004	-.0003
132.76	-.0005	[illegible]	[illegible]	[illegible]	-.0032	0.0065	-.0059	0.0007	0.0069	0.0061	0.0035	0.0016	0.0005	0.0000	-.0002	-.0002
137.68	-.0010	-.0016	[illegible]	-.0003	0.0037	0.0005	-.0020	-.0050	0.0022	0.0056	0.0048	0.0030	0.0015	0.0007	0.0003	0.0001
142.59	-.0002	[illegible]	[illegible]	-.0050	-.0013	-.0079	0.0032	-.0078	-.0043	0.0021	0.0041	0.0036	0.0024	0.0015	0.0008	0.0004
147.51	[illegible]	[illegible]	[illegible]	-.0031	-.0075	-.0057	0.0021	-.0035	-.0055	-.0032	0.0013	0.0025	0.0026	0.0019	0.0017	0.0008
152.43	[illegible]	[illegible]	[illegible]	-.0034	-.0003	0.0025	-.0049	0.0022	-.0075	-.0074	-.0028	0.0005	0.0016	0.0017	0.0015	0.0010
157.34	[illegible]	[illegible]	[illegible]	-.0099	0.0017	0.0017	-.0077	0.0032	-.0018	-.0033	-.0064	-.0027	-.0003	0.0008	0.0010	0.0009
162.26	[illegible]	[illegible]	[illegible]	-.0005	-.0040	-.0050	-.0011	-.0013	0.0040	-.0042	-.0074	-.0054	-.0026	-.0008	0.0001	0.0005
167.18	[illegible]	[illegible]	[illegible]	[illegible]	0.0017	-.0014	0.0065	-.0051	0.0051	0.0022	-.0046	-.0061	-.0045	-.0025	-.0011	-.0003
172.10	[illegible]	[illegible]	[illegible]	0.0029	0.0043	0.0066	0.0040	-.0018	0.0011	0.0057	0.0009	-.0040	-.0048	-.0037	-.0022	-.0012
177.01	[illegible]	[illegible]	[illegible]	[illegible]	-.0009	0.0027	-.0025	0.0057	[illegible]	0.0051	0.0062	0.0006	-.0029	-.0036	-.0029	-.0014
181.93	[illegible]	[illegible]	[illegible]	0.0024	0.0036	-.0023	-.0005	0.0076	-.0016	0.0015	0.0079	0.0054	0.0003	-.0019	-.0025	-.0021
186.85	[illegible]	[illegible]	[illegible]	[illegible]	0.0021	0.0035	0.0061	0.0012	0.0039	-.0028	0.0049	0.0075	0.0048	0.0011	-.0009	-.0016
191.76	0.0002	[illegible]	[illegible]	0.0009	-.0016	0.0034	0.0033	-.0045	0.0066	-.0025	-.0007	0.0062	0.0071	0.0043	0.0015	-.0002
196.68	-.0008	[illegible]	[illegible]	-.0012	0.0015	-.0042	-.0048	-.0015	0.0020	[illegible]	-.0045	0.0011	0.0061	0.0061	0.0038	0.0017
201.60	-.0016	[illegible]	[illegible]	[illegible]	-.0030	-.0017	-.0030	0.0036	[illegible]	[illegible]	-.0039	-.0043	0.0019	0.0053	0.0050	0.0034
206.51	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.0008	0.0025	[illegible]	-.0043	[illegible]	[illegible]	-.0045	-.0035	0.0017	0.0042	0.0040

Table 2. Wave Configuration of a Havelock Source (Depth of Source F=1.97cm, Wave Number K=0.102/cm)

(UNITS IN CM) DEPTH OF SOURCE F=1.97 CM

X \ Y	0.0	4.92	9.83	14.75	19.67	24.58	29.50	34.42	39.34	44.25	49.17	54.09	59.00	63.92	68.84	73.75
-24.59	0.0002	0.0002	0.0002	0.0001	0.0002	0.0001										
-19.67	0.0002	0.0001	0.0001	0.0002	0.0001	0.0001										
-14.75	0.0003	0.0001	0.0004	0.0002	0.0002	0.0002										
-9.83	0.0006	0.0002	0.0003	0.0003	0.0003	0.0002										
-4.92	0.0014	0.0056	0.0007	0.0005	0.0003	0.0003										
0.0	0.0115	0.0026	0.0011	0.0007	0.0004	0.0003	0.0002									
4.92	0.0097	0.0050	0.0016	0.0009	0.0006	0.0004	0.0002									
9.83	-.0056	0.0033	0.0023	0.0011	0.0006	0.0004	0.0002	0.0001								
14.75	-.0095	0.0014	0.0019	0.0011	0.0006	0.0003	0.0002	0.0001								
19.67	-.0078	-.0069	0.0004	0.0005	0.0004	0.0002	0.0001	0.0000	0.0000							
24.59	-.0056	-.0115	-.0033	-.0007	-.0003	-.0001	-.0001	-.0001	-.0001							
29.50	-.0034	-.0078	-.0073	-.0026	-.0011	-.0006	-.0004	-.0002	-.0002	-.0001						
34.42	-.0011	0.0011	-.0082	-.0047	-.0022	-.0012	-.0007	-.0005	-.0003	-.0002						
39.34	0.0010	0.0036	-.0067	-.0056	-.0032	-.0018	-.0011	-.0007	-.0005	-.0003	-.0002					
44.25	0.0027	0.0017	0.0037	-.0039	-.0035	-.0022	-.0014	-.0009	-.0006	-.0004	-.0003					
49.17	0.0036	0.0035	0.0094	0.0008	-.0022	-.0019	-.0014	-.0009	-.0006	-.0004	-.0003	-.0002				
54.09	0.0036	0.0039	0.0066	0.0067	0.0008	-.0008	-.0009	-.0007	-.0005	-.0004	-.0003	-.0002				
59.00	0.0028	0.0028	0.0002	0.0095	0.0046	0.0012	0.0001	-.0003	-.0003	-.0002	-.0002	-.0001	-.0001			
63.92	0.0015	0.0016	-.0007	0.0060	0.0072	0.0036	0.0015	0.0005	0.0002	0.0000	-.0000	-.0001	-.0000			
68.84	-.0001	-.0001	0.0018	-.0015	0.0064	0.0052	0.0029	0.0015	0.0007	0.0004	0.0002	0.0001	0.0000	0.0000		
73.76	-.0016	-.0016	-.0005	-.0062	0.0015	0.0048	0.0037	0.0023	0.0013	0.0007	0.0004	0.0003	0.0001	0.0001		
78.67	-.0026	-.0026	-.0039	-.0041	-.0050	0.0016	0.0031	0.0025	0.0017	0.0011	0.0007	0.0004	0.0003	0.0002	0.0001	
83.59	-.0030	-.0030	-.0029	-.0000	-.0084	-.0033	0.0008	0.0018	0.0016	0.0011	0.0008	0.0005	0.0003	0.0002	0.0001	
88.51	-.0026	-.0026	-.0022	-.0008	-.0058	-.0074	-.0027	-.0000	0.0008	0.0009	0.0007	0.0005	0.0003	0.0002	0.0002	0.0001
93.42	-.0017	-.0017	-.0022	-.0040	0.0001	-.0077	-.0059	-.0025	-.0006	0.0001	0.0003	0.0003	0.0003	0.0002	0.0001	0.0001
98.34	-.0004	-.0004	-.0004	-.0023	0.0031	-.0033	-.0067	-.0046	-.0023	-.0010	-.0003	-.0000	0.0001	0.0001	0.0001	0.0001
103.26	0.0009	0.0009	0.0009	0.0023	0.0010	0.0029	-.0041	-.0052	-.0036	-.0021	-.0011	-.0005	-.0002	-.0001	-.0000	-.0000
108.17	0.0019	0.0019	0.0018	0.0025	-.0014	0.0061	0.0012	-.0034	-.0038	-.0026	-.0018	-.0010	-.0006	-.0003	-.0002	-.0001
113.09	0.0024	0.0024	0.0025	0.0012	0.0010	0.0041	0.0061	0.0007	-.0023	-.0026	-.0020	-.0014	-.0009	-.0006	-.0004	-.0002
118.01	0.0024	0.0024	0.0024	0.0027	0.0048	-.0000	0.0073	0.0051	0.0008	-.0012	-.0016	-.0014	-.0010	-.0007	-.0005	-.0003
122.93	0.0017	0.0017	0.0018	0.0026	0.0037	-.0010	0.0038	0.0073	0.0043	0.0012	-.0004	-.0008	-.0008	-.0007	-.0005	-.0003
127.84	0.0007	0.0007	0.0008	0.0003	-.0006	0.0017	-.0012	0.0056	0.0063	0.0037	0.0014	0.0002	-.0003	-.0004	-.0004	-.0003
132.76	-.0005	-.0004	-.0004	-.0002	-.0013	0.0032	-.0035	0.0007	0.0054	0.0052	0.0032	0.0015	0.0006	0.0001	-.0001	-.0001
137.68	-.0015	-.0015	-.0014	-.0011	0.0002	-.0002	-.0016	-.0042	0.0016	0.0045	0.0040	0.0027	0.0015	0.0007	0.0003	0.0001
142.59	-.0021	-.0021	-.0021	-.0024	-.0016	-.0044	0.0010	-.0056	-.0034	0.0014	0.0033	0.0030	0.0022	0.0013	0.0008	0.0004
147.51	-.0022	-.0022	-.0022	-.0021	-.0036	-.0038	0.0002	-.0030	-.0066	-.0029	0.0008	0.0022	0.0022	0.0016	0.0011	0.0007
152.43	-.0018	-.0018	-.0018	-.0019	-.0017	-.0003	-.0034	0.0006	-.0058	-.0062	-.0026	0.0001	0.0013	0.0014	0.0012	0.0008
157.34	-.0009	-.0010	-.0010	-.0011	-.0005	-.0001	-.0045	0.0013	-.0018	-.0065	-.0054	-.0025	-.0004	0.0005	0.0008	0.0007
162.26	0.0001	0.0001	0.0000	0.0000	-.0008	-.0018	-.0008	-.0010	0.0023	-.0033	-.0060	-.0047	-.0024	-.0008	0.0000	0.0003
167.18	0.0011	0.0011	0.0010	0.0009	0.0009	-.0000	0.0031	-.0027	0.0033	0.0015	-.0036	-.0050	-.0039	-.0023	-.0011	-.0003
172.10	0.0018	0.0018	0.0017	0.0018	0.0021	0.0030	0.0026	-.0004	0.0011	0.0048	0.0008	-.0031	-.0040	-.0032	-.0020	-.0011
177.01	0.0020	0.0020	0.0020	0.0020	0.0016	0.0023	0.0002	0.0038	-.0010	0.0047	0.0049	0.0007	-.0023	-.0030	-.0025	-.0017
181.93	0.0018	0.0018	0.0018	0.0019	0.0020	0.0008	0.0009	0.0048	-.0002	0.0016	0.0062	0.0045	0.0009	-.0014	-.0021	-.0018
186.85	0.0011	0.0011	0.0011	0.0012	0.0014	0.0016	0.0031	0.0014	0.0029	-.0012	0.0039	0.0063	0.0041	0.0012	-.0006	-.0013
191.76	0.0002	0.0002	0.0002	0.0003	0.0002	0.0012	0.0016	-.0017	0.0041	-.0012	-.0001	0.0049	0.0058	0.0038	0.0014	-.0000
196.68	-.0007	-.0007	-.0007	-.0007	-.0004	-.0012	-.0018	-.0008	0.0011	0.0011	-.0029	0.0010	0.0049	0.0051	0.0034	0.0016
201.60	-.0015	-.0015	-.0015	-.0014	-.0015	-.0014	-.0019	0.0009	-.0031	0.0022	-.0027	-.0031	0.0014	0.0043	0.0043	0.0039
206.51	-.0018	-.0018	-.0018	-.0018	-.0018	-.0013	-.0005	-.0008	-.0040	0.0001	-.0004	-.0046	-.0028	0.0013	0.0034	0.0034

Fig. 3. Wave Pattern (Source Submergence 0.98cm)

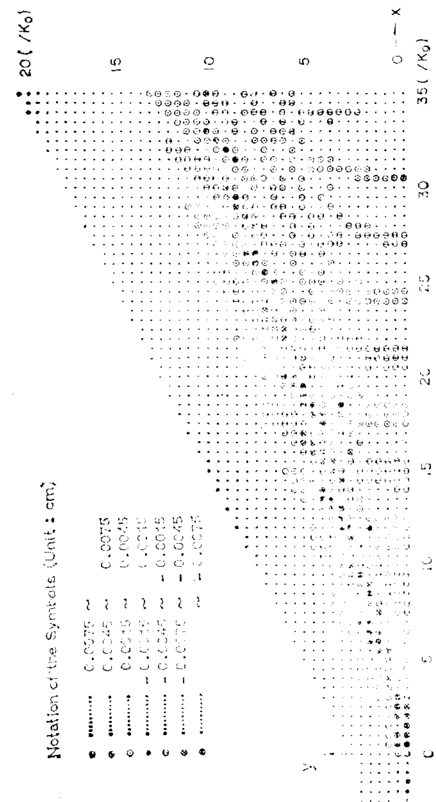

Fig. 4. Wave Pattern (Source Submergence 1.97cm)

$$G_x=\frac{4K}{\pi}\int_0^{\pi/2}\cos\theta\,\frac{\bar{w}}{f^2+\bar{w}^2}\int_0^\infty\frac{r\cos^2\theta-2Kf}{(r\cos^2\theta-Kf)^2+K^2\bar{w}^2}re^{-r}\,dr\,d\theta$$
$$+8K^2\int_0^{\pi/2}\sec^3\theta e^{-Kf\sec^2\theta}\cos(K\bar{w}\sec^2\theta)d\theta$$
$$=\frac{4K}{\pi}\int_0^1\frac{\bar{w}}{f^2+\bar{w}^2}\int_0^\infty\frac{r(1-s^2)-2Kf}{[r(1-s^2)-Kf]^2+K^2\bar{w}^2}re^{-r}\,dr\,ds$$
$$+8K^2\int_0^\infty\sqrt{1+t^2}\cos(Kx\sqrt{1+t^2})e^{-Kf(1+t^2)}dt \tag{25}$$

with the transformation $s=\sin\theta$ and $t=\tan\theta$ in the first and second integrals respectively to obtain the latter expression for G_x. $\bar{w}$ is given by $\bar{w}=x\sqrt{1-s^2}$.

6. Remarks on the Computation and Further Comments

The expressions for the x-derivative of the Green function given for the individual cases in the previous section can readily be used for numerical calculation of its value at an arbitrary point on the water plane with some standard quadrature. The most efficient integrating scheme adopted in the quadrature may not be the same for each integral, and this can be determined only by experience.

The merit of the chosen contour and the subsequent developments seems to be that the evaluation of the integrand is very economical. It is to be noted that the poles not fully removed by the contour integration (see eq. 14) have been finally removed in eq. 18.

The examples of the actual numerical computation are presented in the form of wave elevation, this being obtained by eq. 2, in Table 1, Table 2, and for better visual effect, in Fig. 3 and Fig. 4. The wave contours could also be plotted if necessary. The general appearance of the wave system seems quite realistic.

The inner of the double integrals has been evaluated by Gauss-Laguerre quadrature and the outer by Clenshaw-Curtis method. A quadrature based on Simpson's method with interval subdivision proved adequate for the evaluation of the single integral.

The time needed to compute the value at a single point depends on the position of the point, the submergence of the source and the wave number. On average, some fractions of a second of CUP time was spent in the cases of the example presented.

The y-and z-derivatives of the Green function can also be manipulated in much the same fashion as the x-derivative to provide the means of evaluating the velocity components induced by the singularity. Plotting the values of those derivatives however would not suggest as direct analogy to the reality as that of the x-derivative.

Quite apart from the manipulation of the Green function, it seems definitely impractical to use the singularity as the basic flow generating mechanism for the ship wave-making problem by Hess and Smith's [8] type of approach at the present stage of computing technology. The singularity and the method of evaluating its flow inducing property just exist there perhaps waiting for the cheaper in CPU time and faster computing machine. It should also be born in mind that the singularity offers the soluting of only the linearised problem to which the reason of discrepancy between the experimental results and the theoretical predictions is partially attributed.

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