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Calculation of Wave-making Resistance by Guilloton's Method Applied to Slender Ships*

by

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Synopsis

This paper deals with Guilloton's method for wave-making resistance calculation. Ship is considered as slender in this paper. Guilloton's method requires a large and fast computer, while mini-computer is good enough for the present method. Present method is practical as well, as prismatic curves along with other principal particulars are requirements for the calculation.

Unless the ship is thin, Z-transformation is difficult to carry out, but this can be done smoothly in the present method by considering the flow around the bottom of the ship

As an example of this method, corresponding real hulls of Maruo's least wave-making resistance ship forms are calculated.

Introduction

Wave-making resistance occupies a large part of the total resistance for small or fast ships, moreover, it varies with the ship form. Therefore, it is important to investigate the relation between wave-making resistance and ship form especially from the energy saving point of view.

The theory of wave-making resistance is first introduced by Michell in 1898 [1]. Michell's theory is also known as the thin ship theory for the apparent reason. Havelock and others developed the theory of wave-making resistance after Michell and established the foundation of the linear theory.

After then, Wigley, Weinblum and others examined the theory by experiments. As a result, it has been known that the theory agrees well with the experiment only for ships whose breadth is extremely small compared with the length. Otherwise it does not agree quantitatively.

Recently, researches are directed to the nonlinear theories, such as the perturbation analysis method, low speed theory and strained coordinates method by Guilloton, etc.

These nonlinear theories improved considerably the defects in conventional theories. So agreement with the experiment is at least better than Michell's or Havelock's theories.

Guilloton's method is first introduced by Guilloton in 1964 [2] without theoretical background. Later in 1974, it was verified by Noblesse [3] and Dagan [4] that the method satisfies boundary conditions to the second order, while continuity and irrotational condition are satisfied only to the first order. Guilloton's method for wave-making resistance calculation drew serious attention to researchers in this field as well as to engineers who have to calculate wave-making resistance, because it is more simple and practical than other nonlinear theories. Further, the conventional theories of Havelock and Michell can be applied to Guilloton's method. Calculations by

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the present method were carried out by Emerson, Gadd [5], Kitazawa and others, and it is recognized that Guilloton's method is accurate and practical.

However, numerical calculation is still enormous and there are difficulties such as handling of flow around the bottom of ship and often the calculation would not converge. This calculation has been carried out for several mathematical models at Nagasaki Institute of Applied Science [6] and it has been pointed out that there are good agreements between the present result and experiment in some cases, and that the greatest problem lies on the handling of the flow around the bottom of ship.

In this paper, ship is taken as slender. Wave-making resistance of a ship is mostly affected by gradient of the prismatic curve and not by the shape of frame lines [7]. With this, it is assumed that the wave-making resistance of the transformed ship whose length, draft and sectional area curve are the same as those of the given ship is almost equal. The precision of this assumption is studied.

Calculation becomes much simpler and the difficulty of handling the flow around the bottom is removed. The other merit is that input for the calculation are prismatic curves along with main particulars.

Finally, the Guilloton's transformation is performed for Maruo's least wave resistance ship forms [8] as an application example.

Theoretical Approach

A rectangular Cartesian coordinate system is used with the origin at F.P. The Z-axis is vertically upward and X-axis is positive in the direction of the free stream at infinity as shown in Fig. 1.

Let q (u,v,w) be the velocity at a point $Q(X, Y, Z)$, with M as a point on the free surface and $Z=E(X, Y)$ as the free surface elevation. Then free surface condition is

$$w(M)=u(M)\frac{\partial E(X,Y)}{\partial X}+v(M)\frac{\partial E(X,Y)}{\partial Y} \quad (1)$$

The condition that the pressure is constant over the free surface is

$$u^2(M)+v^2(M)+w^2(M)+2gE(X,Y)=U^2 \quad (2)$$

N be a point on the hull surface and $Y=F(X,Z)$ be the equation of hull. Hull surface condition is

$$v(N)=u(N)\frac{\partial F(X,Z)}{\partial X}+w(N)\frac{\partial F(X,Z)}{\partial Z} \quad (3)$$

The equation of continuity is

$$\frac{\partial u(Q)}{\partial X}+\frac{\partial v(Q)}{\partial Y}+\frac{\partial w(Q)}{\partial Z}=0 \quad (4)$$

The equation of irrotationality is

$$\frac{\partial u(Q)}{\partial Y}-\frac{\partial v(Q)}{\partial X}=\frac{\partial v(Q)}{\partial Z}-\frac{\partial w(Q)}{\partial Y}$$
$$=\frac{\partial w(Q)}{\partial X}-\frac{\partial u(Q)}{\partial Z}=0 \quad (5)$$

Fluid is assumed as non-viscous. Besides this, the radiation condition that waves do not radiate ahead of ship has to be satisfied. The velocity potential ϕ is chosen as

$$q=-\nabla\Phi(X,Y,Z)=U\vec{i}-\nabla\phi$$

where $\vec{i}$, $\vec{j}$, $\vec{k}$, are unit vectors.

Ship is assumed to be thin and terms of perturbed velocities greater than second order are neglected in (1), (2) and (3). They become

$$-\frac{\partial\phi}{\partial Z}=U\frac{\partial E(X,Z)}{\partial X} \quad \text{on } Z=0 \quad (1)'$$

$$-U\frac{\partial\phi}{\partial X}+gE(X,Y)=0 \quad \text{on } Z=0 \quad (2)'$$

$$-\frac{\partial\phi}{\partial Y}=U\frac{\partial F(X,Z)}{\partial X} \quad \text{on } Y=0 \quad (3)'$$

Michell [1] obtained a solution satisfying (1)′, (2)′, and (3)′ along with the radiation condition. It is

$$\phi=-\frac{1}{2\pi}\iint_S\frac{\partial F(X,Z)}{\partial X}\,G(X,0,Z;\bar{X},0,\bar{Z})\,d\bar{X}d\bar{Z} \quad (6)$$

where the expression of G is omitted and the symbol S refers the centerplane of hull.

Later, Havelock [9] arrievd at the same solution by obtaining the potential of a point source and then distributing the source on the centerplane of ship

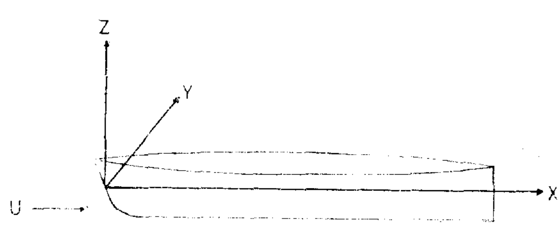

Fig. 1. Coordinate System

with source strength

$$\sigma = \frac{1}{2\pi} \frac{\partial F(X, Z)}{\partial X} \quad (0 \leq X \leq L) \tag{7}$$

When the velocity potential is found, free surface elevation is given from (2)′ as

$$E = \frac{U}{g} \frac{\partial \phi}{\partial X} \text{ on } Z=0 \tag{8}$$

wave-making resistance R [10] is

$$R = \frac{4\rho g^2}{\pi U^2} \int_1^\infty (I^2 + J^2) \frac{\lambda^2}{\sqrt{\lambda^2 - 1}} d\lambda \tag{9}$$

where

$$\left.\begin{matrix} I \\ J \end{matrix}\right\} = \iint_S \frac{\partial F(X, Z)}{\partial X} \begin{matrix} \cos \\ \sin \end{matrix} (K_o X \lambda) e^{K_o Z \lambda^2} \, dXdZ \tag{10}$$

$$K_o = \frac{g}{U^2}$$

Wave-making resistance and wave profile obtained by the present theory agrees with experiment for very thin ships, but it is not the case for ordinary ships. Guilloton [2] presented the method for wave-making resistance calculation by appropriately transforming the position of the source obtained by Havelock's theory. His result agrees exceptionally well with the experiment. Emerson, Gadd [5], Kitazawa and others presented calculated results that seem to support the Guilloton's presentation.

Following is the brief review of the above method in accordance to Gadd [5]. It is assumed that same amount of time is required for a fluid particle to travel from 0 to a (X, f, Z) through the path of the isobar and to $b_0(X_0, 0, Z_0)$ on the centerplane. The velocity of fluid paticle through the isobar is $U+u$, while on the centerplane is U (Fig. 2).

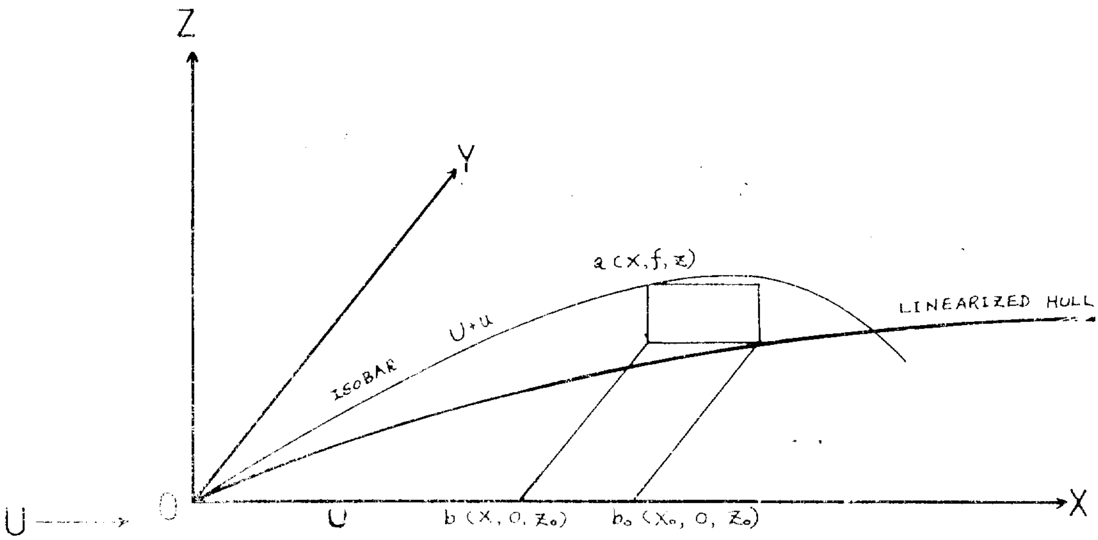

Fig. 2. Schematic Illustration of the Transformation

The relation between the real hull and the linearized hull in X-axis is

$$X = X_0 + \xi = \int_o^{b_o} \frac{1 + \frac{u}{U}}{1 + \alpha^2/2} dX_0 \tag{11}$$

where X is that of the real hull, X_o is of the linearized hull and $\alpha = \frac{\partial Y}{\partial X}$.

The lineaized hull is mentioned later. The relation in Y-axis

$$Y = 2\pi \int_o^{b_o} \sigma_o dX_0 \tag{12}$$

where σ_0 is the Harvelock's source strength.

The relation in Z-axis

$$Z = Z_o + \zeta = Z_o - \frac{uU}{g} \tag{13}$$

ξ and ζ of (11) and (13) are X-axis and Z-axis strainings of Guilloton's transformation. Guilloton's source strength is then

$$\sigma_o=\frac{1}{2\pi}\left\{\frac{1+\frac{u}{U}}{1+\alpha^2/2}\frac{\partial Y}{\partial X}-\frac{U}{g}\frac{\partial U_0}{\partial X_o}\frac{\partial Y}{\partial Z}\right\} \tag{14}$$

The linearized hull is obtained by the iteration procedure.

Noblesse [3] and Dagan [4] used the strained coordinates method to investigate Guilloton's method theoretically.

The perturbation parameter ϵ is the breadth-length ratio(=B/L). Velocity(u, v, w), coordinate(X, Y, Z), free surface elevation E and the hull function F are expanded with respect to ϵ as followings [3].

$$\left.\begin{aligned}
u(Q)&=u+u_1(Q_0)+u_2(Q_0)+\cdots\\
v(Q)&=v_1(Q_0)+v_2(Q_0)+\cdots\\
w(Q)&=w_1(Q_0)+w_2(Q_0)+\cdots\\
X&=X_0+X_1(Q_0)+X_2(Q_0)+\cdots\\
Y&=Y_0+Y_1(Q_0)+Y_2(Q_0)+\cdots\\
Z&=Z_o+Z_1(Q_0)+Z_2(Q_0)+\cdots\\
E(X, Y)&=E_1(X_0, Y_0)+E_2(Xo, Yo)+\cdots\\
F(X, Y)&=F_1(X_0, Y_0)+F_2(X_0, Y_0)+\cdots
\end{aligned}\right\} \tag{15}$$

Q_0 is the coordinate of a water particle when there is no disturbance due to the ship and Q is the corresponding coordinates after being perturbed. ϵ is omitted and subscript number indicates the order.

Substituting (15) into (1) to (5) and arranging them in accordance to each order of ϵ, equations(1) to (5) are established in the corresponding order. It is omitted in this paper.

When X, Y and Z are chosen as the following,

$$X=X_0+X_1,\quad Y=Y_0+Y_1,\quad Z=Z_0+Z_1$$

$$\phi=\frac{1}{2\pi}\iint_S G(X_0, 0, Z_0;\ \bar{X}_0, 0, \bar{Z}_0)\frac{\partial F_1(\bar{X}_0, \bar{Z}_0)}{\partial X_0}\,d\bar{X}_0 d\bar{Z}_0 \tag{16}$$

Boundary conditions (1), (2) and (3) are satisfied to the second order while field equations (4) to (5) are satisfied to the first order.

Wave-making resistance becomes

$$R=\frac{4\rho g^2}{\pi U^2}\int_1^\infty (I^2+J^2)\frac{\lambda^2}{\sqrt{\lambda^2-1}}\,d\lambda \tag{17}$$

where

$$\left.\begin{matrix}I\\J\end{matrix}\right\}=\iint_S\frac{\partial F_1(X_0, Z_0)}{\partial X_0}\begin{matrix}\cos\\\sin\end{matrix}(K_0X_0\lambda)e^{K_0Z_0\lambda^2}\,dX_0dZ_0$$

F_1 in the above equations are equivalent to the linearized hull mentioned in the last section. That is

$$F_1(X_0, Z_0)\ F\ (X, Z) \tag{18}$$

The gradient of the linearized hull of a thin ship is, by using Gadd's expression [5] of Guilloton's method, as below.

$$\frac{\partial Y_0(X_0)}{\partial X_0}=\left(\frac{\partial Y}{\partial X}\Big|_{X_0}+\xi\frac{\partial^2 Y}{\partial X^2}\Big|_{X_0}\right)\left(1+\frac{\partial\xi}{\partial X}\right) \tag{19}$$

It should be noted that ships of small $\frac{\partial Y}{\partial X}$ at bow is not suitable for the present method as can be seen above. $\xi\frac{\partial^2 Y}{\partial X^2}\Big|_{X_0}$ is generally small and neglected

$$\frac{\partial Y_0(X_0)}{\partial X_0}=\frac{\partial Y}{\partial X}\left(1+\frac{u}{U}\right) \tag{20}$$

Thus Guilloton's method corresponds to putting sourcce strength σ as

$$\sigma=\frac{U}{2\pi}\left(1+\frac{u}{U}\right)\frac{\partial F}{\partial X} \tag{21}$$

in Michell-Havelock's theory.

Difficulty in Guilloton's Method

The Guilloton's method interpreted by Gadd is based on thin ship. When this method is applied to ordinary ships, $\frac{\partial Y}{\partial Z}$ near the bottom can often be extremely large and the calculation does not converge. Appropriate manipulations are done near the bottom in this paper.

Application of the Slender Ship Theory and Handling of the Flow around the Bottom

Ship is generally slender and the principal factor affecting wavemaking resistance is the gradient of prismatic curve. In the present paper, ship is transformed into the wall-sided ship of same length, same draft and same sectional area curve. It is assumed that wave-making resistance of the given and the transformed ship are almost same. This assumption is studied by calculating wave-making resistance by Michells's theory on both ships.

The calculation become very simple for the transformed ship as $\frac{\partial S}{\partial X}$ is used instead of $\frac{\partial F}{\partial X}$ (S is the cross sectional area). With this method, the flow around the ship's bottom can be included in the calculation by considering the isobar at ship's bottom.

Isobar at the bottom is written as $\zeta(-T)$ and the increase in sectional area due to this at certain longitudinal cross section whose breadth is $2b$ is $2b\zeta(-T)$. The flux passing through the bottom is

$$2b(U+u)\frac{\partial\zeta(-T)}{\partial X}=(U+u)\frac{\partial\Delta S}{\partial X}$$

where

$$\Delta S=2b\zeta(-T) \qquad (22)$$

The flow around the bottom of the ship is included in this calculation by adding ΔS to the cress section.

Numerical Calculation

Given ships are transformed into the wall-sided ship of same length, same draft and same sectional area curve. Further, for the numerical calculation, this is divided into fifty sections in the longitudinal direction each on which the source strength is constant. $\frac{\partial\phi}{\partial X}$ of uniform source distribution on a plate, which corresponds to single section of ship's centerplane, is calculated by using Havelock's theory and tabulated in a table. Wave profile and isobar are calculated by using this table. Wave-making resistance calculation is done by replacing Havelock's source strength with Guilloton's one in Michell's theory. This procedure is iterated till the calculation converges. The flow chart is shown in Fig. 3. Computers YHP 2100 and YHP 3000 are used

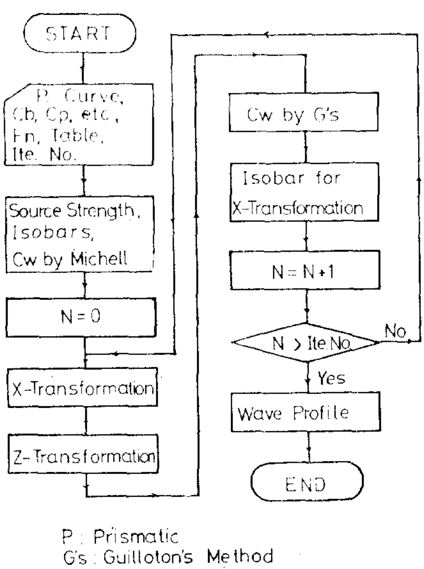

Fig. 3. Flow Chart of Computation

Calculation Result

Principal particulars of models are as in Table 1 and Table. 2. Prismatic curves of models in Table 1 are given by equation

$$S(x)=\sqrt{1-\frac{2(x-l)}{L}}\left[1+\partial\left\{\frac{2(x-l)}{L}\right\}^2-(1+\partial)\left\{\frac{2(x-l)}{L}\right\}^4\right] \qquad (23)$$

where

$$l=L/2,\ \partial=\frac{32}{\pi}C_p-7 \text{ and } 0\leq x\leq L$$

Table 1. Principal Particulars of Models

M. NO.	L (m)	B (m)	d (m)	Cb	Cp	Cpf	Cpa	Cm
331	4.0	0.5	0.167	0.5	0.6	0.6	0.6	0.833
337-F	4.0	0.5	0.167	0.5	0.6	0.5	0.7	0.833
337-A	4.0	0.5	0.167	0.5	0.6	0.7	0.5	0.833
339	4.0	0.5	0.167	0.417	0.5	0.5	0.5	0.833
340	4.0	0.5	0.167	0.583	0.7	0.7	0.7	0.833

Table 2. Principal Particulars of Series 60 with C_b=0.60

LWL (ft)	B (ft)	d (ft)	Cb	Cp	Cpf	Cpa
406.7	53.33	21.33	0.60	0.614	0.581	0.646

M331 is the parent model. M337-F and M337-A are longitudinally non-symmetrical models. M339 and M 340 are C_p series. Towing results of these models are taken from the reference [11]. C_w is the coefficient of wavemaking resistance obtained by the wave analysis and

$$C_w=\frac{R}{\frac{1}{2}\rho U^2L^2}$$

In addition, C_w of Cb=0.60 of series 60 is calculated, whose principal particulars are given in Table 2. C_w of this model is obtained by using the Schoenherr line with the addition of +0.0004 for roughness allowance.

Fig. 4 is the calculated C_w by Michell's theory for the parent model. Solid line is calculated C_w by

Michell's theory while broken one is Michell's theory calculated on the transformed ship. It can be seen that both values are identical. Thus, the assumption that wave-making resistance of transformed wall-sided ship of same length, same draft and same sectional area curve is almost same as that of the given ship is verified, and further calculation can be carried out based on this verification.

Calculation of C_w by Guilloton's method without Z-transformation on the parent model is plotted in

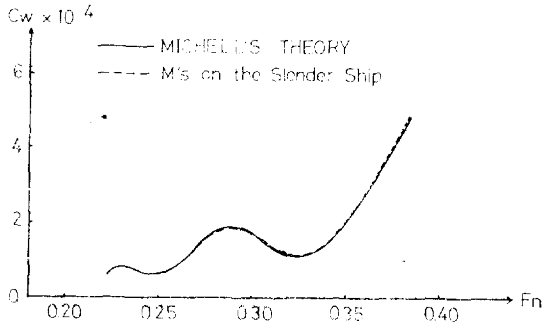

Fig. 4. Calculated C_w for M331 by Michell's Method

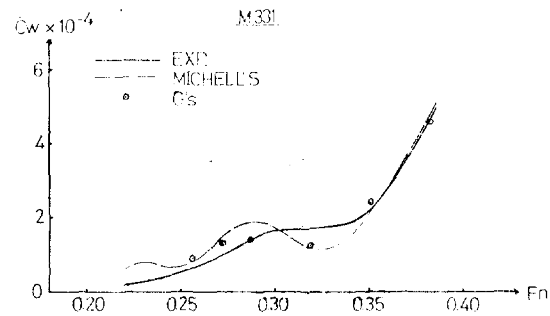

Fig. 5. Calculated C_w for M331 by Guilloton's Method without Z-Transformation

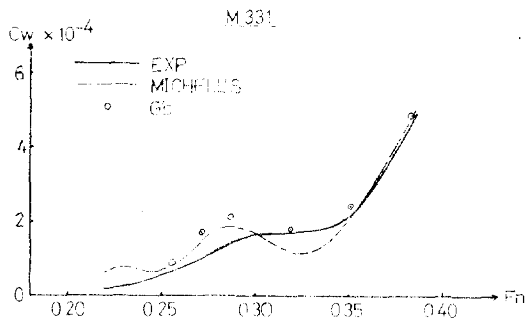

Fig. 6. Calculated C_w for M331 by Guilloton's Method with Z-Transformation

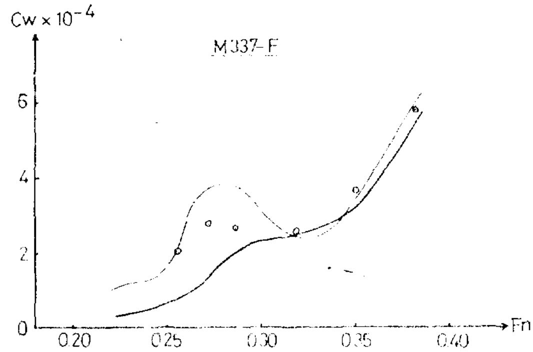

Fig. 7. Calculated C_w for M337-F by Guilloton's Method

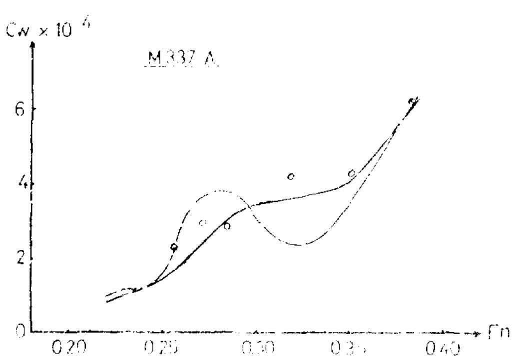

Fig. 8. Calculated C_w for M337-A by Guilloton's Method

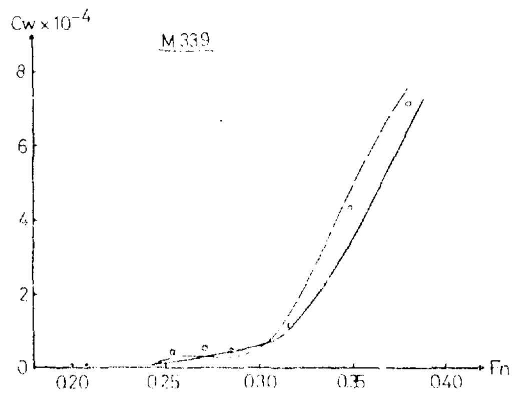

Fig. 9. Calculated C_w for M339 by Guilloton's Method

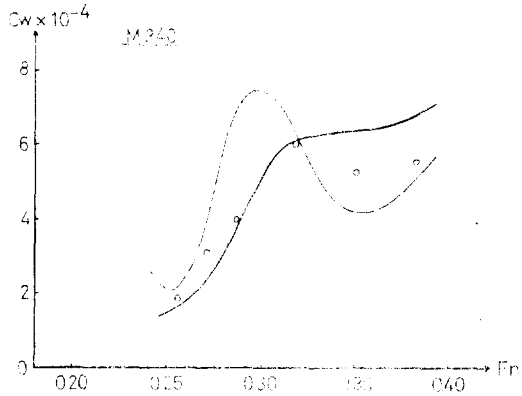

Fig. 10. Calculated C_w for M340 by Guilloton's Method

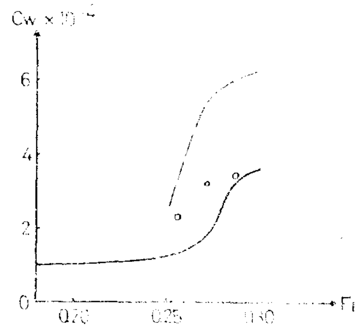

Fig. 11. Calculated C_w for Series 60 with $C_b=0.6$

Fig. 5. It can be seen that the present method is better than Michell's theory in some F_n, especially in hump. Iteration number is 5. Result with Z-transformation on the same model is given in Fig. 6. In this calculation, results are very good for F_n greater than 0.30.

For further models, F_n smaller than 0.30 is done without Z-transformation, while for other F_n the transformation is included. Calculation for six models in this manner are plotted in Fig.7~Fig. 11.

Wave profile calculation is compared with Havelock's theory and experiment. Fig. 12 is wave profile calculation carried out on the parent model at $F_n=0.319$ without Z-transformation, and Fig. 13 is with the Z-transformation. It can be seen clearly that calculation with the transformation is better than the one without. Fig. 14 to Fig 18 are wave profiles of different models and F_n, all with Z-transformation. It can be seen that the present method is better than Havelock's one in most cases.

Lastly, the corresponding real hull of Maruo's least wave resistance ship forms are calculated, for three F_n, as in Fig. 19

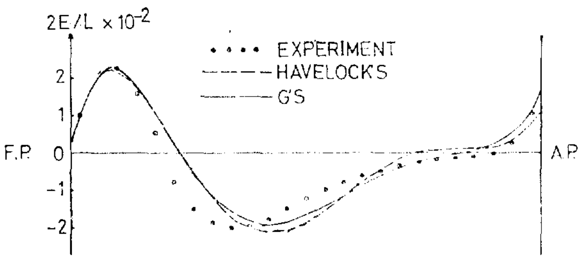

Fig. 12. Wave Profile of M331 at $F_n=0.319$ by Guilloton's Method without Z-Trans.

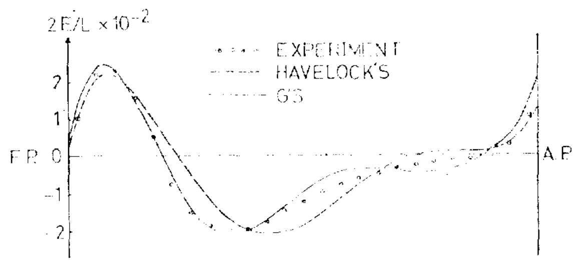

Fig. 13. Wave Profile of M331 at $F_n=0.319$ by Guilloton's Method with Z-Trans.

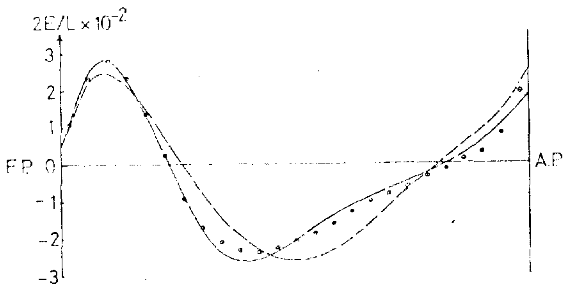

Fig. 14. Wave Profile of M331 at $F_n=0.351$

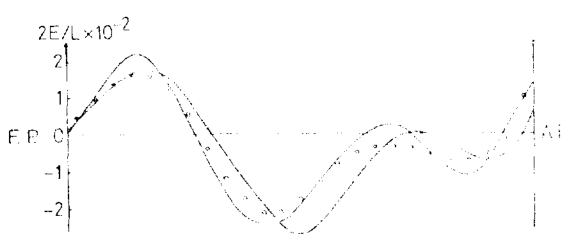

Fig. 15. Wave Profile of M337-F at $F_n=0.319$

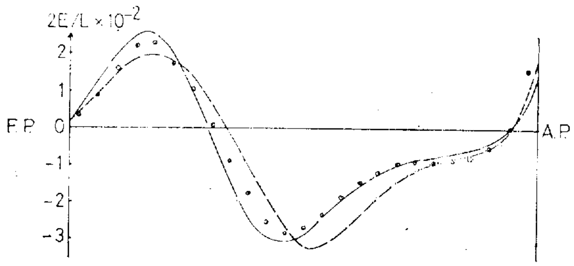

Fig. 16. Wave Profile of M337-F at F_n=0.351

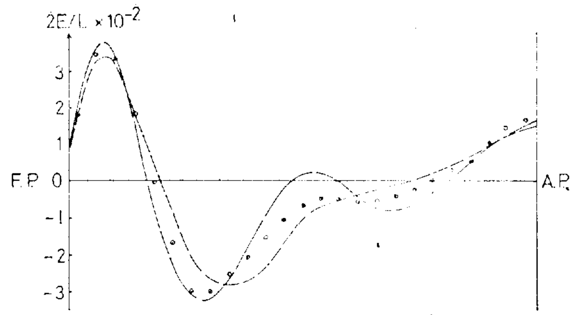

Fig. 17. Wave Profile of M337-A at F_n=0.319

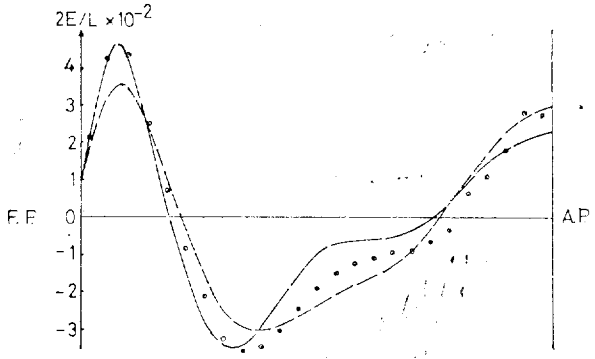

Fig. 18. Wave profile of M337-A at F_n=0.351

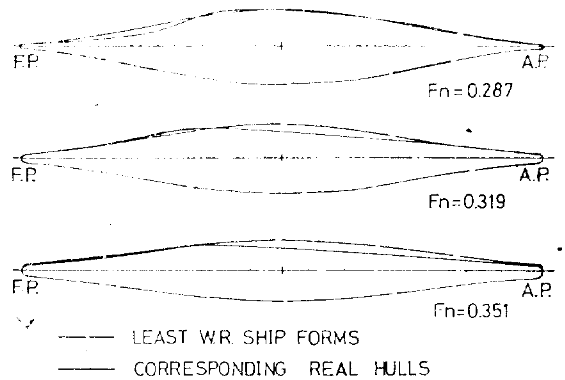

Fig. 19. Real Hull Forms of Maruo's Least Wave-Making Resistance Ships

Conclusions

Wave-making resistance calculated by Michell's theory on the transformed wall-sided ship of same length, same draft and same sectional area curve is almost equal to that of the given ship.

The calculation of wave-making resistance by Guilloton's method applied to the slender ship theory leads to the following conclusions:

1. C_w calculated without Z-transformation agrees well with experiment in some Froude numbers.

2. Calculation of C_w and wave profile with Z-transformation agrees well with the experiment for F_n greater than 0.30.

Z-transformation should be carried out for all F_n and further study is required.

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