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A Scismic Excitation on Floating Platforms

by

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Abstract

The method of computing a hydrodynamic force induced by a random seismic motion of boundary is presented and a sway force acting on a platform during an earthquake is shown.

Nomenclature

D=domain of water
∂D=boundary of water
H=depth of water
Γ_w=rigid walls ab and cd
Γ_b=boundary of platform
$\Gamma=\Gamma_b+\Gamma_w$

Introduction

A floating platform inside a semi-circular breakwater is subject to a sway hydrodynamic excitation due to a horizontal motion of the breakwater during an earthquake. Since the excitation induces a motion of the platform as well as a force, the present study is undertaken to study the sway exciting force during and after an earthquake.

An earthquake is transient in nature, and the problem is formulated with a time-dependent Green's function in a linear wave theory and the random seismic acceleration is applied for the boundary condition. The sway exciting force is to be evaluated numerically by solving an integral equation.

Formulation

We consider a platform is floating in a harbor closed by breakwaters. Initially the platform and water are at rest. As an earthquake occurs and proceeds, the walls of the breakwaters induces a motion of water and the platform becomes excited. In order to evaluate the sway force, we treat the present problem two-dimensionally.

The motion of the fluid, which is assumed irrotational, may be described by means of a velocity potential $\phi(\underset{\sim}{x},t)$, $\underset{\sim}{x}=x(,z)$, satisfying Laplace's equation

$$\Delta\phi=\phi_{xx}+\phi_{zz}=0 \text{ for } \underset{\sim}{x}\in D,\ t>0 \tag{1}$$

and the linearized boundary condition on the undisturbed water surface

$$\phi_{tt}(\underset{\sim}{x},t)+g\phi_z=0 \text{ at } z=0 \tag{2}$$

$$\left.\begin{aligned}\frac{\partial}{\partial n}\phi(\underset{\sim}{x},t)&=a_n(\underset{\sim}{x})T(\dot{t}) \text{ for } \underset{\sim}{x}\in\Gamma_w\\ &=0 \quad \text{at } z=-H \text{ and } x\in\Gamma_b\end{aligned}\right\} \tag{3}$$

Here n is the unit normal pointing into the domain. and Γ_b is the submerged body boundary. The fluid is assumed to start from rest

$$\phi=\phi_t=0 \quad \text{at } t=0 \tag{4}$$

The hydrodynamic pressure is determined by

$$p(\underset{\sim}{x},t)=-\rho\phi_t$$

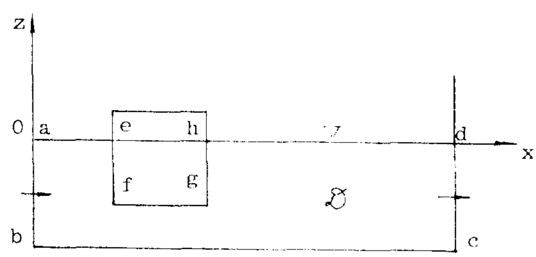

Fig. 1 Schematic diagram

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The problem defined by (1) to (4) can be conveniently treated with a acceleration potential $\phi=\phi_t$. It is easy to verify that ϕ satisfies Laplace's equation and the following boundary conditions which are analogous to (2) and (3):

$$\phi_{tt}(\underline{x},t)+g\phi_z(\underline{x},t)=0 \qquad \text{at } z=0 \tag{5}$$

$$\left.\begin{aligned}\frac{\partial}{\partial n}\phi(\underline{x},t)&=a_n(\underline{x})\dot{T}(t) \qquad \text{for } x\in\Gamma_w\\ &=0 \qquad \text{at } z=-H \text{ and } x\in\Gamma_b\end{aligned}\right\} \tag{6}$$

where

$$\dot{T}(t)=\frac{d}{dt}T(t)$$

The dynamic pressure p is given in terms of the acceleration potential as

$$p(\underline{x},t)=-\rho\phi(\underline{x},t)$$

Solution via Green's Function

Let $G(\underline{x},\underline{\xi},t)$, $\underline{x}=(x,z)$, $\underline{\xi}=(\xi,\zeta)$, be a time-dependent Green's function which satisfies the following:

$$\left.\begin{aligned}&\Delta G=\delta(\underline{x}-\underline{\xi},\ y-\zeta)\\ &G_{tt}+gG_z=0 \qquad \text{for } z=0\\ &G_z=0 \qquad \text{for } z=-h\\ &G=G_t=0 \qquad \text{for } t=\tau,\ z=0\end{aligned}\right\} \tag{7}$$

Such $G(x,\xi,t)$ exists and is given explicitly by the following expression:

$$G=\frac{1}{2\pi}\log(R/R') -\frac{1}{\pi}\int_0^\infty\frac{e^{-KH}\sinh(Kz)\sinh(K\zeta)\cos(Kr)dK}{K\cosh(KH)} -\frac{1}{\pi}\int_0^\infty\frac{\cosh K(z+H)\cosh K(\zeta+H)[1-\cos\omega(t-\tau)]}{K\cosh^2(KH)\tanh(KH)}\cos(Kr)dK \tag{8}$$

where
$$R=[(x-\xi)^2+(z-\zeta)^2]^{\frac{1}{2}}$$
$$R'=[(x-\xi)^2+(z+\zeta)^2]^{\frac{1}{2}}$$
$$r=|x-\xi|$$
$$\omega^2=gK\tanh(KH)$$

We apply the Green's theorem to the functions $\phi(x,t)$ and $G(\underline{x},\underline{\xi},t-\tau)$

$$2\pi\phi(\underline{x},t)=\int_{\partial D}[G(\underline{x},\underline{\xi},t-\tau)\phi_n(\underline{\xi},t) -G_n(\underline{x},\underline{\xi},t-\tau)\phi(\underline{\xi},t)]ds_\xi \quad \text{for all } \underline{x}\in D). \tag{9}$$

where ∂D is the boundary of water. Integrating (9) with respect to t between $t=0$ and $t=\tau$ we find

$$2\pi[\phi(\underline{x},\tau)-\phi(\underline{x},0)] =\int_0^\tau\int_{\partial D}[G(\underline{x},\underline{\xi},t-\tau)\phi_n(\underline{\xi},t) -G_n(\underline{x},\underline{\xi},t-\tau)\phi(\underline{\xi},t)]ds_\xi dt. \tag{10}$$

Using the initial and free surface conditions, we write (10) as

$$2\pi[\phi(\underline{x},\tau)-\phi(\underline{x},0)]=\int_0^\tau\int_\Gamma[G(\underline{x},\underline{\xi},t-\tau)a_n(\underline{\xi})\dot{T}(t) -G_n(\underline{x},\underline{\xi},t-\tau)\phi(\underline{\xi},t)]ds_\xi dt.x \tag{11}$$

Now we differentiate equation (11) with respect to τ. Then we have

$$2\pi\phi(x,\tau)=\int_\Gamma[G(\underline{x},\underline{\xi},0)a_n(\underline{\xi})\dot{T}(\tau) -G_n(\underline{x},\underline{\xi},0)\phi(\underline{\xi},\tau)]ds_\xi +\int_0^\tau\int_\Gamma[G_{nt}(\underline{x},\underline{\xi},t-\tau)\phi(\underline{\xi},t) -G_t(x,\xi,t-\tau)a_n(\xi)\dot{T}(t)]ds_\xi dt. \tag{12}$$

Next we let $x\in D$ in (12) approach a point on the boundary Γ. This results in the following integral equation for the boundary potential:

$$\lambda(\underline{x})\phi(\underline{x},\tau)+P.V.\int_\Gamma G_n(\underline{x},\underline{\xi},0)\phi(\underline{\xi},\tau)ds_\xi =\int_0^\tau\int_\Gamma G_{nt}(\underline{x},\underline{\xi},t-\tau)\phi(\underline{\xi},t)ds_\xi dt+c(\underline{x},\tau), \tag{13}$$

where

$$c(\underline{x},\tau)=-\int_\Gamma G(\underline{x},\underline{\xi},0)a_n(\underline{\xi})\dot{T}(\tau)ds_\xi +\int_0^\tau[\dot{T}(t)\int_\Gamma G_t(\underline{x},\underline{\xi},t-\tau)a_n(\underline{\xi})ds_\xi]dt.$$

Here P.V. indicates the Cauchy principal value integral and $\lambda(x)/2\pi$ is the part of an infinitesimal circle centered at the boundary point $\underline{x}$, which is contained in D, i.e. $\lambda=\pi$ if the boundary is straight. The convolution with respect to time in the right-hand side of (13) is approximated by some quadrature formula, e.g. the trapizoidal rule:

$$\int_0^\tau\int_\Gamma G_{nt}\ (\underline{x},\underline{\xi},t-\tau)\phi(\underline{\xi},t)\ ds_\xi dt\approx \frac{\Delta t}{2}\int_\Gamma G_{nt}\ (\underline{x},\underline{\xi},0)\phi(\underline{\xi},\tau)ds_\xi +\Delta t\sum_{l=1}^{L-1}\int_\Gamma G_{nt}(\underline{x},\underline{\xi},l\Delta t)\Psi(\xi,(L-l)\Delta t)ds_\xi$$

where $L=\tau/\Delta t$.

Next we discretize (13) with respect to space by dividing the boundary Γ into intervals Δ_i, $1\le i\le N$. We use a piecewise constant approximation to Ψ; the constand values are $\Psi_i^l=\Psi(\underline{x}_i,\ l\Delta t)$ where x_i is the center of Δ_i.

With this discretization (13) becomes a system of N linear equations

$$\underline{A}\phi^L = \Delta t \sum_{l=1}^{L-1} \underline{B}^l \phi^{L-l} + \gamma^L \tag{14}$$

where

$$\phi^l = \begin{pmatrix} \phi_1^l \\ \vdots \\ \phi_N^l \end{pmatrix}$$

and A is an $N \times N$ constant matrix whose entries are

$$A_{ij} = \delta_{ij}\lambda(\underline{x}_i) + \text{P.V.} \int_{\Delta_j} G_n(x_i, \xi, 0) ds_\xi$$

$$-\frac{\Delta t}{2} \int_{\Delta_j} G_{nt}(\underline{x}_i, \underline{\xi}, 0) ds_\xi$$

B_l is an $N \times N$ matrix whose entries are

$$B_{i,j}{}^l = \int_{\Delta_j} G_{nt}(\underline{x}_i, \xi, l\Delta t) ds_\xi$$

γ^L is an N vector whose components are

$$\gamma_i^L = c(\underline{x}_i, \ L\Delta t) = -\alpha_i \dot{T}(L\Delta t)$$

$$-\frac{\Delta t}{2} [\dot{T}(0)\beta_i{}^L + \dot{T}(L\Delta t)\beta_i{}^o]$$

$$-\Delta t \sum_{l=1}^{L-1} \dot{T}((L-l)\Delta t)\beta_i{}^l$$

where we have introduced

$$\alpha_i = \int_\Gamma G(\underline{x}_i, \underline{\xi}, 0) a_n(\underline{\xi}) ds_\xi,$$

$$\beta_i^l = \int_\Gamma G_t(x_i, \xi, l\Delta t) a_n(\xi) ds_\xi.$$

Now we take H=13.7m, $\overline{ae}$=9.1m, $\overline{bc}$=315m, $\overline{ef}$= 9.46m and fg=114.4m. The horizontal seismic acceleration along ab and cd is given in Fig. 2. The computed sway force per unit length is shown in Fig. 3.

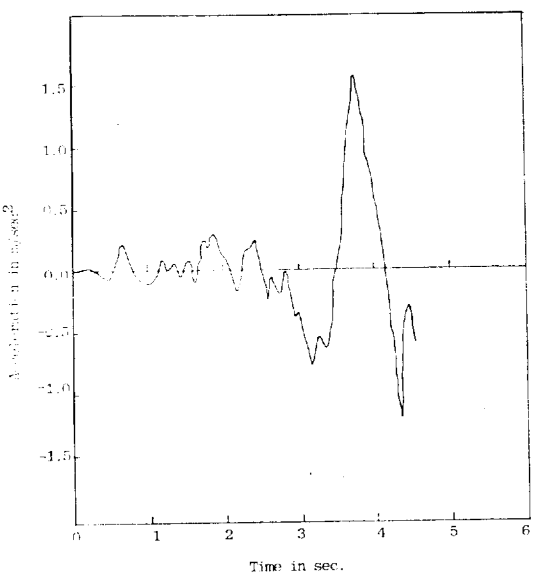

Fig. 2. Horizontal seismic acceleration along Γ_w

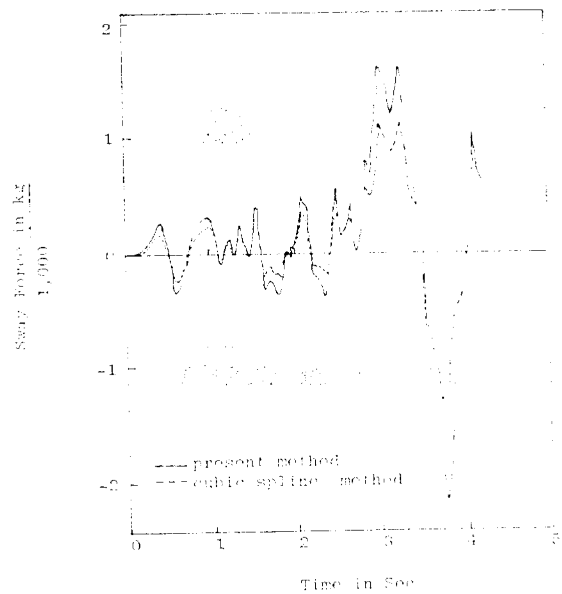

Fig. 3. Sway force on platform

Discussion and Conclusions

We observe that the matrix A and the vector $\underline{\alpha}$ are independent of time and therefore have to be computed only once. The number of operations needed to compute L time-steps of the potential on Γ is proportional to L^2N^3. The quadratic dependence on the number of time steps is due to the need to evaluate the convolution integral in (13). Hence, the present method requires a large memory allocation because of this convolution.

It was tacitly assumed that the walls are rigid and the seismic acceleration is only time-dependent. Hence two walls are in the same motion during the earthquake and the present work is an extension of the classical dam theory.

The sway exciting force in Fig. 3 agrees fairly well with the spline method [3]. The histories of the acceleration in Fig. 2 and the sway force in Fig. 3 are closely similar except for the sign. It was found from computations that the maximum sway force occurs during the earthquake, while the maximum surface wave appears after the earthquake and interacts with the platform.

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