Development of Two Dimensional Filter for the Reconstructive Image Processing

(映像 再構成 處理를 爲한 二次元 필터의 構成)

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要 約

單純逆投射에 依하여 얻어진 鮮明치 않은 映像에서 斷層映像을 再構成할 수 있는 二次元 필터들을 周波數領域에서 考察하고 이의 性能을 比較하였다. 이 필터 函數들은 斷層映像시스템의 傳達函數로부터 求하였으며 그 모양은 ramp 函數에 window 函數를 곱한 形態로 되어 있다. 이 window 函數는 映像 再構成에서의 不必要한 雜音畵像을 줄이는 役割을 하는 것이 模擬映像 데이터를 利用한 computer simulation에서 밝혀졌다.. 또한 simulation의 結果로 二次元 映像 再構成 方法이 一次元 convolution method에 依한 映像 再構成에 比해서 畵質은 나쁘지만 計算時間은 十倍 以上 短縮됨을 發見하였다. 二次元 映像 再構成에서는 window 函數가 變함에 따라 再構成된 映像의 畵質도 달라진다. 즉 映像의 解像度를 높일 수록 雜音이 많이 섞인 畵像을 얻게 되는 것을 再構成한 映像들을 通하여 考察하였다.

Abstract

Two dimensional kernels which reconstruct a tomographic image from a blurred one formed by simple back-projection are investigated in the frequency domain and their performances are compared. The kernels are derived from a point spread function of the tomographic system and have the form of a ramp filter modified by several window functions to suppress ringings or artifacts in the reconstruction. Computer simulation using computer-generated phantom image data with different filter functions has been carried out. In this simulation, it is found that the computation time for 2-D reconstruction is much less than that of 1-D convolution method by a factor of ten or more whereas the reconstructed image quality of the former is far poorer than the latter. In 2-D reconstruction heavy windowing results in less noisy reconstruction but details smear out in this case. The trade-offs between these points are considered.

I. Introduction

The 3-D reconstruction techniques have been of great interest to many researchers who are engaged in medical diagnostic imaging and many other related fields such as nondestructive analysis. A more recent and very dramatic advance in this area has been the development of transaxial computerized tomography in which a single plane is isolated and viewed in the 2-D image plane. The actual 3-D reconstruction of an object then can be obtained by a series of 2-D cross sections of the object. Each 2-D cross section is reconstructed by proper summation (filtering) of a series of

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projection data resulting from different projection angles. In the process of 3-D imaging, the 2-D reconstruction of the tomographic image plays a key role.

There are two convolution methods in 2-D reconstruction. The widely used one in practical X-ray computed tomography is the 1-D convolution method[1] in which the 1-D projections are convolved with 1-D correction function and then projected back into a reconstruction plane.

Another one which needs further study is the 2-D convolution method which involves 2-D correction kernels applied to the image formed by simple back-projection. The 2-D correction functions (kernels) have not yet been extensively studied though it has been suggested earlier.[2, 3] Therefore this study concerns problems involved in implementation of actual 2-D filters in 2-D image reconstruction.

Following the introduction, 2-D reconstruction algorithms are presented and compared with each other by computer simulation using a computer-generated phantom in Section II and III, respectively. Then in Section IV, conclusions are made on the performances of the different 2-D correction functions.

II. Two Dimensional Reconstruction of Tomographic Image

Since the 2-D image will be reconstructed from the 1-D projection data at different angles, the filtering for reconstruction can be performed either in 1-D space on 1-D projection data or in 2-D space on the back-projection plane. Since, for 1-D filtering, deblurring functions and their effects on reconstruction have been thoroughly compared elsewhere,[4, 5] this paper is restricted to 2-D filtering.

1. Projection and Back-Projection

The data avilable for 2-D reconstruction are multiple 1-D projections. A simple and approximate method of reconstructing a section image is simply to project the projections back to an image plane, called back-projection. In this back-projectioned image, one finds that the details of the original image of the object are smeared out and the important diagnostic information can not be obtained since they have density differences of orders of only one to several percents compared with the background density. This is because the summation image (back-projected image) obtained by simple back-projection is blurred by a function propotional $1/r$ where r denotes the distance from the image point.[6] To recover the original details from the blurred image, two basic principles are used for reconstruction cited in Section I. They are 1-D filtering and 2-D filtering, respectively.

1-D filtering of projection data and back-projection of these filtered data yield a good reconstructed image when the projection is performed with parallel beam scanning. But if the projection data are obtained by the scanning of fan beams, or curved rays, or randomly oriented chords from positron annihilation events, etc., 1-D filtering of projection data requires several extra processes which are, for example, reordering of projection data, additional suitable weighting in 1-D filtering, and reordering for back-projection. These processes need considerable amount of extra computing time compared with the case of parallel beam scanning tomographic system.

If the 2-D filtering method is incorporated in reconstruction, the reconstruction would be initiated from the back-projected image plane. Thus the reconstruction does not depend on whether the scanning is performed by parallel beams or otherwise.

2. Two-Dimensional Deblurring Functions

Since the back-projection yields an image which is blurred by a $1/r$ type function, the inverse operation (deconvolution) is required to reconstruct the original image. To obtain the deblurring function, the Fourier transform of 1/r is taken to obtain

$$F[r^{-1}] = 2\pi \int r^{-1} J_0(2\pi\rho r) r dr = \rho^{-1}, \qquad (1)$$

where J_0 is the zero order Bessel function and ρ is the frequency radius. As a result, the 2-D filter function becomes a ρ type function in frequency domain which is called the "rho-filter". Taking the inverse Fourier transform of this function results in a 2-D deconvolution kernel which can be applicable to the back-projected image array in space domain. The blurred image may be filtered in two ways, either by filtering in the space domain or filtering in the frequency domain. The former spatial domain method by 2-D convolution can also be useful to reconstruct the original image, it requires, however, much more time than the latter frequency domain method to obtain the image of practical significance. It means that the length of the 2-D kernel for convolution should be long enough to ensure complete reconstruction and the processing of this convolution necessarily requires much longer computation time than the frequency domain method where the latter requires only two 2-D FFT's and one multiplication of the image array in frequency domain by the rho-filter.

When the rho-filter is applied directly to the Fourier transformed image array, the resulting image will have more artifacts than desired. These artifacts in the reconstructed image could be avoided by using a simple rho-filter if several apodizing (windowing) functions are appropriately incorporated. In contrast to the removal of noisy features in the reconstructed image, the apodizing functions tend to smooth the sharp edges of the original image. Trade-offs between them therefore should be made by choosing a suitable window function.

ORIGINAL

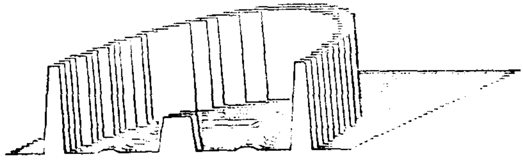

Fig. 1. Original phantom for computer simulation.

SB-BACKPROJ-128-64-1

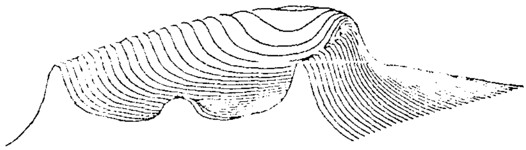

Fig. 2. A reconstruction of a phantom with simple back-projection.

SBF2-FBPROJ-REAL

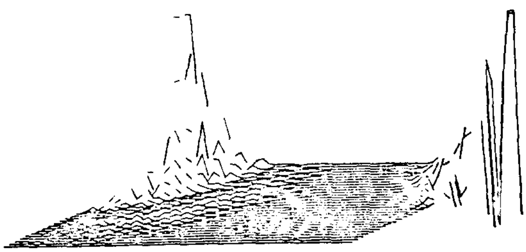

Fig. 3. Fourier transform of a simple back-projected data; real part.

SBF2-FBPROJ-IMAG

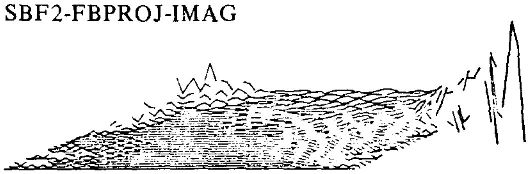

Fig. 4. Fourier transform of a simple back-projected data; imaginary part.

III. Simulation Results and Discussion

Computer simulation of the 2-D reconstruction method was done on Data General NOVA

830 minicomputer. To simulate this method one phantom image is first obtained as shown in Fig. 1 using the same method as that of Shepp and Logan.[1, 8] After getting a series of projection data which was projected at 128 directions around the angle of o to π, the simple back-projection of this data resulted in a 64 x 64 blurred image array (Fig. 2). 2-D fast-Fourier-transform of this blurred image array (Fig. 3 for the real part and Fig. 4 for the imaginary part) is multiplied by a composite rho-filter and then 2-D inverse FFT of the filtered array is performed. This procedure is performed by using the program of Sheep and Logan[1, 8] for simple back-projection and the FFT program in IBM scientific subroutine package for 2-D FFT and 2-D inverse FFT. The resulting image is then the recontructed image. To obtain the perspective view of this resulting 64 x 64 image array onto the graphic plane of a Tektronix 4010-1 graphic terminal, 3-D plot routines have been programmed in assembly language.

The apodizing functions (window functions) used in this simulation are as follows;

A. Rectangular

$$A(\rho) = \mathrm{rect}\,(\rho/2\rho_m) = \begin{cases} 1, & \text{if } \rho \leqslant \rho_m \\ 0, & \text{if } \rho > \rho_m. \end{cases} \quad (2)$$

B. Hamming

$$A(\rho) = \begin{cases} 0.54+0.46\cos(\pi\rho/\rho_m), & \text{if } \rho \leqslant \rho_m \\ 0, & \text{if } \rho > \rho_m. \end{cases} \quad (3)$$

C. Hanning

$$A(\rho) = \begin{cases} 0.5\,[1.0+\cos(\pi\rho/\rho_m)], & \text{if } \rho \leqslant \rho_m \\ 0, & \text{if } \rho > \rho_m. \end{cases} \quad (4)$$

D. Butterworth

$$A(\rho) = \begin{cases} 1/\ \ [1 + (\rho/\rho_c)^{2N}\,]^2, & \text{if } \rho \leqslant \rho_m \\ 0, & \text{if } \rho > \rho_m. \end{cases} \quad (6)$$

E. Shepp and Logan

$$A(\rho) = \begin{cases} (2/\pi)\sin(\pi\rho/2\rho_m)/\rho, & \text{if } \rho \leqslant \rho_m \\ 0, & \text{if } \rho > \rho_m. \end{cases} \quad (7)$$

Here, ρm is the maximum digital frequency. The responses of these composite filters are shown in Fig. 5 and the reconstructed images are shown in Fig. 6 - Fig. 11. In those figures, it is evident tht reducing artifacts by heavy apodizing also results in degradation of resolu-

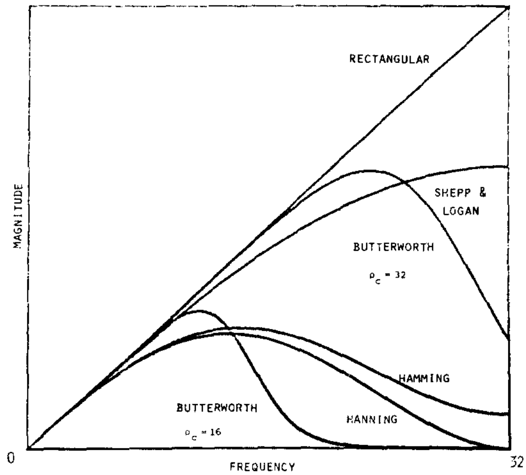

Fig. 5. Responses of composite filters in frequency domain.

SBF2FR-WRECT

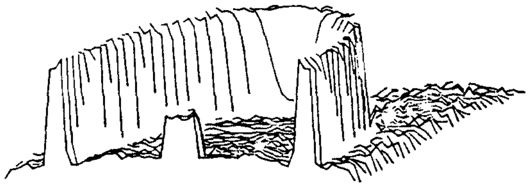

Fig. 6. A 2-D reconstruction by a rho-filter with a rectangular window.

SBF2FR-WHAMM

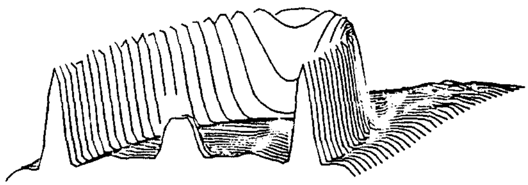

Fig. 7. A 2-D reconstruction by a rho-filter with a Hamming window.

SBF2FR-WHANN

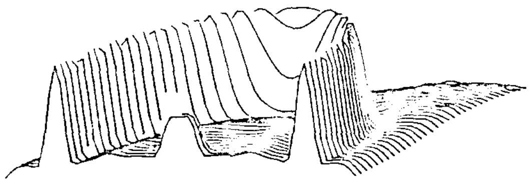

Fig. 8. A 2-D reconstruction by a rho-filter with a Hanning window.

SBF2FR-WBUFF8-1

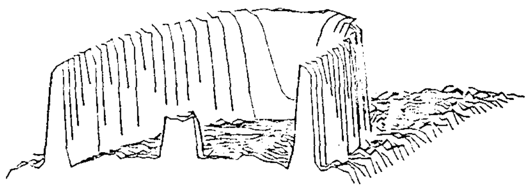

Fig. 9. A 2-D reconstruction by a rho-filter with a Butterworth window. ($\rho_c = \rho_m$)

SBF2-FR-WBUFF8-1/2

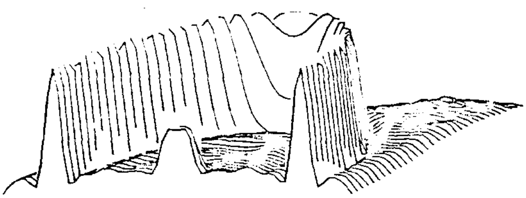

Fig. 10. A 2-D reconstruction by a rho-filter with a Butterworth window. ($\rho_c = \rho_m/2$)

SBF2FR-WSHEP

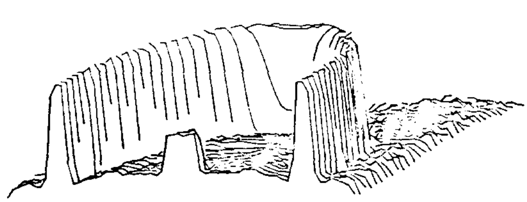

Fig. 11. A 2-D reconstruction by a rho-filter with a Sheep & Logan window.

tion. This effect is clearly shown in Fig. 9 and Fig. 10 in the case of Butterworth window with the cutoff frequencies (ρ_c) of ρ_m or $\rho_m/2$, respectively.

IV. Conclusions

2-D filters in frequency domain for tomographic image reconstruction are obtained by windowing the simple rho-filter with several window functions. In the reconstruction process, two 2-D FFT's and one filter multiplication are required and the computation time for 2-D reconstruction is therefore much less than that of 1-D filtered back-projection reconstruction. A speed advantage by a factor as much as ten or more using a general-purpose 16-bit minicomputer was observed. This speed advantage is inherent of the algorithms used in 1-D and 2-D reconstruction method. Specifically the number of multiplication of 2-D filtering (16 x 6 x 64 x 2, complex) is much smaller than that of 1-D filtering (128 x 128 x 128) and ther other operations are almost the same. But the reconstructed image quality in this case is far poorer than that of 1-D convolution method. The compromise between the image quality and the reconstruction time are also examined to determine which reconstruction method should be used as well as the resolution and the effect of noisy artifacts in 2-D filtering in frequency domain. In the case of the image array of 128 x 128 elements, the image quality of 2-D reconstruction in frequency domain is expected to be the same as that of 1-D convolution reconstruction image quality.

The main reason why the quality of the reconstruction in 2-D frequency domain is poorer than space domain method seems to be in the 2-D filtering. The simple 2-D circular expansion of the 1-D filter function used in this simulation gives at best sub-optimum results whereas the 1-D filter function is optimum in 1-D space. That is because the 2-D filter function obtained in this manner is not the 2-D equivalent of the 1-D filter function.[7]

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