論文 79-16-5-2

M-ary DPSK Error Performances with Noise and Interference

(雜音 및 干涉波에 依한 M相 DPSK 시스템의 誤率 特性)

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要 約

DPSK信號帶域內에 相加性 雜音과 干涉波가 存在할 경우, 多相 DPSK信號의 受信誤率에 對한 理論的 解析을 行했다. 이에는 受信合成位相의 確率密度函數(pdf)를 利用, 多相 DPSK信號의 受信誤率를 求하기 爲한 簡潔한 一般式을 導出하여 2相, 4相 및 8相에 對해 數値計算후 相應한 CPSK 結果와 比較檢討를 行했다.

Abstract

This paper presents the investigation of the theoretical symbol error performances of an M-ary differential phase shift-keyed (DPSK) system in an interference environment. A simple method is presented which yields the exact probability of error for an M-ary DPSK signal which is transmitted over a nondistorting channel, but additively corrupted by Gaussian noise and cochannel interference. Computed DPSK symbol error performance results for M=2, 4, and 8 are compared with the corresponding curves for coherent phase-shift-keyed (CPSK) system as a function of carrier-to-noise power ratio (CNR) with carrier-to-interferer power ratio (CIR) as a parameter. Comparisons between DPSK and CPSK systems reveal, as we might expect, that DPSK system suffers more degradation.

1. Introduction

Multi-phase digital modulation proves to be an efficient technique for trading bandwidth for signal-to-noise ratio to reduce spectrum congestion. In a phase-shift-keyed (PSK) digital radio transmission system, while some factors contribute to the overall system performance, two major sources of performance degradation are the thermal noise and cochannel interference.

Lately, the error performance of differential phase-shift keyed (DPSK) transmission systems in an interference environment has been investigated by many authors. However, most of the earlier studies used the bounding technique by which tightness of error rate could not be obtained without studying specific cases.

Among the results obtained, for the case of Gaussian noise only, explicit expression of the

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接受日字 : 1979年 7月 19日

probability of error is given by Cahn[1], and for the case of Gaussian noise and angle-modulated interference, the error rate has been obtained by Rosenbaum[2].

In treating the problem of interference for calculation of the exact probability of error, it is desirable to have a simple but adequate representation. Accordingly, we use an approach that is different from Rosenbaum's method, and derive in this paper an alternative probability density function (*pdf*) of the phase difference of composite receiver signals by using the characteristic function method[3,4]. This approach yields a simple and exact probability of error.

Using the derived equation we evaluate the theoretical symbol error performances of an M-ary DPSK system in the presence of noise and cochannel interference for M=2,4, and 8. Also, performance comparison will be made between coherent phase-shift-keyed (CPSK) and DPSK systems with respect to interference immunity.

2. Model and Assumption

In this paper, we assume that there is a steady received signal which is corrupted by random Gaussian noise and cochannel interference. The transmission channel is assumed to be nondistorting, and stationary white Gaussian noise is assumed to be introduced at the front end of receiver with interference.

A typical DPSK receiver for our analysis is shown in Fig. 1. The receiver consists mainly of a bandpass filter and a standard DPSK detector. The bandwidth of the bandpass filter is assumed to be sufficiently wide to prevent any signal distortion.

In an M-ary DPSK system, transmitted information is conveyed with the phase transition of $2\pi j/\mathrm{M}$ ($j=0, 1,\cdots, \mathrm{M}-1$) between adjacent pulses rather than with the absolute phases of the pulses.

If we assume that each signal transmitted has a duration T, Nth transmitted signal can be represented as

$$s_N(t) = S\cos(\omega_s t+\zeta),\quad N\mathrm{T}\leq t\leq (N+1)\mathrm{T} \tag{1}$$

where S is the amplitude of signal and ω_s the angular frequency which is assumed to be an integer multiple of $2\pi/\mathrm{T}$.

If symbol j is to be transmitted during the $(N+1)$th interval, then the transmitted signal during the $(N+1)$th interval is given by

$$s_{N+1}(t)=S\cos\left(\omega_s t+\zeta+\frac{2\pi}{\mathrm{M}}j\right),\quad (N+1)\mathrm{T}\leq t\leq (N+2)\mathrm{T}. \tag{2}$$

Here, all message symbols are assumed to be equally likely. Then the composite signals at the bandpass filter output can be written as respectively,

$$z_N(t)=s_N(t)+n_N(t)+i_N(t),\quad N\mathrm{T}\leq t\leq (N+1)\mathrm{T}, \tag{3-a}$$

$$z_{N+1}(t)=s_{N+1}(t)+n_{N+1}(t)+i_{N+1}(t),\quad (N+1)\mathrm{T}\leq t\leq (N+2)\mathrm{T}. \tag{3-b}$$

where $n(t)$ and $i(t)$ are referred to the presence of noise and interference.

$n(t)$ is the result of the passage of zero-mean stationary white Gaussian noise through a bandpass filter, and can be written as[3]

$$n(t)=n_C(t)\cos\omega_s t - n_S(t)\sin\omega_s t, \tag{4}$$

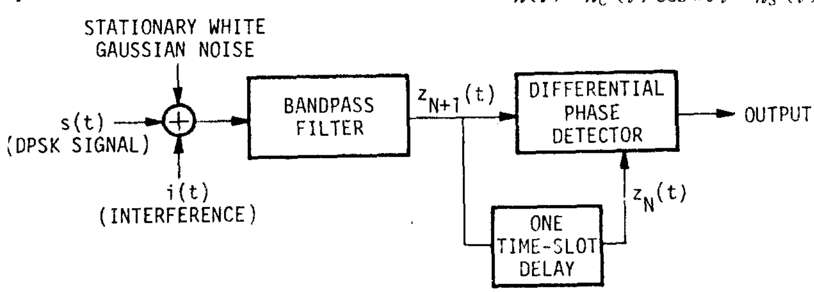

Fig. 1. DPSK receiver.

where $n_C(t)$ and $n_S(t)$ are zero-mean independent stationary lowpass Gaussian random processes with powers equal to σ_n^2. Here, the noise is assumed to be independent between symbol intervals.

Let the bandpass interference at the output of the receiver filter be written as

$$i(t)=I\cos[\omega_s t+\lambda(t)], \quad (5)$$

such that it has a constant envelope I and phase λ with respect to the carrier. For the interference we assume that the phase λ has a uniform probability density function (pdf) of $(2\pi)^{-1}$. Also we assume that the interference is independent between symbol intervals. Thus the interference as well as the noise possesses circular symmetry in the two-dimentional signal space.

In the presence of additive Gaussian noise and interference when each j symbol is assumed to be transmitted with equal probability, DPSK system is characterized by a phase reference which suffers random perturbations.

3. Error Probability Analysis

we show in Fig. 2 the phasor diagram of two successively received composite signals during Nth and $(N+1)$th intervals. In that figure S_N and S_{N+1} are the PSK signals transmitted during Nth and $(N+1)$th intervals respectively, I_N and I_{N+1} the interferers, N_N and N_{N+1} the narrowband Gaussian noises, Z_N and Z_{N+1} the composite signals. Also, θ_N and θ_{N+1} are the phase angles of composite signals, $2\pi j/M$ the phase transition between adjacent PSK signals, ζ the phase angle of the Nth transmitted PSK signal, and ϕ is the phase difference between two composite signals to be detected by the phase detector as a message. The noise and interference introduce distortion to each PSK signal both in amplitude and in phase as shown in Fig. 2.

With the two resultant phasors, Z_N and Z_{N+1}, the differential phase detector acts as follows: It measures the phase difference ϕ between two phasors, Z_N and Z_{N+1}, and quantizes this angle ϕ to the nearest $(2\pi k)/M$ ($k=0, 1, \cdots, M-1$), then it decides k was transmitted.

Now, without any loss of generality, we assume that the phase angle ζ of the Nth transmitted signal is zero and zeroth symbol, i.e., $j=0$, is transmitted in $(N+1)$th signal, an error then occurred in the Nth symbol if and only if ϕ of reduced modulo 2π is greater than $\pm\frac{\pi}{M}$. With this assumption the probability of error depends only on ϕ, and can be calculated by integration of the probability density function (pdf) of ϕ, $p_\phi(\theta)$, over the error region.

Then, from the previous assumptions that the components of the noise and interference at the same instant of time are independent of each other and are also independent of the phase

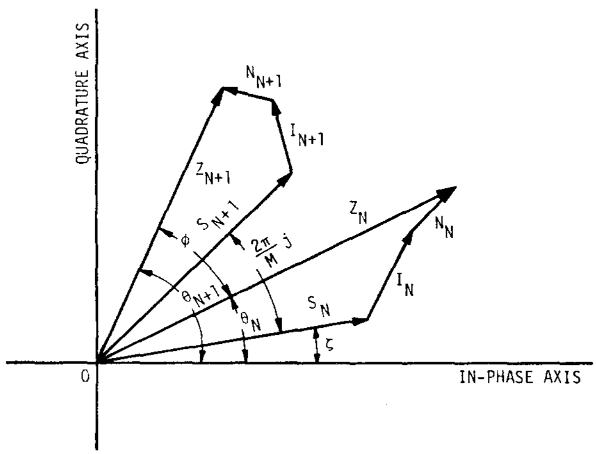

Fig. 2. Phasor diagram of received composite signals.

modulation of any other instant of time, we know that the probability density functions of phase angle θ_N and θ_{N+1} of the received composite signals, Z_N and Z_{N+1}, are the same. Accordingly, the probability density function $p_{\theta_N}(\theta)$ is derived by the characteristic function method as follows[4,5]:

$$p_{\theta_N}(\theta)=p_{\theta_{N+1}}(\theta)=\frac{1}{2\pi}+\frac{1}{\pi}\sum_{m=1}^{\infty}a_m\cos m\theta, \quad (6)$$

where

$$a_m\triangleq\sum_{\ell=0}^{\infty}\left(\frac{m}{2}\right)\left[\Gamma\left(\frac{m}{2}+\ell\right)/\Gamma(m+1)\right](-1)^{\ell}\frac{r^{-\ell}}{(\ell!)^2}\alpha^{\frac{m}{2}+\ell}\cdot{}_1F_1\left(\frac{m}{2}+\ell\,;m+1\,;-\alpha\right)$$

$\alpha(=S^2/2\sigma_n^2)$ is carrier-to-noise power ratio (CNR) and $r(=S^2/I^2)$ is the carrier-to-interferer power ratio (CIR). $\Gamma(\cdot)$ is the Gamma function, and ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function.

Because θ_N and θ_{N+1} are independent, the *pdf*, $p_\phi(\theta)$, of the phase angle ϕ as detected by an ideal phase-comparator is given by convolution of $p_{\theta_N}(\theta)$ and $p_{\theta_{N+1}}(\theta)$ as[6]

$$p_\phi(\theta)=\int_0^{2\pi}p_{\theta_N}(\mu)\,p_{\theta_{N+1}}(\theta+\mu)\,d\mu. \quad (7)$$

Substituting (6) into (7) yields

$$p_\phi(\theta)=\int_0^{2\pi}\left[\left\{\frac{1}{2\pi}+\frac{1}{\pi}\sum_{m=1}^{\infty}a_m\cos m\mu\right\}\left\{\frac{1}{2\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}a_n\cos n(\theta+\mu)\right\}\right]d\mu$$
$$=\frac{1}{2\pi}+\frac{1}{\pi}\sum_{m=1}^{\infty}a_m^2\cos m\theta. \quad (8)$$

Accordingly, we can calculate the desired symbol error probabilities $P_e(M)$ of an M-ary DPSK system by integrating $p_\phi(\theta)$ over the error region $[\frac{\pi}{M}\leq|\theta|\leq\pi]$ as

$$P_e(M)=2\int_{\frac{\pi}{M}}^{\pi}p_\phi(\theta)d\theta=2\int_{\frac{\pi}{M}}^{\pi}\left[\frac{1}{2\pi}+\frac{1}{\pi}\sum_{m=1}^{\infty}a_m^2\cos m\theta\right]d\theta$$
$$=1-\frac{1}{M}-\frac{2}{\pi}\sum_{m=1}^{\infty}a_m^2\frac{1}{m}\sin\left(\frac{m}{M}\right)\pi. \quad (9)$$

(9) is a general equation for the symbol error performance of an M-ary DPSK signal corrupted by Gaussian noise and cochannel interference.

4. Numerical Results

Using the derived expression (9), symbol error probabilities of an M-ary DPSK system in the presence of noise and cochannel interference have been computed numerically for M=2, 4, and 8. Also, for an M-ary CPSK system, we can calculate the symbol error probabilities directly by integrating (6) over the error region $[\frac{\pi}{M}\leq|\theta|\leq\pi]$. The calculated results are plotted in Figs. 3-5, as a function of CNR with CIR as a parameter. The curves are in excellent agreement with those obtained earlier by different methods[2,7].

In Figs. 3-5, we see that as CNR increases, each error curve is decreasing monotonically. For high CNR and high CIR, it is seen that DPSK system yields about a 3-dB degradation as compared with CPSK system except for M=2. With M=2 the degradation becomes negligible with high CNR, i.e., less than 1 dB for the error rate below

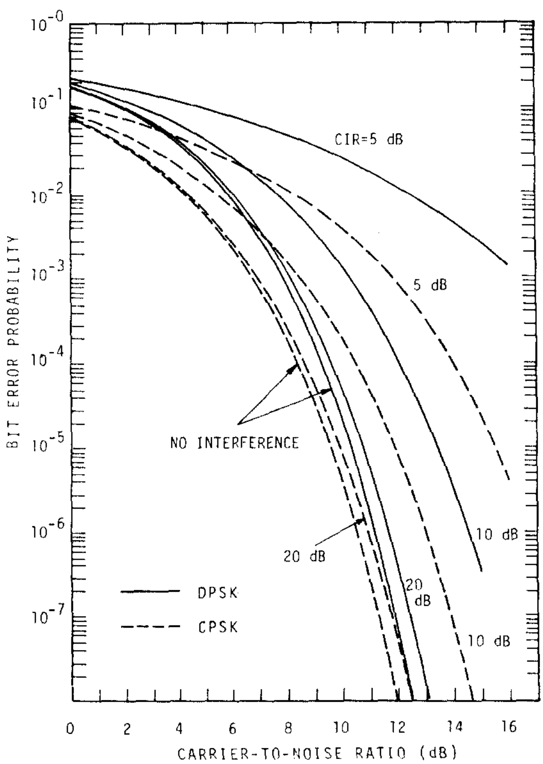

Fig. 3. Bit error probabilities of binary PSK system.

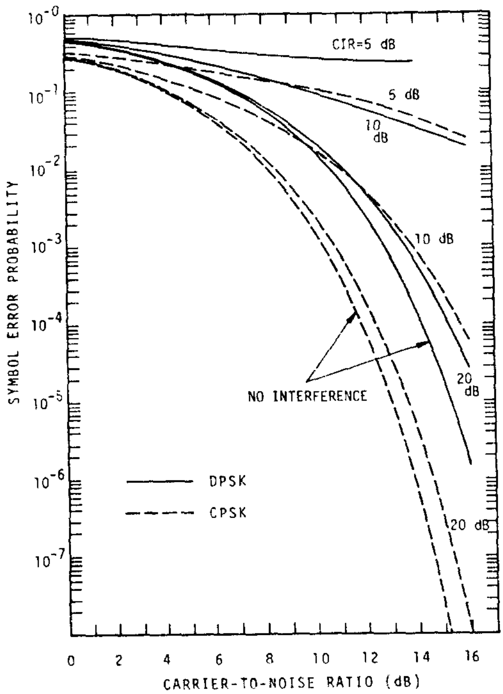

Fig. 4. Symbol error probabilities of quaternary PSK system.

10^{-3}. Also, it is seen that for high CNR and relatively low CIR, the effect of the interference dominates that of the noise.

Comparing CPSK and DPSK systems reveals, as we might expect, that CPSK system has a greater immunity for interference, i.e., for the same probability of error, a DPSK system requires a larger increase in CNR when interference is present over that needed in no interference case.

5. Conclusion

Presently, PSK appears to be a probable choice of system engineers for high-speed radio and waveguide digital transmission. Therefore it is important to know the error performance of PSK systems in an interference environment.

In this paper, we have presented a simple method for the performance analysis of an M-ary DPSK system that is different from the earlier bounding techniques. This method yielded the exact probability of error for an M-ary DPSK

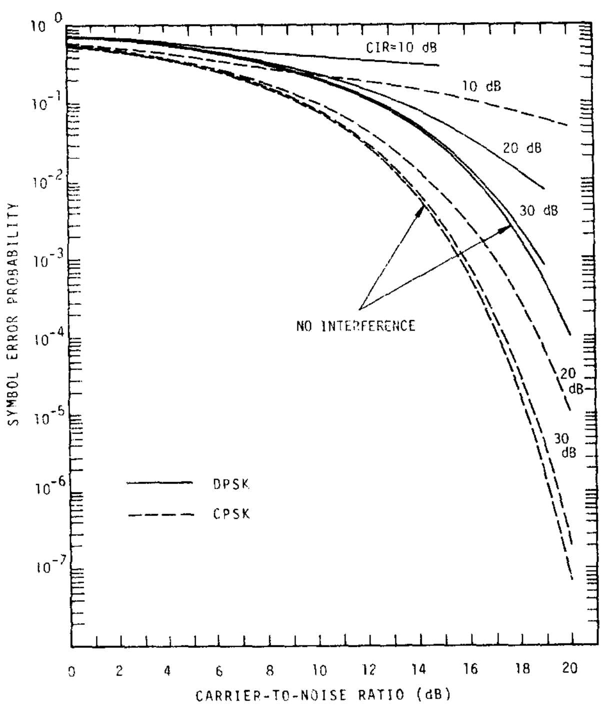

Fig. 5. Symbol error probabilities of 8-phase PSK system.

system in the presence of noise and cochannel interference. Using the derived expression, we have calculated the symbol error rates of an M-ary DPSK system for M=2, 4, 8 and comparison has been done with the corresponding CPSK results.

From the results obtained in this paper, one can know the theoretical symbol error performances of an M-ary DPSK system in an interference environment.

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