

# 線路開閉狀態를 包含하는 電力系統 狀態推定 및 同定

論 文
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## Power System State Estimation and Identification in Consideration of Line Switching

박 영 문\* · 유 석 한\*\*  
(Young Moon Park, Suk Han You)

### Abstract

The static state estimation are divided into two groups; estimation and detection & identification. This paper centers on detection and identification algorithm. Especially, the identification of line errors is focused on and is performed by the extended W.L.S. algorithm with line switching states. Here, line switching states mean the discrete values of line admittance which are influenced by unexpected line switching. The numerical results are obtained from the assumption that the noise vector is independent zero mean Gaussian random variables.

### 1. Introduction

Recently, it has been indispensable to give a reliable data base for power system control and security monitoring e.t.c. in the large scaled power system.

To give the reliable data base, the W.L.S.(weighted least square) algorithm has been widely used in the power system state estimation.

Firstly, Schweppe F.C. suggested the use of W.L.S. algorithm in for that purpose.<sup>1)2)3)</sup>

But one of the dominant problems in the field of W.L.S. algorithm in power system is the presence of bad data in either the telemetered measurements or the power system configuration and unaccurate results are obtained under the influence of bad data.

So it is important to detect and identify the

bad data. To detect and identify the bad data, Schweppe Suggested a bad data suppression algorithm (especially to aid in identification)<sup>4)</sup>.

But especially in cases of large bad data or large line errors, bad data suppression algorithm could lead to oscillations of the estimates and a consequent diversion of the solution.

Lately, a statistical bad data detection algorithm was suggested, but an identification algorithm of line errors was performed by trial and error method<sup>5)6)</sup>.

In order to identify line errors, this paper employs the extended W.L.S. algorithm which includes suspected conditions of line switching in power system state, and also to save computer storage and computing time it introduces the P-Q, Q-V Decoupling method<sup>7)</sup>.

### 2. The Basic W.L.S. Algorithm

The problem is to find the best estimate  $\hat{X}$  of the state  $X$  of the power system(voltage modules

\* 正會員 : 서울대工大電氣工學科教授. 工博(當學會總務理事)  
\*\* 正會員 : 서울대工大 電氣工學科  
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and angles at every bus of the power system)

i.e. the one which minimizes the weighted sum of squared residuals;

$$J(\hat{x}) = \min [Z - h(x)]^T R^{-1} [Z - h(x)] \quad (2.1)$$

To obtain the best estimate  $\hat{x}$  which satisfies (2.1)

$$\frac{\partial}{\partial x} J(x) = -2H^T(x)R^{-1}[Z - h(x)] = 0 \quad (2.2)$$

The  $\hat{x}$  is determined by (2.2), using an approximate Newton optimization technique

i.e.

$$x_{i+1} = x_i + [H^T(x_i)R^{-1}H(x_i)]^{-1}H^T(x_i)R^{-1}[Z - h(x_i)] \quad (2.3)$$

where  $x_i$  is the computed values of  $x$  in the  $i$ -th iterative stage and  $x_0$  is initially guessed.

But the above basic W.L.S. algorithm requires the inversion of a gain matrix  $\Sigma(x_i)$

$$\Sigma(x_i) = H^T(x_i)R^{-1}H(x_i)$$

Hence, the above algorithm requires large computer memory storage and much computing time.

For overcoming these difficulties, the P-Q and Q-V decoupling method is used in this paper.

### 3. P-Q, Q-V Decoupling Method

Let  $N$  be the number of buses of the power system whose state is

$$x^T = [\theta^T, v^T]$$

where

$$\theta^T = [\theta_1, \theta_2, \dots, \theta_{N-1}] \quad v^T = [v_1, v_2, \dots, v_{N-1}]$$

represents the bus voltage angles and modules, respectively.

Note that the  $N$ th bus is the reference bus with  $Q_N = 0$  and  $V_N$  is assumed to be 1.

Then, the measurement vector  $Z$  is partitioned into the active and reactive parts:

$$Z = \begin{bmatrix} Z_a \\ Z_r \end{bmatrix} = \begin{bmatrix} f(v, \theta) \\ g(v, \theta) \end{bmatrix} + \begin{bmatrix} \epsilon_a \\ \epsilon_r \end{bmatrix} \quad (3.1)$$

With the above definitions, we obtain the Jacobian:

$$H(\theta, v) = \begin{pmatrix} \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial v} \\ \frac{\partial g}{\partial \theta} & \frac{\partial g}{\partial v} \end{pmatrix} \sim \begin{pmatrix} \frac{\partial f}{\partial \theta} & 0 \\ 0 & \frac{\partial g}{\partial v} \end{pmatrix} \quad (3.2)$$

Accordingly, from (2.3) we obtain

$$v_{i+1} = v_i + \left[ \frac{\partial g^T}{\partial v} \cdot R_r^{-1} \frac{\partial g}{\partial v} \right]^{-1} \frac{\partial g^T}{\partial v} \cdot R_r^{-1} [Z_r - g(v, \theta)] \quad (3.3)$$

$$\theta_{i+1} = \theta_i + \left[ \frac{\partial f^T}{\partial \theta} \cdot R_a^{-1} \frac{\partial f}{\partial \theta} \right]^{-1} \frac{\partial f^T}{\partial \theta} \cdot R_a^{-1} [Z_a - f(v, \theta)] \quad (3.4)$$

where

$$R^{-1} = \begin{bmatrix} R_a^{-1} & 0 \\ 0 & R_r^{-1} \end{bmatrix}$$

The optimal state estimate  $[\hat{\theta}^T, \hat{v}^T]$  is obtained by interlacing iterations of (3.3), and (3.4) and iteration process terminates if  $\|\Delta\theta\|_\infty = \delta_a$  and  $\|\Delta v\|_\infty = \delta_r$ , consecutively, where  $\delta_a, \delta_r$  reflect desired accuracies in  $\theta$  and  $v$ , respectively.

### 4. Bad Data Detection

Let  $\tau$  be the  $m \times 1$  residue vector

$$\tau = Z - h(\hat{x})$$

then,

$$J(\hat{x}) = \tau^T R^{-1} \tau \quad (4.1)$$

If random error  $\epsilon$  is zero mean, Gaussian random variables, then the residue vector  $\tau$  is also zero mean Gaussian random variable i.e.  $E[\tau] = 0$  and have a chi-square distribution with degree of freedom  $K = m - n$ . Hence detection can be performed by the following hypothesis testing with two hypothesis:

$H_0$ : The measurement is not a bad data and is not related to any line errors

$H_1$ :  $H_0$  is not true

Accept  $H_0$  if  $J(\hat{x}) = \tau_i$

Reject  $H_0$  otherwise

Where  $\tau_i$  is threshold level which is determined from false alarm probability  $P_a$  (the probability of rejecting  $H_0$  when  $H_0$  is true) and degree freedom  $K$

Assuming that  $\epsilon$  is Gaussian (and  $\tau$  is small), the distribution of  $J(\hat{x})$  can be evaluated and  $P_a$  computed for any  $\tau_i$ . By the above hypothesis testing, we can detect the measurement bad date or line errors occurring in the power system.

### 5. Measurement Bad data detection and Identification

#### 5.1 Detection Criteria

As  $E[\tau.\tau^T] = WR = \Sigma$ ,  $\tau = W\epsilon$  (see ref. 5)

where  $W = I - H(H^T R^{-1} H)^{-1} \cdot H^T R^{-1}$

We can normalize the residue into zero mean random variables with unit variance. i.e.

$$r_i^1 = \frac{r_i}{\sqrt{\Sigma_{r,ii}}}$$

where  $\Sigma_{r,ii}$  is the  $i$ th diagonal element of  $\Sigma_r$  matrix

Consequently, we can detect bad data by performing the hypothesis test on each of the  $m$  residues (see ref. 5).

Since the residual sensitivity matrix  $W$  always has many large off-diagonal elements, in the event of a large interacting bad data or line errors, through interaction, the magnitudes of all the residues will be raised, increasing the size of the set of suspected bad data.

Because of these important drawbacks, which are due to the assumption of being random errors of known variance in the mathematical derivations, the following algorithm is suggested by N.Q. Le. (see ref. 6).

a) The suspected measurement must have the largest residue for the particular measurement type (since the errors in voltage and power, for example, can't be directly compared). That is,

$$|r_i^k| > |r_j^k| \text{ for all } r_j^k \text{ in measurement type } K$$

Here, the measurements are divided into five types; voltage, active power flow, active power injection, reactive power flow and reactive power injections.

b) As well as satisfying criterion a) for the absolute value of  $r_i^k$ , the same test must be satisfied as followings.

$$\left| \frac{r_i^k}{Z_i^k} \right| > \left| \frac{r_j^k}{Z_j^k} \right| \text{ for all } r_j^k \text{ in measurement type. } k.$$

In the above algorithm, at each stage, there exist at most five suspected bad data.

In the case of multiple suspected bad data, the one which has the largest residue is selected as the first suspected bad data to test.

### 5.2 Detection and Identification Scheme

Since, in the 5.1 criteria, the suspected bad data set contains at most five suspected bad data, one from each type of measurements, there is the possibility of the less severe error being

left out of the set (especially if this is the same type as the more severe error).

However, since a new selection of suspected data is made after each identification, it is sufficient that the less severe bad data is included in the suspected data set after one or more identification.

Since all off-diagonal elements of  $W$  were found to be smaller than 0.5,  $J(x_s)/2$  was chosen as the threshold of detection for multiple bad data.

The measurement bad data detection and identification scheme is shown in Fig. 1.

## 6. Line error Detection and Identification

### 6.1 Line Error Detection

After measurement bad data detection and identification is completed by means of bad data scheme, in the case that  $J(\hat{x})$  is greater than the threshold level (for reduced degree of freedom), line error detection is performed by the following algorithm.

a) selection level 1

Since a line error directly affects the power flow on that line, the largest flow residue corresponds to the line in error.

The selection level 1 selects the line with the largest flow residue as the suspected line.

b) selection level 2

In the case that not all the lines are monitored, geometrical correlation of the residues is necessary to locate the area containing the line in error.

Geometrical correlation is a procedure which pool the subscripts of all the large residues.

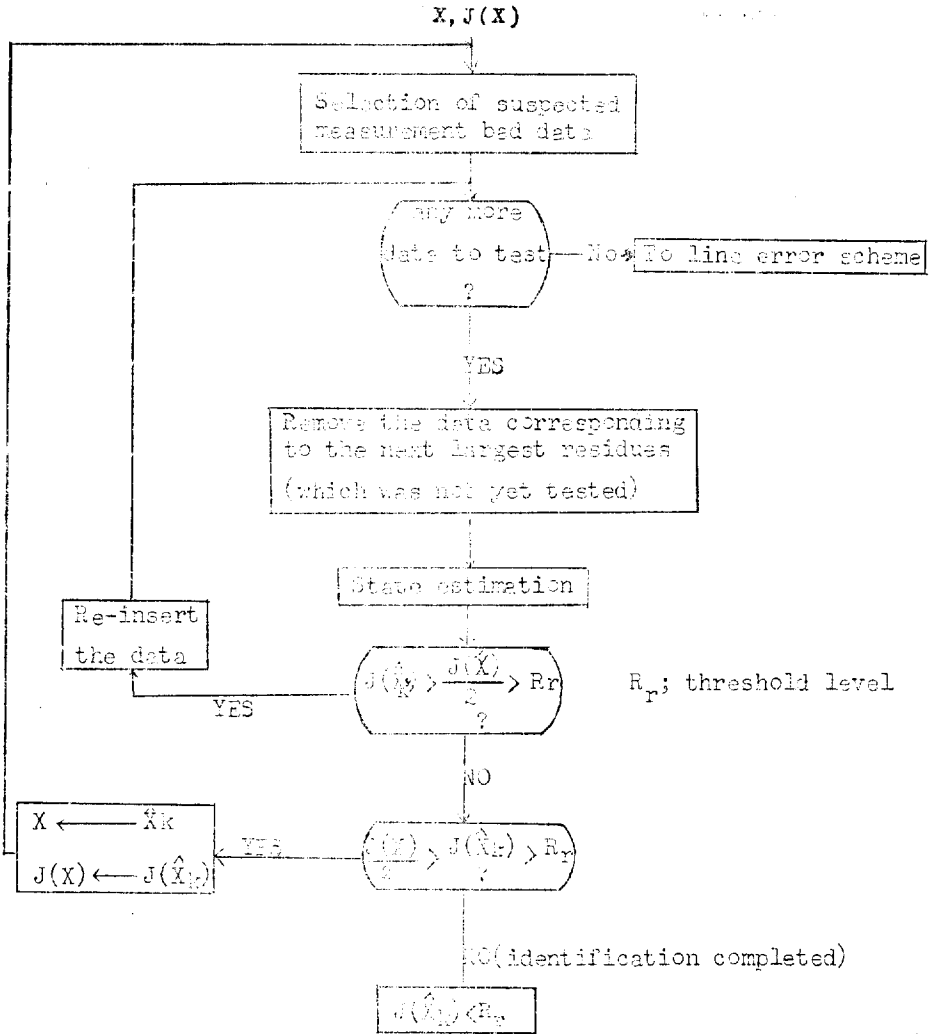
### 6.2 Line Error Identification

After the set of suspected line error is determined by line error detection algorithm, line error identification can be performed by the following extended W.L.S. algorithm.

$$\text{Let } Z_{..} \triangleq \left\{ \frac{Z_a}{Y} \right\}$$

$$Z_{..} \triangleq \left\{ \frac{Z_r}{Y} \right\}$$

where  $Y$ ; the nominated admittance vector of the suspected line error



\* Measurement bad data identification scheme

Fig. 1

Then from (3.3), (3.4)

$$\begin{bmatrix} V_{i+1} \\ Y_{i+1} \end{bmatrix} = \begin{bmatrix} V_i \\ Y_i \end{bmatrix} + \begin{bmatrix} \frac{\partial g}{\partial v} & \frac{\partial g}{\partial Y} \\ 0 & I \end{bmatrix}^T \begin{bmatrix} R_s & 0 \\ 0 & M \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial g}{\partial v} & \frac{\partial g}{\partial Y} \\ 0 & I \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \frac{\partial g}{\partial v} & \frac{\partial g}{\partial Y} \\ 0 & I \end{bmatrix}^T \begin{bmatrix} R_s & 0 \\ 0 & M \end{bmatrix}^{-1} [Z_{ii} - g_s(V, \theta, Y)] \quad (6.1)$$

$$\begin{bmatrix} \theta_{i+1} \\ Y_{i+1} \end{bmatrix} = \begin{bmatrix} \theta_i \\ Y_i \end{bmatrix} + \begin{bmatrix} \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial Y} \\ 0 & I \end{bmatrix}^T \begin{bmatrix} R_s & 0 \\ 0 & M \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial Y} \\ 0 & I \end{bmatrix}^{-1}$$

$$\begin{bmatrix} \frac{\partial f}{\partial \theta} & \frac{\partial f}{\partial Y} \\ 0 & I \end{bmatrix}^T \begin{bmatrix} R_s & 0 \\ 0 & M \end{bmatrix}^{-1} [Z_{ii} - f_s(V, \theta, Y)] \quad (6.2)$$

where

$g_s(V, \theta, Y) = g(V, \theta, Y) : (m_s + k) + 1$  vector

$f_s(V, \theta, Y) = f(V, \theta, Y) : (m_s + k) + 1$  vector

$M$ ;  $k \times k$  weighting diagonal matrix such that

$M_{ii} \gg r_{a_{ij}}$  and  $M_{ii} \gg R_{r_{ii}}$  for all

$1 = i = k, 1 = j = m_s, 1 = l = m_s$ .

But in the actual case, it is assumed that line error is caused by the abrupt change of admittance (i.e. unexpected line switching) and is assumed that the maximum number of parallel circuits on each line is known.

For example, the number of the possible value of line admittance is limited to three states in

the case that the maximum number of parallel circuits is two.

$$i.e. \quad Y_i = \begin{cases} Y_i' \\ Y_i'/2 \\ 0 \end{cases}$$

Hence from the calculated results by (6.1), (6.2), the value of the estimated admittance can be readjusted by the following method.

$$Y_i = \begin{cases} \frac{m}{lm} \cdot Y_i & \text{if } \frac{2n-1}{2lm} \cdot Y_i \leq Y_i \leq \frac{m}{lm} \cdot Y_i \\ \frac{n-1}{lm} \cdot Y_i & \text{if } \frac{n-1}{lm} \cdot Y_i \leq Y_i \leq \frac{2m-1}{2lm} \cdot Y_i \end{cases}$$

where  $lm$ ; the maximum number of parallel circuits on  $i$ th line in the suspected line error set.  
 $n$ ; integer,  $1 \leq n \leq lm$

### 7. Sample Test

Several cases involving bad data and single line errors are performed on a simulated 5 bus 7 line system shown in Fig. 2. A set of 24 measurements are used, comprising 4 voltages, 8 power injections and 12 line power flows as shown in Fig. 2.

The computer program is divided into three blocks.

1) Given the desired network and bus powers, establish the true bus voltages by a conventional load flow.

2) Construct the meter readings by adding random errors which is generated from random generator.

3) Calculate the best mean square estimate  $\hat{x}$

In this paper, the standard deviation of measurement  $\sigma$  is 0.02 (in the case of no bad data) and 1.4 (in the case of bad data) Some results are shown in Table 1, 2, 3, 4, Fig. 3, 4, 5.

Table 1.

Bus #	Voltage Module		Voltage Angle		% Error	
	True	Estimate	True	Estimate	V	Q
1	0.9849	0.9851	-0.0551	-0.0551	0.02	0.182
2	0.9583	0.9584	-0.098	-0.099	0.01	1.02
3	0.9556	0.9557	-0.104	-0.1049	0.01	0.865
4	0.9557	0.9564	-0.122	-0.1213	0.073	0.574

$$J_p(\hat{x}) = 5.16 \quad r_p = 12.6 (P_e = 0.05)$$

$$J_q(\hat{x}) = 5.36 \quad r_q = 18.3 (P_e = 0.05)$$

\*\* The case of no measurement bad data and no line error

Table 2. Before detection

Bus #	Voltage Module		Voltage Angle		% Error	
	True	Estimate	True	Estimate	V	Q
1	0.9849	0.9848	-0.055	-0.0557	0.01	1.273
2	0.9583	0.9583	-0.098	-0.0992	0.0	1.21
3	0.9556	0.9552	-0.104	-0.1063	0.042	2.212
4	0.9557	0.9558	-0.122	-0.123	0.01	0.82

$$J_p(\hat{x}) = 39.9 \quad r_p = 12.6 (P_e = 0.05)$$

$$J_q(\hat{x}) = 5.51 \quad r_q = 18.3 (P_e = 0.05)$$

\*\* The case of measurement bad data ( $P_{2-3}$ ) and no line error

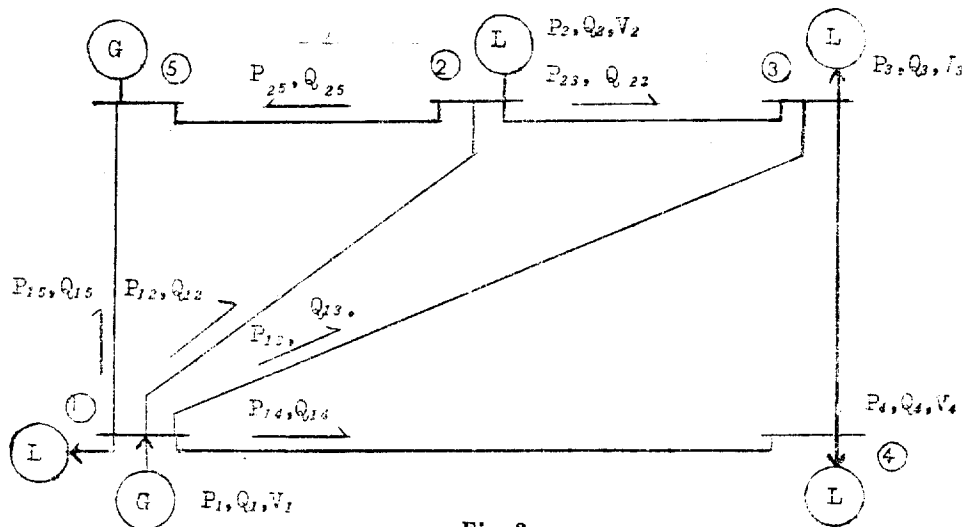


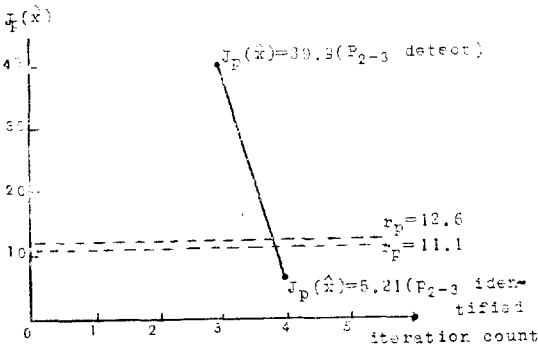
Fig. 2

**Table 2-2.** After detection

Bus #	Voltage Module		Voltage Angle		% Error	
	Ttrue	esti- mate	True	Estimate	V	Q
1	0.9849	0.9851	-0.055	-0.0549	0.02	0.182
2	0.9583	0.9584	-0.098	-0.09895	0.01	0.97
3	0.9556	0.9558	-0.104	-0.1046	0.021	0.577
4	0.9557	0.9565	-0.122	-0.1211	0.0837	0.738

$J_p(x)=5.21$        $r_p=11.1 (P_r=0.05)$   
 $J_q(x)=5.43$        $r_q=16.9 (P_r=0.05)$

\*\* The case of measurement bad data ( $P_{2-3}$ ) and no line error



\*\* The case of measurement bad data ( $P_{2-3}$ ) and no line error

**Fig. 3**

**Table 4.** The case of line error ( $Y_{12}$ )

Bus #	Voltage Module		Voltage Angle		% Error			
	A	B	A	B	A		B	
					V	Q	V	Q
1	0.9869	0.9863	-0.05493	-0.0553	0.2	0.13	0.14	0.55
2	0.9531	0.9579	-0.1064	-0.0991	0.54	8.57	0.042	1.12
3	0.9510	0.9554	-0.1123	-0.1048	0.48	7.98	0.021	0.77
4	0.9550	0.9564	-0.1243	-0.1211	0.07	1.89	0.73	0.74

A: Before detection and identification

$J_p(x)=114.5$   
 $J_q(x)=13.6$

B: After detection and identification

$J_p(x)=5.2$   
 $J_q(x)=8.2$   
 True  $Y_{12}=5.2075$   
 Estimate  $Y_{12}=5.213$

\*\* The case of measurement bad data ( $P_{2-3}$ ) and line error ( $Y_{12}$ )

**Table 3-1.** Before identification

Bus #	Voltage Module		Voltage Angle		% Error	
	True	Esti- mate	True	Estimate	V	Q
1	0.9849	0.9857	-0.055	-0.054	0.081	1.818
2	0.9583	0.9534	-0.098	-0.1072	0.522	9.388
3	0.9556	0.9516	-0.014	-0.1117	0.419	7.404
4	0.9557	0.9555	-0.122	-0.1229	0.021	0.738

$J_p(x)=48.2$        $r_p=12.6 (P_r=0.05)$   
 $J_q(x)=9.1$        $r_q=18.3 (P_r=0.05)$

\*\* The case of line error ( $Y_{12}$ )

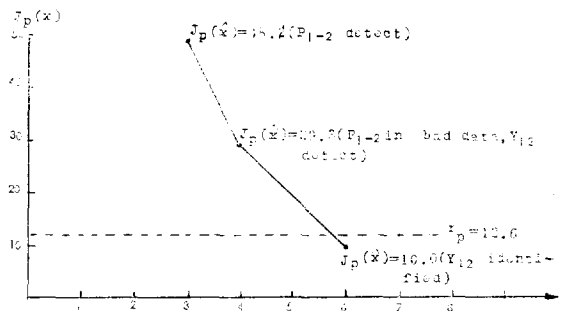
**Table 3-2.** After identification

Bus #	Voltage Module		Voltage Angle		% Error	
	True	Esti- mate	True	Estimate	V	Q
1	0.9849	0.9849	-0.055	-0.05495	0.0	0.091
2	0.9583	0.9578	-0.098	-0.09893	0.0522	0.0949
3	0.9556	0.9553	-0.104	-0.1048	0.0314	0.769
4	0.9557	0.9563	-0.122	-0.1213	0.0628	0.574

$J_p(x)=10.0$        $r_q=12.6 (P_r=0.05)$   
 $J_q(x)=6.2$        $r_q=18.3 (P_r=0.05)$

True  $Y_{12}=5.2705$       Estimat  $Y_{12}=5.28$

\*\* The case of line error ( $Y_{12}$ )



**Fig. 4**

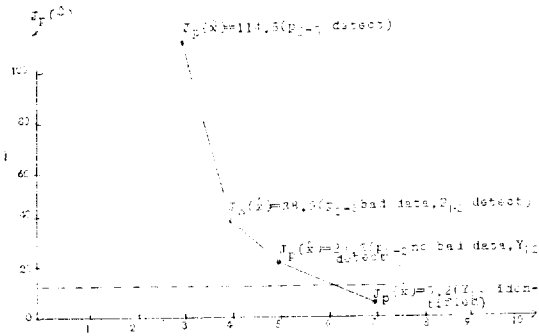


Fig. 5 The case of measurement bad data ( $Z_{p_{-1}}$ ) and line error ( $X_{L_{-1}}$ )

Fig. 5

8. Conclusions

from the test result it could be concluded that

1) though the line admittance was changed by unexpected line switching, the power system state with the suspected line admittance could be accurately be accurately estimated.

2) and it is possible to estimate without incoasing of the matrix dimension and decreasing of the redundancy.

To sum up, it could be said that the algorithms for identification of line errors operate efficiently and satisfactorily.

Nomenclature

Z; a given redundant mxl measurement vector set of measurements (the active and reactive line power flows, the bus injections and voltage magnitudes).

X; nxl state vector of the power system (the voltage magnitude and angle at each bus).

$\epsilon$ ; mxl measurement noise vector of zero mean Gaussian random variables

R; mxm covarince matrix of the measurement noise vector and is assumed to be diagonal

$h(x)$ ; mxl vector (the active and reactive line power flows, the bus injections and the voltage magnitudes which are the function of the state)

$H(x)$ ; mxn Jacobian matrix

$$\left( H(x) = \frac{\partial h(x)}{\partial x} \right)$$

ma; the number of the active line power flow and the active injection measurements

mr; the number of the reactive line power flow the reactive bus injection and the voltage magnitude measurements

Za; mxl vector of the active line power flow and the active bus injection measurements

Zr; mxl vector of the reactive line power flow, the reactive bus injection and the voltage magnitude measurements

k; the number of the line admittances of the suspected line error

Zae; (ma +k)xl vector of the active line power flow, the active bus injection measurements and the admittances of the suspected line error set

Zre; (mr+k)xl vector of the reactive line power flow, the reactive bus injection measurements, the bus voltage magnitude measurements and the admittances of the suspected line error

Y; k+1 admittances vector to be estimated.

9. References

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**Appendix**

The state estimate  $\hat{x}$  is a value which minimizes (A.1)

$$I(x) = [x - h(x)]^T \cdot R^{-1} [z - h(x)] \quad (A.1)$$

Thus

$$\left. \frac{\partial I(x)}{\partial x} \right|_{x = \hat{x}} = H^T(\hat{x}) \cdot R^{-1} \cdot [z - h(\hat{x})]$$

$$= H^T(\hat{x}) r^{-1} [h(x_i) + \varepsilon - h(\hat{x})] = 0$$

where  $x_i$  is the true state value

The linearization of  $h(x_i)$  using a Taylor series expansion about  $\hat{x}$  yields

$$H^T(\hat{x}) \cdot r^{-1} \cdot [h(\hat{x}) + H(x) \cdot (x_i - \hat{x}) + \varepsilon - h(\hat{x})] = 0$$

$$x_i - \hat{x} = -[H^T(x) R^{-1} H(x)]^{-1} \cdot H^T(x) R^{-1} \varepsilon$$

$$r = z - \hat{z}$$

$$= h(x) + \varepsilon - h(\hat{x})$$

$$\doteq h(\hat{x}) + H(\hat{x}) \cdot (x - \hat{x}) + \varepsilon - h(\hat{x})$$

$$= [I - H(\hat{x}) \cdot (H^T(\hat{x}) R^{-1} H(\hat{x}))^{-1} H(\hat{x})^T R^{-1}] \varepsilon$$

$$= W \varepsilon$$