時間遲延시스템의 最短時間制御에 對한 研究

論 文
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A Note on the Time Optimal Control of Dynamic Systems with Time Delay

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Abstract

The time optimal control of dynamic systems with time delay is studied with emphasis on the practical realization of controllers. An extensive survey on various methods of control is included and a result for a time optimal regulator with single delay in control is presented and simulated on a digital computer.

1. Introduction

In the mathematical formulation of physical processes, a large class of physical processes can be satisfactorily described by a system of ordinary difefrential equations. The assumption is that the future behavior of the system depends only on the present state and not at all upon its past history, and also, that the influence of the present state is instantaneous. But there are control processes that involve nonnegligible time delays, which exist when changes occuring at one point in a system are reproduced at another point after a finite interval of time. These can be described by systems of delay differential equations[34]. The significance of these equations lies in the ability to describe processes with aftereffect, which appear also in various branch of technology, economics, biology, and medical science.

One example of time delay system is the cold rolling mill [82,83] where the incoming sheet is rolled down through several rollers.An X-ray thickness gauge following the final roll measures the final thickness. The time delay in the system arises due to the spacing between the roll and the gauge, which is proportional to the spacing divided by the speed of the sheet. Another examples of time delay systems are the viscosity blender, the heat exchanger [83], the catalytic cracker [82], and the automatically controlled furnace where the material strip is heat-treated passing through the furnace [91].

Time delay in the system often causes an undesirable system performance such as oscillbtion, lengthy settling time or even breakdown. A feedback controller applies the corrective action based on the present state of output, therefore the automatic control of a process containing such a time delay is obviously difficult. Considering the frequency response of such a system, the time delay introduces phase lag without attenuation, thuse prmitting only low values of loop gain and the closed loop control becomes poor.

The study of specific time optimal system has been a major influence in the development of

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modern control theory. In section 2, the time optimal control problem for systems characterized by delay differnetial equations has been surveyed. Some results can be obtained by the extension of the Pontryagin's maximumrinciple and Neustadt's method in an abstract framework. Also, the problem with target in function space is discussed, which is a more realistic approach for the practical purpose of regulation. Problems wthi delayed control variable and no delay in the state variable are discussed, and many papers concerning the realization of such systems are discussed, based mainly on the theory of time optimal control for nondelayed case.

In section 3, the time optimal feedback control of linear systems with pure delay in the control path is discussed. Using the Smith's linear predictor scheme, the system with single delay in control can be transformed to the system without delay, and the time optimal bang-bang principle is applied and solved for the nondelayed system, which combines to become a practical implementation of the time optimal control of delayed control variable case. The results of simulation are discussed in section 4.

2. Summary of Previous Results

2-1. Delay-in-state case

Control problems involving the time delay sys tems have been investigated by many researchers [12,22], which are described by the following general delay differential equation (or differential equation with retarded argument) [14]

$$\frac{dx(t)}{dt}=f[x(t),\ x(t-\theta_1),\cdots\cdots,\quad (t-\theta_p),\ u(t),u(t-\nu_1),\cdots\cdots,\ u(t-\nu_q),t]\quad t\geqq t_0 \quad (1)$$

where $x(t)$ is an n-dimensional state vector

$u(t)$ is an r-dimensional control vector

and $0<\theta_1<\theta_2<\cdots\cdots<\theta_p,\ 0<\nu_1<\nu_2<\cdots\cdots<\nu_q$

together with the initial functions

$$x(t)=\phi(t),\ t_0-\theta_p\leqq t\leqq t_0 \quad (2)$$

$$u(t)=\psi(t),\ t_0-\nu_q\leqq t\leqq t_0. \quad (3)$$

It is the simplest and most natural type of functional differential equation (or differential equation with deviating argument) [12,34]

$$\frac{dx(t)}{dt}=f(x(\cdot),u(t),t) \quad (4)$$

where $x(\cdot)$ denotes the dependence of f on some of the values $x(s)$, $\alpha<s\leqq t$ with $\alpha\leqq t_0$=initial time. This functional differential equation is used to describe hereditary systems, whose dynamics are described by equation in which the rate of change of presents tate depends essentially on past data of the system, such as past control, or past state derivative [9,13,42].

When constant time delays appear only in the argument of the state variables of the system, (1) is reduced to

$$\dot{x}(t)=f(x(t),x(t-\theta_1),\cdots\cdots,\ x(t-\theta_p),u(t)) \quad (5)$$

withinitial conditions on $x(t)$ specified by (2). The problem is to minimize the objective functional (or cost functional) I represented by

$$I=G(x(t_f))+\int_{t_0}^{t_f}F[x(t),x(t-\theta_1),\cdots,x(t-\theta_p),\ u(t)]dt \quad (6)$$

by choosing the optimal control $u^*(t)$ within the admissible control set of $u(t)$ i.e., $u(t)\in\Omega$.

For the optimal control problem given by (5), the optimal control $u^*(t)$ maximizes the Hamiltonian defined by

$$H=F+\lambda^t f \quad (7)$$

where the adjoint vectrs $\lambda(t)$ are given by [77]

$$\frac{d\lambda^t}{dt}=\begin{cases}-\left\{\dfrac{\partial H(t)}{\partial x(t)}+\displaystyle\sum_{i=1}^{p}\left[\dfrac{\partial H(\tau)}{\partial x(\tau-\theta_i)}\right]_{\tau=t+\theta_i}\right\} & 0<t<t_f-\theta_p\\ -\left\{\dfrac{\partial H(t)}{\partial x(t)}+\displaystyle\sum_{i=1}^{p-k}\left[\dfrac{\partial H(\tau)}{\partial x(\tau-\theta_i)}\right]_{\tau=t+\theta_i}\right\} & t_f-\theta_{p+1-k}<t<t_f-\theta_{m-k}\\ & k=1,2,\cdots\cdots,\ p-1\\ -\dfrac{\partial H(t)}{\partial x(t)} & t_f-\theta_1<t<t_f.\end{cases} \quad (8)$$

Considering the equation (8), to obtain the $\lambda(t)$, a knowledge of $x(t)$ and $\lambda(t+\theta_i)$, $i=1,2,\cdots,p$ is required. Therefore computational schemes resulting from the undelayed maximum principle can not always possibly applied to the time delayed case.

This type of problem with delay-in-state has received the major attetnion. Kharatishvili[51] extended Pontryagin's maximum principle [75] for the case of single time delay in the state variable. Chyung [25,27] and Connor [29] has derived necessary conditions for single delays in linear systems which is represented as

$$\dot{x}(t)=A(t)x(t)+B(t)x(t-\tau)+\phi[u(t),t]. \quad (9)$$

Chyung and Lee [23, 24] have derived existence and unqueness theorems as well as necessry and sufficient conditions for optimality when the cost functional is either linear, quadratic, or satisfies certain convexity conditions. Controllability and stability of time delay system are treated by Lee and Manitus [62].

Banks [8] considered a nonlinear system with variable time dependent lags and derived an integral form maximum principle. Banas and Vacroux [6] considered the case with variable time dependent lags also. Connor [30] proposed a simple degradation method which is iterative for the solution of time delay systems having a state inequality constraint. Optimal control problems formulated with discrete-time systems with time lag is considered by Chyung[26] where the existence and the maximum principle are proved.

The time-optimal problem for systems characterized by linear delay differential equations has been investig ated by Haratišvili [44], also pontryagin et. al. [75], Shimemura [79], and Oğuztöreli [72, 73, 74]. Oğutztöreli (72, 73] has derived existence theorems and necessary conditions and generalized this work in his book[74] to the system with the equation

$$\dot{x}(t)=\sum_{k=0}^{p}A_k(t)x(t-\theta_k)+\int_{t_0}^{t}K(\tau,t)x(\tau)d\tau + B(t)u(t)+f(t) \quad (10)$$

$$0=\theta_0<\theta_1<\cdots\cdots<\theta_p$$

where K is a continuous matrix

$f(t)$ is a given continuous n-dimensional column vector with the control region

$$R=\{u \mid |u_k(t)|\leqq 1,\ k=1,\cdots\cdots,\ r\}. \quad (11)$$

He showed that the problem can be reduced to an integral equation using the kernal matrix representation, and the bang-bang principle due to La-Salle[58, 43] is extended for this problem, which is stated as follows; "If there is an optimal control, then there is always a bang-bang control that is optimal." Wells and Kashiwagi [92] used the Kernel matrix for synthesizing the time optimal control function for a second order linear systems with a constant state delay. Neustadt [68] described a method for synthesizing time optimal controllers for linear systems of ordinary differential equations, which is adapted to treat certain class of linear hereditary systems [10]. Jacobs and Pickel [50] applied the Neustadt's abstract variational theory [69, 70] to certain time optimal control problems involving neutral delay differential equations.

2-2. Delay in both state and control case

When the system has constant delays in both the state and the control, the system can be described by the equation(1) with initial condtions (2), (3). The objective functionial to be minimized is

$$I=G[x(t_f)]+\int_{t_0}^{t_f}F[x(t),\ x(t-\theta_1),\cdots,\ x(t-\theta_p),\ u(t),\ u(t-\nu),\cdots\cdots,\ u(t-\nu_q)]dt \quad (12)$$

by choosing $u^*(t)$, $t_0\leqq t\leqq t_f$.

Kharatishvili [52] dervied a maximum principle to an optimal control problem with delays both in the controls and in the state variables, which is the principle result for non-linear systems. Ichikawa [48] derived the same result by transforming the equation into an infinitely high order differential form. Lee[61] derived a maximum principle for the system

$$x(t)=Ax(t)+Bx(t-1)+Cu(t)+Du(t-a) \quad (13)$$

with convex cost functionals. Sufficiency condition and result on the existance have also been given. Chyung [28] applied the optimization technique to the discrete form.

Bate [14] considered a system by differential-integral equation and derived a necessary condition. Using the kernel matrix representation, he derived necessary conditions for quadratic problem, time optimal problem, and minimum effort control problem. He also applied the result to linear systems with delay in state and control. Halanay [41] established the maximum principle for systems with delay in both state and control and for systems with time-variable lags. Budelis and Bryson [19] derived some results for this system and presented an analytic solution for a linear system with a quardratic performance index. Ray and Soliman[77] surveyed the delay-in-state case and delays in both state and control. He extended the result to the case with variable timerdelays, and proposed a conjugate gradient control vector iteration algoriahm and tested on a continuous stirred tank reactor. Teo and Mcore [87] proved an integral maxiumum princple following the approach based onithe theory of quasiconvex families of functions [8, 38]. This result is used to obtain a pointwise maximum principle of the Pontryagin type. Hughes [47] extended a maximum principle by Hestenes [45] to control problems which involve delays in both the state and the control variable.

2-3. Function space target problem.

Many authors have dealt with problems of controlling a system to a target point in R^n- the Eucledian n-dimensional space. Thus the usual optimal control problem investigated was that of finding a controller which steers the output of a time delay system from a given inital function to a final point while minimizing the given cost functional. *In* systems without time delays, once the system has reached a target point, it is usually possible to remain at that state thereafter. However, when there is a time delay in the system, reaching a final point does not guarantee that the system can be remained at such a state thereafter. In most of the practical control systems, the objective is to change the present system state to a new state and to keep it in the new state. The regulator problem is one of the typical examples for such problems.

A more realistic approach for the control of time delay systems is the problem of controlling an object, subject to delay dynamics, not to a single point in R^n but to a target which is a point in a function space. For the practical purpose of regulation, this is a more realistic approach. The prescribed target may be a function defined on an interval of length which is a maximum delay of given system.

$$x(t,\phi,u)=\xi(t) \qquad t_f-\theta\leqq t\leqq t_f \qquad (14)$$

The problem of optimal control of time delay system to a final fuction target are studied by several researchers. In Banks [7] and Jacobs[49] no magnitude constraints in control variable were imposed. Banks and Kent [11] derived necessary and sufficient conditions in integral form for the system of retaded and neutral type. In Banks [11] and in Charrier [21], it is shown that the bang-bang property of LaSalle[58] does not hold in the strict sense but in some approximate sense. Using degeneracy of linear autonomous delay differential equations studied by Popov [76], Charrier [21] pointed out a new behavior of some controlled delay systems, which is called the loss of memory phenomenon.

Bien [16] derived a necessary condition in the form of a pointwise maximum principle whit a nontrivial adjoint solution. The result is applicable for the cases where there are magnitude constraints on the control variables. Westdal and Lehn [93] considered a time-optimal regulator problem with increasing the dimesion of the system which transforms to a nondelayed problem, and solved an example using the iterative methods on a computer.

2-4. Application

Considering the application of time optimal control theory to the practical systems, it is restricted to relatively simple systems such as the systems with delayed control variable and no delay in the state variable. This is due to the lack of results on the development of computational methods for optimal control synthesis. Much of the realization processes arebased on the theory of time optimal control for nondelayed case.

The theory for time optimal control for nondelayed system has been established rather completely [4, 15, 20, 39, 40, 46, 53, 57, 75, 89], and it is implemented to the real processes or simulated in many papers. Discrete time domain approaches are also done by many researchers [2, 3, 31, 32, 71, 95]. Burnovsky [17, 18] has established the theory for the closed loop time optimal control and solved in dimension two. Wang[90] has proposed the analytical method for practical design of electrohydraulic servomechanisms. A dual mode approch is used which involves opening the valve ports as wide as possible during the initial acceleration (bang-bang control), followed by controlled closure (conventional feedback con trol) during deceleration. A dual mode approach is also discussed in the paper by Freguson [35].

Fujisawa [36] has and Knudsen [54] have established an iterative procedure for solving the time-optimal regulator problem suitable for digital computers with the proof of exponential convergence. Baba [5] found the algorithms for the on-line closed-loop computer control of a linear process based on a time-suboptimal control for a linear discrete system. Sephaban [78] has designed experimental sing-axis time-optimal attitude control computer using the digital-differential analyzer technique. Mohleji and Thomas [66,67] designed the optimal third-order bang-bang systems using delayed switching, and presented a simple digital technique and takes into consideration any change in system parameters. Szabados et al [85,86] provided for the practical switching characteristics for minimum-time position control using a permanent-magnet motor and designed a time-optimal digital position controller for that system. Mellichamp [94] considered a feedback controller with the fast-time model which determines the control input each time by considering the state feedback information as initial conditions for the model using a hybrid computer.

For high-order systems, the direct application of optimal control theories offer considerable difficulties' since the calculation and implementation of the optimal controller is usually complex. In practice however, most plants are of high-order and there remains a need to adapt optimal control plant to such plants. One approach to this problem is that the high-order plant is approximated by a low-order system together with a pure delay. Optimal control theory can then be applied to the appropriate plant, even though it is of infinite order [36,60]. These approximated systems can be represented by

$$\dot{x}(t)=f(x(t),\ u(t-\tau)) \qquad (15)$$

Therfore the importance of optimality of contol systems having time delays in the control vector arises.

Fuller [37] centered attention on the case when the plant has only one control input and is represented by a pure delay followed by a linear time-invariant system. He showed that the optimal controller and optimal performance can ofte be caclulated, provided the corresponding result in the delay-free case are known. Examples for the case with the performance index

$$I=\int_0^\infty q(x_1,\cdots\cdots,\ x_n)dt \qquad (16)$$

and the minimum settling time are given. Some sub-optimal controllers are also discussed. Banks et. al. [10] studied this control delay systems systematically. The topics such as controllability, existance of solution, uniqueness, sufficient conditions are covered. Soliman [84] stated a new necessary condition for optimality of non-singular control problems with time delay. A necessary conditions for optimality of singular control problem (i.e., $\partial^2 H/\partial^2 u(t-\nu_1)=0$ for all $t\in[t_0, t_f]$) is also obtained.

Many researchers have developed techniques for synthesizing time optimal controllers for control delay case. Some of them applied such techniques to real processes. A number of exam ples of processes having transport delay and methods for quantitative determination of time lags are given by Ziegler and Nichols [97].

Thomasson and Cook [88] has described a method of constructing a swtiching curve for a second-order system with a delayed control vari able. The method is restricted to systems with distinct poles on the negative real axis. Latour and Koppel [55,59,60] has extensively studied the transportation lag systems. A two-position programmed controller, based on switching time is synthesized on an analog computer [55]which has experimented on the water temperature process [59]. Also, a feedback, time-optium, switching controllers is reported with emphasis on their practical design characteristics for process control [60]. Miller [65] has derived a control algorithm for basis weight of a wet-end dynamics, with limitations in real-time measurements of product qualities, which operates on a profile average feedback and is adaptive in terms of machine speed. Kurzweil [56] presented a digitally oriented technique for the control of processes in which a primary characteristic of the process is a set of pure transport delays associated with the input dynamics of the process. Zahr and Slivinsky [96] has constructed an algorithm to compensate time delay effects in the process and verified on third order systems with two inputs and two outputs.

An alternative approach for time delay compensation in feedback control systems, the analytical predictor has been developed by Moore et. al. [99]. In their approach a process model is used to predict the future output from current measurements and the predicted value is sent to the controller. Meyer et. al. [98] used this technique for experimental and simulation studies.

3. Time-Optimal Feedback Solution for Single Delay in Control Case

In this section, the attention is restricted to the time-optimal feedback control of the linear systems with pure delalys in the control paths only.

As mentioned previously, conventional feedback controller posseses many problems such as oscillation, low values of loop gain etc. But the design of the feedback controller for time delay system is generally very difficult. In addition, even though the design produre has been finished, the practical implementation can not be done in many cases because of hardware difficulties. The advent of computers, and recently, the wide use of microprocessors have decreased burdens on the hardware implementation of control systems. This section covers the application of one of schemes for compensating the time delay [33,94], viz., linear predictor control [82,83] to time-optimal feedback control and describes the methods to simulate these scheme on a digital computer. This controller can be implemented directly on the microcomputer based control system.

The conventional feedback process control system is shown in Fig. 1 with a forward transfer function of the process with pure delay written as

$$\frac{C(s)}{U(s)}=G_p(s)=KG_0(s)e^{-\tau s} \tag{17}$$

where K=steady-state gain factor of the proce-

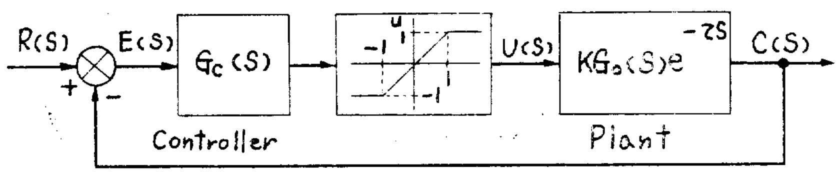

Fig. 1. The conventional feedback process control system with pure control delay

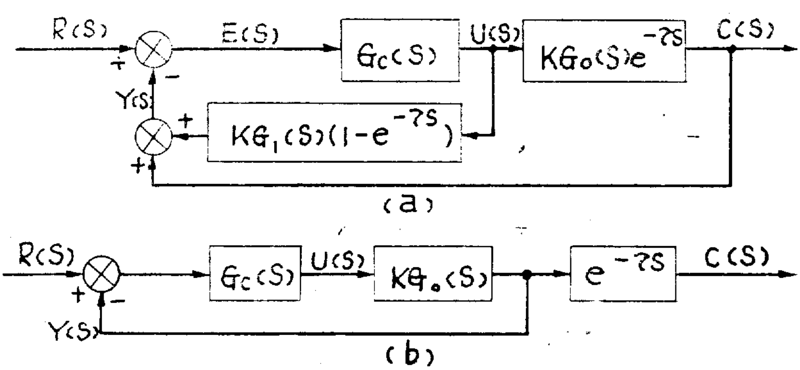

Fig. 2. Linear Predictor Control Scheme

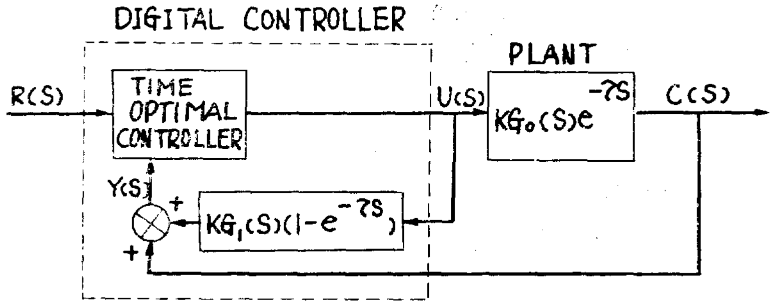

Fig. 3. Time optimal feedback regulator

sses and τ=pure time-delay constant, with input constraints $|u(t)|\leqq 1$.

In the linear predictrol Control suggested by Smith [81, 82, 83] the information based the present state of the output is analyzed, the state after a time equal to the dead time is anticipated and a corrective action is applied immediately, by providing a minor feedback loop around a conventional controller with the transfer function (Fig. 2-a)

$$G_x(s)=KG_1(s)(1-e^{-\tau s}) \qquad (18)$$

where $G_1(s)$=process model transfer function= $G_0(s)$.

Singh [80] have shown that using an approximate analog model of dead time synthesized from electronic components, a controller using the linear predictor approach is quite practicable. Mee [63] extended this predictor control to the problem with a quadratic criterion.

If the nondelayed state output can be obtained, it is possible to construct a feedback time-optimal controller $u(t)$ based on the switching surface $(y_1(t),\ y_2(t))$ in the state-space. In Fig. 2, $Y(s)$ can be represented by

$$Y(s)=KG_1(s)\ (1-e^{-\tau s})\ U(s)+C(s) \qquad (19)$$

Assuming the process model transfer function $G_1(s)$ is equal to the plant transfer function $G_0(s)$, $Y(s)$ becomes

$$Y(s)=KG_1(s)U(s)-KG_1(s)e^{-\tau s}U(s)+KG_0(s)e^{-\tau s}U(s)$$
$$=KG_1(s)U(s) \qquad (20)$$

So, $Y(s)$ can be considered as the desired nondelayed state output as in Fig. 2-b. By using this state $y(t)$, the time-optimal controller can be implemented as in Fig. 3. If $y(t)$ is not available, it is required that $u(t)$ must be determined by the future values of $c(t)$, considering the equation (17) for the feedback configuration. But, $u(t)$ can be obtained as a function of $y(t)$ since $y(t)$ can be obtained from $c(t)$ and prior values of $u(t)$.

The control of processes whose dynamic behavior can be adequately represented by the second-order plus dead time transfer function

$$\frac{C(s)}{U(s)}=G_p(s)=\frac{K\exp(-\tau s)}{(s+p_1)\ (s+p_2)} \qquad (21)$$

is of interest since this class of process includes many operations in the process industries[55, 60]. In this case, equation (19) is represented by

$$\frac{Y(s)}{U(s)}=\frac{K}{(s+p_1)\ (s+p_2)} \qquad (22)$$

whose state variable representation is

$$\dot{y}(t)=Ay(t)+Bu(t)$$

or
$$\begin{bmatrix}\dot{y}_1(t)\\ \dot{y}_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ -p_1p_2 & -p_1-p_2\end{bmatrix}\begin{bmatrix}y_1(t)\\ y_2(t)\end{bmatrix}+\begin{bmatrix}0\\ K\end{bmatrix}u(t) \qquad (23)$$

The time optimal control of this problem for a step input of $r(t)$ can be stated as follows: given the system (23) with initial condition $u(t)=0,\ -\tau\leqq t\leqq 0$, find the minimum time t^* and the optimal trajectory y^* which renders the system state $y_1(t)=d,\quad t\geqq t^*$ in minimum time t^* with the admissible control set $|u(t)|\leqq 1$, for all t.

By using the similarity transformation[4]

$$z(t)=P^{-1}y(t) \qquad (24)$$

where

$$P=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix}$$

and λ_1, λ_2 are the eigenvalues of the matrix A, which become

$$\lambda_1=-p_1,\ \ \lambda_2=-p_2$$

for this case. It is convenient to define the state variables $x_1(t)$ and $x_2(t)$ by the relations

$$x_1=(t)-\frac{\lambda_1(\lambda_2-\lambda_1)}{K}z_1(t)$$

$$x^z(t)=\frac{\lambda_2(\lambda_2^{\kappa}-\lambda_1)}{K}z_2(t) \qquad (25)$$

Then $x_1(t)$ and $x_2(t)$ satisfy the differential equation

$$\dot{x}_1(t)=\lambda_1 x_1(t)+\lambda_1 u(t)$$

$$\dot{x}_2(t)=\lambda_2 x(t)+\lambda_2 u(t) \qquad (26)$$

Using the Pontryagin's maximum principle, the optimal control must be

$$u(t)=\Delta=\pm 1 \qquad (27)$$

and the optimal trajectory is described by

$$x_2(t)=-\Delta+(\xi_2+\Delta)\left[\frac{x_1(t)+\Delta}{\xi_1+\Delta}\right]^{\alpha} \qquad (28)$$

where $\alpha\triangleq\lambda_2/\lambda_1$. Assuming that $0>\lambda_1>\lambda_2$, the trajectoreis for this problem is drawn as in Fig. 4

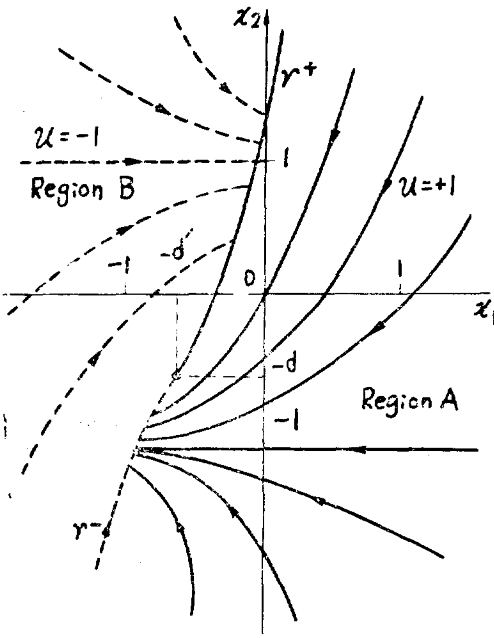

Fig. 4. Time-optimal Trajectoreis

The coordinates of the state planecan be transformed into each other by the relation

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}=\frac{1}{K}\begin{bmatrix}-\lambda_1\lambda_2 & \lambda_1\\-\lambda_1\lambda_2 & \lambda_2\end{bmatrix}\begin{bmatrix}y_1\\y_2\end{bmatrix} \qquad (29)$$

and $$\begin{bmatrix}y_1\\y_1\end{bmatrix}=\frac{K}{\lambda_2-\lambda_1}\begin{bmatrix}-1/\lambda_1 & 1/\lambda_2\\-1 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} \qquad (30)$$

The time optimal feedback control law for this case can be implemented as follows;

i) transform y_1, y_2 into x_1, x_2 usng the relation (29)

ii) If x_1, x_2 is in the region A or on the line γ^+ then $u(t)=1$.

iii) If x_1, x_2 is in the region B or on the line γ^- then $u(t)=-1$.

iv) For each time slice, repeat steps i), ii) and iii)

4. Simulation Results

Three kinds of controllers —the conventional linear feedback controller, the linear prediction controller, and the time-optimal feedback controller are simulated using a digital computer Nova 830. The controller and the plant dynamics are discretized using the Euler approximation of the derivatives [1]. All the programs are written in BASIC language, and the out puts are drawn on the graphic terminal using the adequate software routines.

The conventional linear controller used the differencess between the reference signal $u(t)$ and the feedback signal $c(t)$ as an input as shown in Fig.1. The linear prediction controller uses a compensated output state $y(t)$ for the feedback singal as in Fig. 2. The time optimal feedback controller accepts informations about the input $u(t)$ as well as the output state $c(t)$. On calculating the compensated term $y(t)$, the controller decides $u(t)$ with the information $r(t)$ and $y(t)$ using the time optimal control law represented in the last part of section 3 (refer Fig. 3). All inputs are bounded by $|u(t)|\leqq 1$.

The simulated results for the output state for the step input of $r(t)$ are shown in Fig. 5. In this simulation, the discretized time interval is set to 0.01 sec, and the time delay is set to 1 sec, and finally the controller gain $G_c(s)$ is set to unity. Simulations are done with various parameters of the plant constants i.e., by varying the forward gain factor K, and the values of poles P_1, P_2. Typical results for the parameter $K=1$, $P_1=1$, $P_2=2$ are shown in Fig. 5-a, and $K=10$,

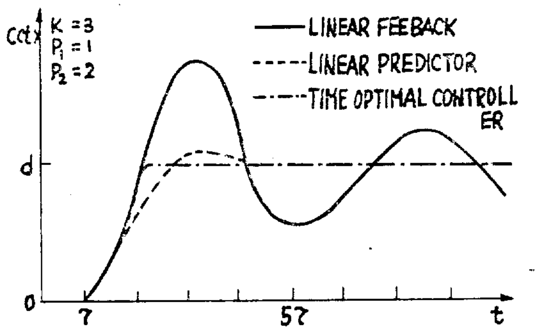

Fig. 5-a. Simulation Results

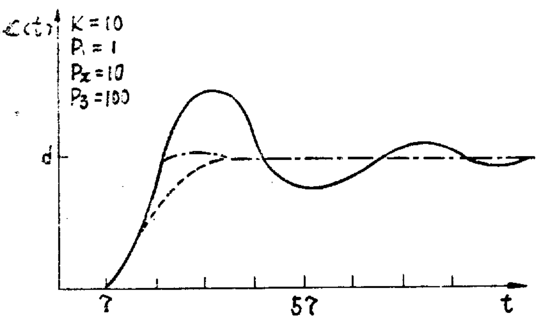

Fig. 5-b. Simulation Results

$P_1=1$, $P_2=10$ are shown in Fig. 5-b. From this result, it is verified that the performance of the time optimal feedback controller are superior than the other controllers. The linear prediction controller can eliminate the time delay effect, which causes oscillatory output for the conventional feedback controller.But the response time of the linear prediction controller is somewhat long, and this settling time can be minimized by adopting the time optimal controller. It is obvious from the figure that the responses are appeared τ seconds later after the inputs are applied for all three cases, which is not different from the physical intuition.

In the above simulation, it is assumed that the dynamics of model is exactly identical to the dynamics of plant. But the accurate description is very complicated even if it can be obtained. Therefore, the approximate models of the plant dynamics are used generally. In addition, the plant dynamics may be represented by higher order systems-say, more than 2nd order in many physical systems. In this case, the calculation of optimal solution is rather difficult and consumes much of time. Following these reasons, it is ncessary to consider the case when the model is not identical to the plant, or furthermore, when the model is a low order say, second order approximation of the higher order plant dynamics.

In the time optimal scheme suggested in section 3, it can be verified that even if the model is inaccurate, the steady state valueis equal to the case with the accurate model. Referring Fig. 3, the term $1-e^{-\tau s}$ becomes zero as the system goes to steady state since the output state remains constant in steady state. So, the minor feedback term $KG_1(s)$ $(1-e^{-\tau s})$ has no effect and the feed back signal $y(t)$ is equal to $c(t)$ in the steady state. It follows from this consideration that the steady state value is not affected by the inaccurate model. Only the transient response may be affected.

Simulations are done for the case when there is a third-order plant with poles P_1, P_2, P_3, and using the approximate second-order model with poles P_1, P_2 which is obtained by neglecting the pole P_3 the pole with largest value among the three. Simulated results for this case are shown in Fig. 6. Fig. 6-1 shows the results for the case with parameters $K=3, P_1=1$, $P_2=2$, $P_3=20$, and Fig. 6-b shows for $K=10$, $P_1=1$, $P_2=10$,

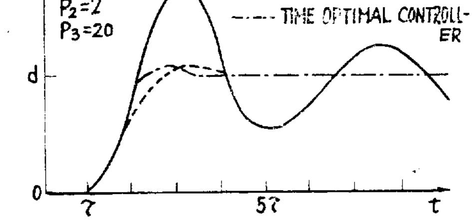

Fig. 6-a. Simulation Results with Approximated Model

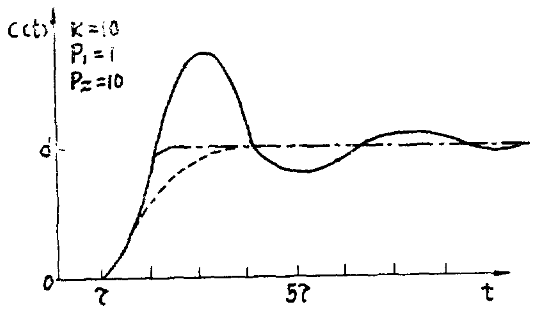

Fig. 6-b. Simulation Results with Approximated Model

$P_3=100$. From this result, it is shown that the performances of the time optimal feedback controller are superior also in this case than the other controllers. The responses are faily good though the transient response for this case are somewhat deteriorated due to the modeling error. The results of simulation for various plant parameters show that the responses become better as the third pole P_3 is further from the second larger pole P_3, or three poles become fairly distinct.

For simpler cases with the 1st order plant and the model, simulations are done and the results are similar to the above discussions as expected.

5. Conclusion and Topics for Future Research

The time optimal control problem for systems characteizeyed by delay differential equation has been surved. Also, the time optimal feedback regulator is synthesized and simulated for the case with pure delay in the control paths only. The results have shown that this controller can be implemented on the real-time microcomputer based control system. The prior task for the implementation of this system is the appropriate modeling of the plant dynamics.

The accurate solution based on the necessary and/or sufficient condition for time optimal contrl of dynamic systems with time delay is, in general, very difficult. In addition, even though the accurate solution may be obtained, the implementation to the real process posseses many hardware difficulties, and even if digital computer is used for the controller,the time limitation arises due to the calculation time of the computer. So the development of the approximate syn thesizing algorithm and implementation technique should be necessary, which is realized to become a sub-optimal controller with slight degradation of performances.

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