Journal of the Radiation Protection
Vol. 4, No. 1, July, 1979

<Original>

Calculation of the Air-Scattering Dose Rate by the Single Scattering Approximation

Chong Chul Yook

Department of Nuclear Engineering, Han Yang University, Seoul, Korea

Chung Woo Ha, Jai Ki Lee, Philip S. Moon

Korea Atomic Energy Research Institute, Seoul, Korea

ABSTRACT

A calculation is presented of air-scattered gamma rays using the modified single-scattering approximation. The air-scattered tissue dose rates are calculated for a general purpose taking into account (a) the buildup and exponential attenuation, (b) the energy spectrum at the position of question and (c) the geometrical scattering volume in three dimensions. These calculations have been further modified to render them applicable to a typical field irradiation facility which is surrounded by a shield wall and in which the source is fitted with a beam collimating device. The results of the calculation include the energy spectra, angular distribution and tissue does rates at source-receiver separation distances of from 35m to 300m. The comparison shows that the present method developed may be generally adequate for the gamma-ray air-scattering problems in field irradiation facilities if energy and angular distribution at the shield are unimportant.

1. Introduction

In the design of large gamma field with purposes of agriculture it is an importance that the evaluation of the exposure rates due to the air-scattered gamma-ray at various distances from the source is made, since these may be included in uncontrolled areas such as public roads, residential areas and laboratory buildings. Even though fairly thick concrete walls surrounding the irradiation facilities of the gamma field could reduce the primary radiation to negligible levels, scattering by the infinite medium of air may result in significant radiation levels compared to the limited doses for the general public, if not due considerations in the shield design. Hence it is necessary to estimate these levels as a part of environmental surveilance programs.

General methods for the determination of the air-scattering contribution from any medium have been described in details by Stephenson[1]. A simple approximation has been proposed by Keller et al[2]. for neutrons scattered in air isotropically in the center-of-mass system, and by Trubey[3] for gamma-rays as well. They neglected the exponential attenuation and buildup, and thus considered only a line beam of monoenergetic gamma rays from a point source to assess the effect of direction. The gamma-ray air-scattering problems in field irradiation facilities have heen worked out by Cowan and Meinhold[4], and Kitazume[5]. However, these values can not be directly applied to positions outside a gamma fields which is bounded by a protective wall.

In this paper, the air-scattered exposure rates are theoretically calculated for a general purpose taking into account (a) the buildup and exponential attenuation, and (b) the erergy spectrum at the position of question. These calculations have further been modified to render them applicable to a typical field irradiation facility which is surrounded by a shield wall and in which the source is fitted with a beam limiting device. The calculated exposure rates at various distances from the source are presented in figures.

2. Theory

2-A. General Methods

It has been hypothesized that a good approximation to the scattered flux in air is the single-scattered flux with exponential attenuation and buildup, and that the scattered radiation under consideration arises from Compton scattering.

The photons emitted from a monoergetic isotropic point source (S) with an azimuthal angle ϕ are scattered by the electrons in an infinitesimal volume dV to a receiver located at P, as shown in Fig. 1. The photon flux incident upon the volume element is given

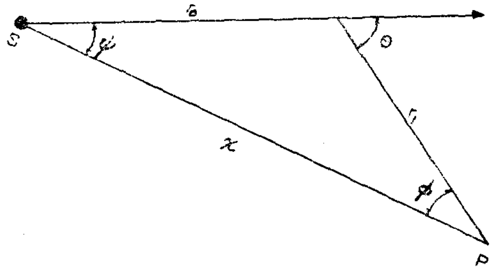

Fig. 1. Schematic representation of single scattering.

by:

$$\Phi_0=\frac{S(E_0)\exp(-\mu_0 r_0)}{4\pi r_0^2} \qquad \cdots\cdots(1)$$

The differential photon flux at the receiver, $d\phi(E)$, attributable to the photons scattered within the volume dV through an angle θ is:

$$d\Phi(E,\psi,X)=\frac{B(\mu_1 r_1)\Phi_0\frac{d\sigma}{d\Omega}\exp(-\mu_1 r_1)NdV}{r_1^2}$$

where $\cdots\cdots(2)$

$B(\mu_1 r_1)$=the buildup factor for photons of energy E;

μ_0 and μ_1=linear attenuation coefficients of air for the primary and scattered energies

r_0=distance from the source to the volume element

r_1=distance from the volume element to the receiver

$S(E_0)$=point isotropic source strength in photons/sec

$\frac{d\sigma}{d\Omega}$=Klein-Nishina differential scattering cross section for scattering through an angle θ per steradian.

N=number density of electrons in air (3.92 x 10^{20} electrons/cm^3 at STP)

E_0=average energy of the primary radiation (1.25 MeV for ^{60}Co)

There is cylindrical symmetry about X-axis for the scattering so that the volume of the scattering element is

$$dV=(\pi r_0\sin\psi)r_0 d\psi dr_0 \qquad \cdots\cdots(3)$$

where ψ is the half angle of the cone subtended by the scattering volume element at the source position. For scattering through an angle θ, the differential cross section is given bv

$$\frac{d\sigma}{d\Omega}=\frac{R_0^2}{2}(P-P^2\sin^2\theta+P^3) \quad \cdots\cdots(4)$$

where R_0 is the classical electron radius$(=\frac{e^2}{mc^2})$ and p is the energy fraction of the scattered photon of epergy E through an angle θ to the primary photon with energy of E_0, and is defined by

$$P(\theta, E_0)=\frac{E}{E_0}=\frac{1}{1+\alpha(1-\cos\theta)},\quad \alpha=\frac{E_0}{m_0C^2} \quad \cdots\cdots(5)$$

Substitution of Eq. (3), (4) and (5) into (2) yields following expression:

$$d\Phi(E,\psi,X)=3.92\times10^{20}\frac{B(\mu_1r_1)S(E_0)R_0^2(P-P^2\sin^2\theta+P^3)}{8}\times\exp[-\mu_0r_0-\mu_1(E)r_1]\sin\psi d\psi dr_0 \quad \cdots\cdots(6)$$

Using the law of sines and differentiating, it can be shown[1] that

$$dr_0=\frac{r_1^2}{x\sin\psi}d\phi \quad \cdots\cdots(7)$$

and

$$d\Phi(E,\psi,X)=\frac{3.92\times10^{20}S(E_0)R_0^2B(\mu_1r_1)(P-P^2\sin^2\theta+P^3)}{8x}\times\exp[-\mu_0r_0-\mu_1(E)r_1]d\phi d\phi \cdots\cdots(8)$$

The differental scattered tissue dose rate dD, at the point of receiver due to the radiation scattered from a infinitesimal volume dV at a distance r_0 from the source is given by:

$$dD(\theta,X)=d\phi(\theta,X)DF_g(E) \quad \cdots\cdots(9)$$

where $DF_g(E)$ is the gamma-ray-flux-to-dose-rate conversion factor which can be computed using the polynonial analytic fit[6]. This analytic form is useful for the calculation of flux-weighted conversion factors over a specified energy interval. The general form of the analytic function is

$$\ln DF_g(E)=A+BX+CX^2+DX^3$$

where $DF_g(E)$ = flux-to-dose-rate conversion factor at energy E, in (rem/hr)/(photons/cm²-sec)
E = gamma-ray energy in MeV
$X=\ln E$

The coefficients of the polynomial expression are taken from American National Standard[6]. The conversion factors presented in the Standard are intended for use in calculations of whole-body exposure resulting from routine operation of a radiation facility.

Since the energy and angle are related by the well-known Compton formula, the conversion factor, $DF_g(E)$, is a computable function of the angle θ. The total tissue dose rate at the receiver by a single scattering can be obtained by integration of the differential scattered dose rate over all volume from which scattering can occur. Expressing r_0 and r_1 in terms of angle ψ, θ and distance x of the point, P, from the source and integrating over all space, we have

$$D(X)=\frac{3.92\times10^{20}S(E_0)R^2}{8x}\int_0^\pi\int_\psi^\pi B(\mu_1r_1)(P-P^2\sin^2\theta+P^3)\times\exp[-\mu_0x\sin(\theta-\psi)/\sin\theta]\exp[-\mu_1(E)x\sin\psi/\sin\theta]d\theta d\psi \quad \cdots\cdots(10')$$

But $\theta=\psi$: therefore

$$D(X)=\frac{3.92\times10^{20}S(E_0)R^2}{8x}\times\int_0^\pi\int_\psi^\pi B(\mu_1r_1)(P-P^2\sin^2\theta+P^3)\times\exp[-\mu_0x\sin(\theta-\psi)/\sin\theta-\mu_1(E)x\sin\psi/\sin\theta]\times DF_g d\theta d\psi \quad \cdots\cdots(10)$$

This expression can be written as follows:

$$D(X)=C\int_0^\pi f(\psi,x)d\psi \quad \cdots\cdots(11)$$

by defining

$$f(\psi, x)=\int_{\psi}^{\pi} DF_g(P)A(\theta, \psi, x)(P-P^2\sin^2\theta+P^3) \times B(\mu_1 r_1)d\theta \qquad \cdots\cdots(12)$$

where $A(\theta, \psi, x)$

=factor representing the attenuation in air

$$=\exp\left[-\frac{x}{\sin\theta}\left(\frac{\sin(\theta-\psi)}{l_0}+\frac{\sin\psi}{l}\right)\right]$$

(where l_0 and l are the photon mean-free-path lengths in air before and after scattering, respectively)

$$C=\frac{3.92\times10^{20}S(E_0)R_0}{8x}$$

Eq. (11) can be numerically evaluated by applying Simpson's rule. The energy of the scattered photons can vary from 0.01 to 1.25 MeV. Buildup factors have been obtained using linear formula[7] given by

$$B(\mu r)=1+\beta\mu r$$

where β is a fitting parameter. For the purposes of computation this has been expressed as a function of energy in the form:

$$\beta=1.1E^{-0.46} \qquad \cdots\cdots(13)$$

Similarly, attenuation coefficients of air is written as follow:

$$\mu=7.7\times10^{-5}E^{-0.51}cm^{-1} \qquad \cdots\cdots(14)$$

2-B. Application to a Gamma Field

The air-scattering calculation method developed here can be applied to the large gamma field of Geumgok Gamma Field located at Geumgok near Seoul. The gamma field has a cobalt-60 irradiation facility, which is surrounded by a concrete wall. The gamma source of ^{60}Co with activity of about 100 Ci at the time of installation is centrally located at the green house. The source is a cylindrical rod of 1.124cm in diameter and 2.54cm in length, and has a shielding lead plug attached to it on the top so that the plug could shield a cone of 80°. The source container is stored underground at the center of the green house. Right above the container, a hollow cylindrical lead cap is suspended with 2m height from the floor and a aluminum guiding pipe is installed inside the lead cap as shown in Fig.2. The lead cap and lead plug, therefore, attenuate the primary beam and hence the air-scattering. For simplification of shielding calculation without loss of generality, it could be convenient to transform the geometrical complexity into a equivalent spherical shell with solid angle about source position which is subtended by the top of the wall.

The gamma field has a radius of about 33m and it bounded by a ordinary concrete wall of height 3m and thinkness of about 75cm. The wall surrounding the gamma field attenuates the primary radiation. The wall in the present field is equal to about 6.5half value layers. The dose due to the primary radiation just outside the wall is about 0.02mR/hr. It also contributes to

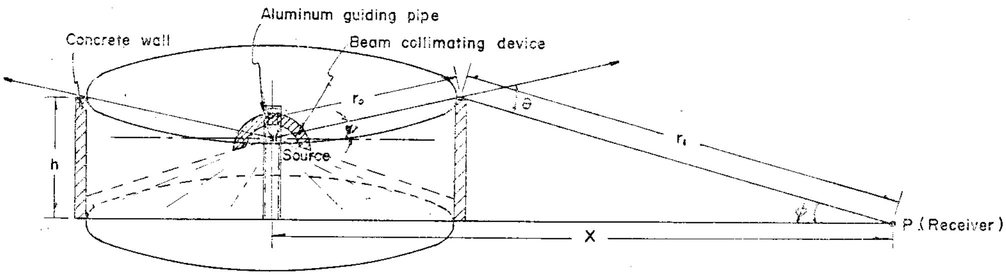

Fig. 2. Geometrical model for calculation of air-scattering dose rate in a typical gamma field irradiation facility surrounded by a wall of height h. and shieided by a beam collimating device.

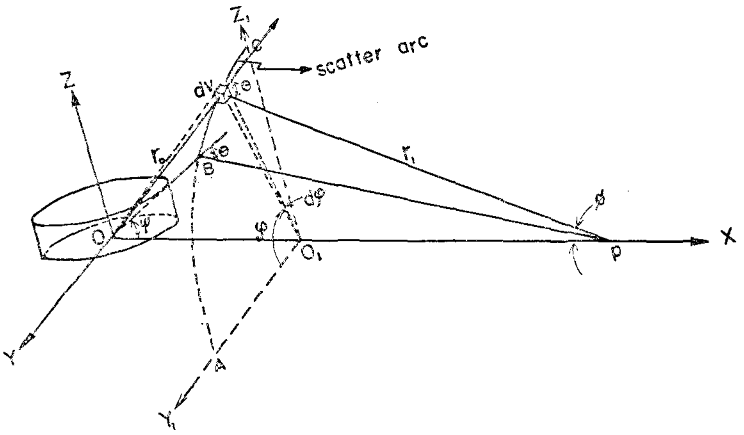

Fig. 3. Geometry used in 3-dimensional calculation.

eliminating the radiation scattered from ground, casts a shadow and reduces the air-scattering exposure dose at point close to the wall. Furthermore, the scattering volume of air should be reduced and consequently the air-scattering as well.

Murthy et al.[8] took into account this shadow effect by considering the apex angle, ω, of the cone of air irradiated and angle ψ and ϕ, where ϕ is the angle subtended by the wall at the receiptor position P. The integrating limits of ψ in Eq. (10) now change from ψ_1 to ψ_2, and of θ from $\psi+\phi_0$ to π where $\psi_2=\pi-\psi_1$, if $\psi_1>\phi_0$ and $\psi_2=\pi-\phi_0$ if $\psi_1<\phi_0$. Thus the effective dose rate was obtained by multiplying $D(x)$ in Eq. (10) by the geometrical factor $\frac{\omega}{2\pi}$:

$$D(x)=\frac{\omega NS(E_0)R^2}{16\pi x}\int_{\psi_1}^{\psi_2}\int_{\psi+\phi_0}^{\pi}B(P-P^2\sin^2\theta+P^3)\ \exp[-\mu_0 x\sin(\theta-\psi)/\sin\theta-\mu_1 x\sin\psi/\sin\theta]DF_g d\theta d\psi \cdots (15)$$

But this expression is only valid for isotropic scattering in which the geometrical factor $\frac{\omega}{2\pi}$ can be applicable. However, the method developed in this paper takes into account the real differential volume in which air-scattering is occurring, resulting in more realistic value with improved accuracy. Geometry for this new model of calculating scatter exposure rate is represented in Fig. 3. In this formulation, it is basically different from the approache by Murthy that the differential scatter volume is represented in three dimension rather that two dimension in Eq. (3);

$$dV=r_0^2 r_1 \sin\psi d\psi d\theta d\varphi \qquad \cdots\cdots (16)$$

Therefore, the three dimensional representation corresponding to Eq. (15) becomes

$$D(x)=\frac{SNR_0^2}{4\pi x}\int_{\psi_1}^{\psi_2}\int_{\varphi+\phi_0}^{\pi}\int_{\varphi_m}^{\frac{\pi}{2}}B(P-P^2\sin^2\theta+P^3)\times\exp[-\mu_0 x\sin(\theta-\psi)/\sin\theta-\mu_1 x\sin\psi/\sin\theta]\ DF_g d\varphi d\theta d\psi \qquad \cdots\cdots (17)$$

where φ_m is the minimum value of angle φ to the lowest point, B, on the arc which can be seen from the source position. This value can be determined by projecting the circumference of the wall on the Y_1-Z_1 plane from the source position, as shown in Fig. 4. Determinig the point of intersection of the arc and the parabola which is the projecting line, one can obtain,

$$\varphi_m=\tan^{-1}\frac{h}{\sqrt{d^2\sin^2\psi-h^2\cos^2\psi}} \qquad \cdots\cdots (18)$$

where d is the distance from the source to shield wall, and h is the height of the wall.

Because the integrand in Eq. (17) is independent of angle φ and φ_m only depends on ψ,

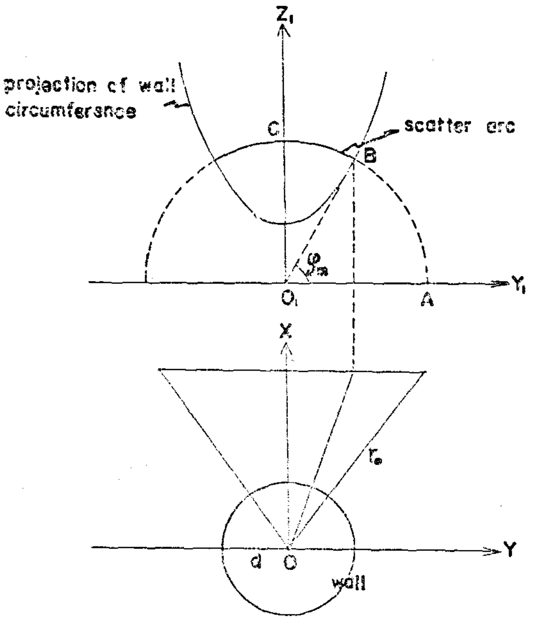

Fig. 4. Diagram for determination of φ_m; parabolar $d^2 z_1^2 - h^2 y_1^2 = h^2 r_0^2 \cos^2\psi$ arc $y_1^2 + z_1^2 = r_0^2 \sin^2\psi$

Eq. (17) finally reduces

$$D(x) = \frac{SN\,R_0^2}{4\pi x}\int_{\varphi_1}^{\varphi_2}\left(\frac{\pi}{2}-\varphi_m\right)\int_{\psi+\varphi_0}^{\pi} B(P-P^2\sin^2\theta+P^3)\exp[-\mu_0 x\sin(\theta-\psi)/\sin\theta-\mu_1 x\sin\psi/\sin\theta]\,DF_g\,d\theta\,d\psi \quad \cdots\cdots(19)$$

3. Results and Discussion

The resulting tissue dose rates in air as a function of distance for the different shield thicknesses of a hemispherical lead-cap are shown in Fig. 5. Fig. 6 shows the calculated air-scattering tissue dose rates in the presence of walls of various heights located at a distance of 33m from the source. The calculated air-scattering tissue dose rates are obtained without consideration of contributions to the exposure levels at the points of interests from primary radiation. With the absence of a wall, the dose rate, as can be seen, decreases continuously with increasing distance from the source. However, it increases rapidly with distance from the wall, reading a maximum at about 7 to 13m depending upon the height of the wall and then decreases thereafter as shown in Fig. 6. The transition positions having the maximum

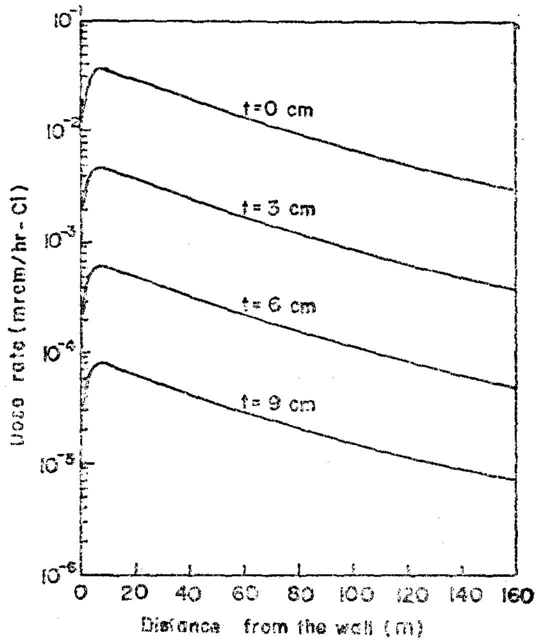

Fig. 5. The variations of the air-scattering dose rate as a function of distance for the different shield thickness.

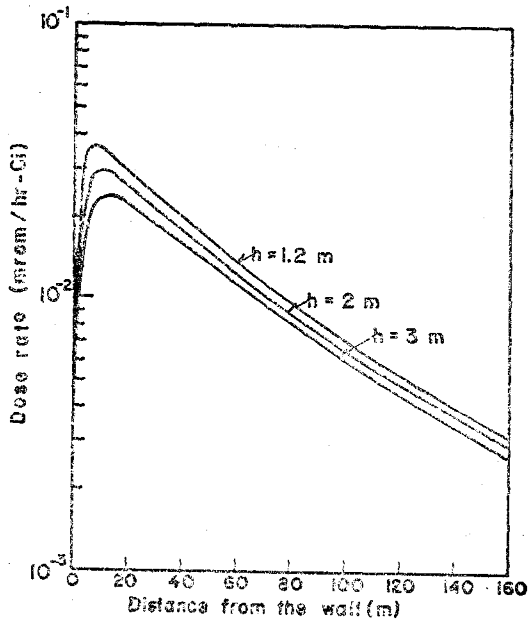

Fig. 6. The exposure dose rate as a function of the distance for different wall heights.

dose rate are moved outward from the wall with increasing the height of the wall due to the shadow effect. Further arralysis manifest the fact that the wall effectively contributes to reducing the scattered radiation outside the boundary of the gamma field and the primary radiation as well. The reduction brought about the wall at positions close to it is attributed to the elimination of both ground scattered and air scattered radiation. At farther positions, the wall will be effective in eliminating only the ground scatter. Since the wall will also attenuate the primary radiation, its usefulness in reducing the exposure levels outside the boundary will be further enhanced. It can also be found that increasing the wall height would not result in significant reduction of exposure dose rates.

The results of a calculation using the method developed here, with appropriate considerations of the geometrical scattering volume, have been compared to results of the Murthy's calculation of the exposure rate in air from a ^{60}Co-gamma rays source, and of actual field measurements. The comparison shows that the Murthy's results are higher as much as a factor of about two as shown in Fig. 7 and 8. on the whole. It is probably due to the fact that Murthy had calculated the exposure rates for an isotropic source by using the two dimensional equation given by Eq. (15) in which the polarized scatterings are unimportant. However, the results presented here have been obtained from the equation in three dimensions given by Eq. (19). A comparison could be made with the actual field measurements by means of a 2"×2" NaI (Tl) scintillation survey meter. The results are a good agreement with the measured values. A discrepancy is observed at the maximum value where the measured

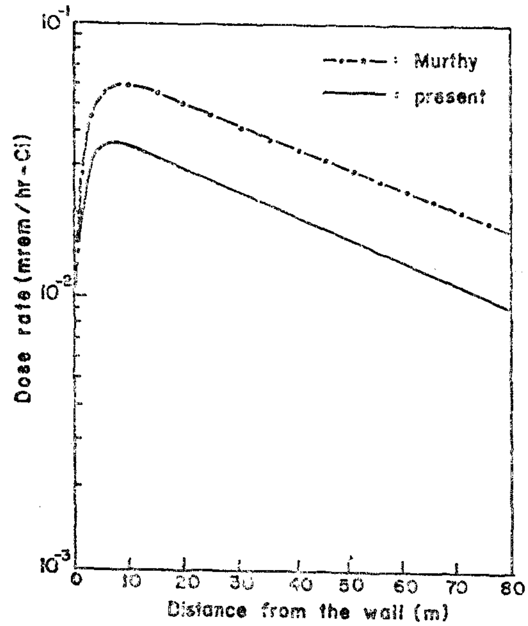

Fig. 7. Comparison of the air-scattering does rates calculated by present method with those of Murthy in the gamma garden surrounded by a wall of height, 1.2 m.

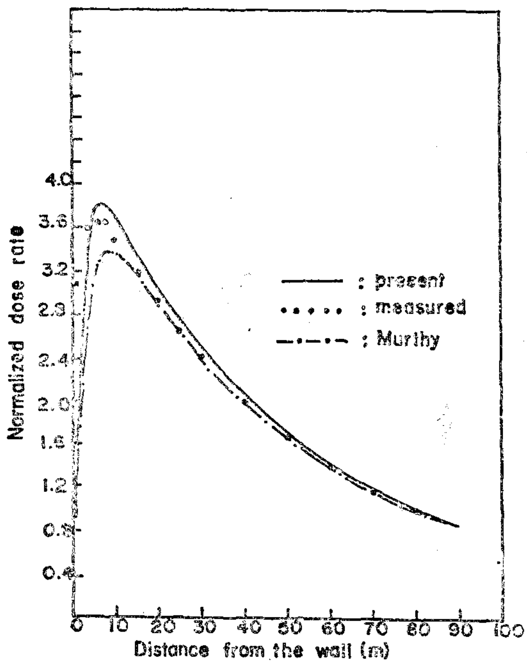

Fig. 8. Comparison of the normalized exposure dose rates calculated by the two different method and those measured by NaI (Tl) scintillation survey meter (Nomalized to data at 80 m from the wall)

value is about 4 percent higher.

It can be concluded that the present method is generally adequate for calculating the dose rate and number flux in air from a monoenergetic gamma-ray source with improved accuracy, if energy and angular distribution at the sky-shine shield are not important.

References

1) R. Stephenson, Introduction to Nuclear Engineering, p. 198, MeGraw-Hill, New York, (1954).
2) P.L. Keller, C.D. Zerly and J. Hilgeman, ORNL-2375 (1958).
3) D.K. Zerby, ORNL-2998 (1960), and also in Nucl. Sci. Eng., **10**, 102 (1961).
4) P.P. Cowan and C.B. Meinhold, Radiat. Bot., **2**, 241 (1962).
5) M. Kitazume, J. Nucl. Sic. Technol., **5**, 464 (1968).
6) American National Standard, "Neutron and Grmma-Ray Flux-to-Dose-rate Factors, ANSI/ANS-6.1.1-1977 (N 666), 1977.
7) A. Martin, G.H. Firmin and J.R.P. Eaton, "The Optimization of Shielding for Direct-cycle steam turbine Plant," Fifth Int. Conf. on Reactor Shielding, Knoxville, Tennesses, U.S.A., (1977).
8) M.S.S. Murthy, A. Vyas, B.K.S. Murthy and Bhuwan Chandra, Int. J. Appl. Radiat. and Isotopes, **24**, 671 (1973).

單一散亂近似法에 의한 空氣中 散亂放射線量의 計算

육 종 철

한양대학교 공과대학 원자력공학과

하정우, 이재기, 문석형

한국원자력연구소 방사선관리실

<개 요>

공기중 산란감마선의 조직등가선량을 수정 단일산란 근사법에 의하여 계산하였다. 공기중 산란 조직등가선량은 축적인자 및 지수감쇄 등 관심위치에서의 에너지 스펙트럼 및 3차원의 기하학적 산란체적을 고려하여 계산하였다. 이 계산방법은 차폐벽으로 둘러쌓인 농학적이용 목적의 대표적 감마조사시설과 시설의 방사선원의 방출감마선이 일정 범위내에만 조사되도록 하는 장치가 부착된 조사시설에 적용할 수 있도록 수정하였다. 선원과 피조사체 사이의 거리가 35m에서 300m 내에 있는 여러 위치에서의 에너지 스펙트럼, 각분포 그리고 조직등가선량율에 대한 계산결과를 얻었다. 3차원 계산방법에 의하여 구한 본 계산결과와 2차원 계산방법에 의하여 얻은 결과를 실제 측정값과 비교한 결과에 의하여 본 결과가 전반적으로 실제 측정값과 좋은 일치를 보여 주었다. 만일에 차폐체에서의 에너지와 각 분포가 중요하지 않다고 하면 3차원 계산방법을 도입한 단일산란근사법은 일반적으로 감마조사 시설에서 공기중 산란방사선 문제들을 취급하는 데에 적합함을 알았다.