Kyungpook Math. J.
Volume 18, Number 1
June, 1978

A NOTE ON LOCAL DISCRETE EXTENSIONS OF TOPOLOGIES

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Let $(X, \mathscr{T})$ be a topological space and let $A \subset X$. The topology $\mathscr{T}[A] = \{U-B \mid U \in \mathscr{T},\ B \subset A\}$ is the local discrete extension of $\mathscr{T}$ by A.

In [2] it is established that regularity, normality and complete regularity are preserved under local discrete extensions to open sets. The purpose of this note is to observe that a much more general result holds, using a simple technique employed in [3].

Let $\mathscr{P}$ be any weakly hereditary topological property for which the Locally Finite Sum Theorem holds. See [1] for a discussion of sum theorems.

THEOREM. *Let* $(X, \mathscr{T})$ *have property* $\mathscr{P}$ *and let* $A \in \mathscr{T}$. *Then* $(X, \mathscr{T}[A])$ *also has property* $\mathscr{P}$.

PROOF. $(X, \mathscr{T}[A]) = (A, \mathscr{T}[A] \cap A) \cup (X-A, \mathscr{T}[A] \cap (X-A))$. The first subspace has property $\mathscr{P}$ since it is discrete. The second subspace has property $\mathscr{P}$ since $\mathscr{T}[A] \cap (X-A) = \mathscr{T} \cap (X-A)$ and $\mathscr{P}$ is hereditary on closed subsets.

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