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A NOTE ON NEARLY COMPACT SPACES

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1. Introduction.

There are well known properties that if S is a compact subset of an arbitrary topological space then every infinite subset of S has an accumulation point in S, and that the intersection of any descending chain of closed susetsts of a compact space is not empty.

The purpose of the present note is to characterize the analogous properties in the nearly compact case using the concept of w-accumulation introduced in [4].

A topological space is nearly compact if and only if every regular open cover of it admits a finite subcover ([2] Theorem 2.1).

2. Characterization of nearly compact spaces.

In [4], Travis Thomson introduced the concept of w-convergence and that of w-accumulation of a filter (of a net) in a topological space in order to characterize nearly compact spaces. In this paper we will use the term "w-accumulation point of a set", and with which concept we are to give the theorems.

DEFINITION 1. [3] Let S be a subset of a topological space X. By a w-accumulation point of S we shall mean a point x in X having the property that every regular open neighbourhood of x intersects S.

DEFINITION 2. [1] A subset S of a topological space (X, τ) is called N-closed relative to τ if any cover of S by τ-open sets admits a finite subcollection the interior of closure of which covers S. Interiors and closures are with respect to τ.

Lemma 2.1 of [1] says that S is N-closed relative to τ if and only if every cover of S by regular open sets in τ admits a finite subcover. It is also noted in [1] that, by this terminology, (X, τ) is nearly compact if and only if it is N-closed relative to τ.

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Other various properties about N-closedness relative to τ can be found in section 2 of [1].

Anywise, we can easily know that every nearly compact subspace (with respect to the relative topology) of (X, τ) is N-closed relative to τ, since the intersection of any τ-regular open set with it is regular open in the relative topology.

DEFINITION 3. A subset S of a topological space is called w-closed if $S=\mathrm{cl}_w S$. where $\mathrm{cl}_w S$ denotes the set of all w-accumlation points of S.

THEOREM 1. *If S is an arbitrary N-closed subset of a topological space, then every infinite subset of S has a w-accumulation point of it in S.*

Proof. Let T be a given infinite subset of S. Suppose that T has no w-accumulation point in S, then to each point $x\in S$ there corresponds some regular open neighbourhood U_x of x such that $(U_x-\{x\})\cap T=\square$. Now the collection of all such regular open sets as given above forms a cover of S. Since S is N-closed, there exists a finite number of points $x_1, \cdots, x_n$ in S such that $U_{x_1}, \cdots, U_{x_n}$ cover S. So we have $T\subset U_{x_1}\cup\cdots\cup U_{x_n}$. But since at most one point of T can be contained in each of $U_{x_k}, k=1, \cdots, n$, T must be finite. This contradiction leads us to the required conclusion.

THEOREM 2. *Let X be a nearly compact space, and let $S_1\supset S_2\supset\cdots\supset S_n\supset\cdots$ be a descending chain of w-closed subsets of X, then the intersection of all S_n for $n=1, 2, \cdots$ is not empty.*

Proof. Choose a point $x_n\in S_n$ for each $n=1, 2, \cdots$, then x_n will have a w-accumulation point x_0 in X by Theorem 1, since X is nealy compact and hence it is N-closed.

On the other hand, for each $k=1, 2, \cdots$, x_0 becomes a w-accumulation point of $\{x_k, x_{k+1}, \cdots\}$ also, hence of S_k. Since each S_k is w-closed, we know that $x_0\in S_k$ for each k, hence that the intersection of all S_k is not empty.

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