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ON THE GAME OF GO

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1. Introduction.

We introduce a two person go-game on a three dimensional go board. Then we discuss to capture, Shicho, life and other problems. There are papers [1, 2, 3, 4 and 5] on the two person game of go on a 2-dimensional board for comuting machines.

2. Definition of a two person go-game on a 3-dimensional board.

Let $\boldsymbol{Z}_+$ be the set of all positive integers. Let $n_i \in \boldsymbol{Z}_+$ with $n_i \geqq 9$ $(i=1, 2, 3)$. Let $I_i = \{n \in \boldsymbol{Z}_+ : n \leqq n_i\}$ and let $B = I_1 \times I_2 \times I_3$. (i, j, k) in B is called a *point* or an *intersection*. Let $S = \{b, w\}$ be a set of two distinct objects b and w. (We call b a *black stone* and w a *white stone.*) Let $y \in S$. If (y, a) is a member of $y \times B$, then $v = (y, a)$ is called a *vertex with y-color* or a *y-vertex.* B is called the *go-board.* $G(y \times B)$ denotes the set of all subsets of $y \times B$ and any member of $G(y)$ of $G(y \times B)$ is called a *y-graph.* Let $u = (i, j, k) \in B$ and let $v \in B$. u and v are *adjacent* if v takes one of the following 18 points:

(1) $(i, j, k+1)$. (2) $(i, j, k-1)$. (3) $(i, j+1, k)$. (4) $(i, j-1, k)$. (5) $(i-1, j, k)$. (6) $(i+1, j, k)$. (7) $(i-1, j-1, k)$. (8) $i+1, j-1, k)$. (9) $(i+1, j+1, k)$. (10) $(i-1, j+1, k)$. (11) $(i, j-1, k+1)$. (12) $(i, j-1, k-1)$. (13) $(i, j+1, k+1)$. (14) $(i, j+1, k-1)$. (15) $(i+1, j, k+1)$. (16) $(i+1, j, k-1)$. (17) $(i-1, j, k+1)$. (18) $(i-1, j, k-1)$.

$G(y) \in G(y \times B)$ is connected if any two vertices of $G(y)$ form an edge. The number $|G(y)| = m$ of all vertices of $G(y)$ will be called the *order* of $G(y)$. A connected graph $G(y) = \{(y, a_i) : i = 1, 2, \cdots, m\}$ of order $m \geqq 3$ is said to be a *simple connected curve graph with two terminal vertices* (y, a_1) *and* (y, a_m) if a_1 is not adjacent with a_m and if $G(y) \setminus (y, a_i)$ $(1 \neq i \neq m)$ is not a connected graph. ($A \setminus B$ means the set of all elements of A which are not in B.) An edge $\{(y, a_1), (y, a_2)\}$ is also called a *simple connected curve graph.*

Let $B_1 = \{(i, j, k) \in B : i = 1$ or $i = n_1\}$, $B_2 = \{(i, j, k) \in B : j = 1$ or $j = n_2\}$,

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$B_3=\{(i,j,k)\in B: k=1$ or $k=n_3\}$ $B(B)=B_1\cup B_2\cup B_3$. $B(B)$ will be called the *border* of B. Let $G(y)$ be a simple connected curve graph with two terminal vertices (y,a_1) and (y,a_m). If $\{a_1,a_m\}\subset B(B)$, then $G(y)$ is called a *weak simple closed curve graph.* A graph $G(y)$ is said to be a *strong simple closed curve graph* if $|G(y)|=m\geq 4$ and if, for each vertex (y,a) of $G(y)$, $G(y)\setminus(y,a)$ is a simple connected curve graph with two terminal vertices. $CG(y\times B)$ denotes the set of all weak and strong simple closed curve graphs. Any member $G(y)$ of $CG(y\times B)$ is called a *simple closed curve graph.* For a graph $G(y)$, define $c(G(y))=\{v\in B:(y,v)\in G(y)\}$ as the coordinate set of $G(y)$. Define $B|_{i=i_0}=\{(i,j,k)\in B: i=i_0\}$, for a fixed i_0. Let $G(y)$ be a simple closed curve graph with $G(y)\subset y\times(B|_{i=i_0})$.

Then $G(y)$ divides the section $B|_{i=i_0}$ into two separated regions $R(G(y))$ and $\bar{R}(G(y))$ such that $R(G(y))\cup\bar{R}(G(y))\cup c(G(y))=B|_{i=i_0}$ and $R(G(y))\cap\bar{R}(G(y))\cap c(G(y))=\phi$ (the empty set).

DEFINITION 1. A simple closed surface graph. Let $G(y)$ be a y-graph and let $(y,(i_0,j_0,k_0))\in G(y)$. $G(y)|_{i=i_0}=\{(y,(i_0,j,k))\in G(y)\}$ is called a *section graph of* $G(y)$ *at* $i=i_0$. Similarly, $G(y)|_{j=j_0}=\{(y,(i,j_0,k)\in G(y)\}$ and $G(y)|_{k=k_0}=\{(y,(i,j,k_0))\in G(y)\}$ are called *section graphs of* $G(y)$ *at* $j=j_0$ *and* $k=k_0$, respectively. $G(y)$ is called a *simple closed surface graph* if every section graph of $G(y)$ is either a simple closed curve graph or a line segment graph including a vertex graph as a special case.

EXAMPLE 1. There are exactly eight simple closed surface graphs $G(y)=\{(y,a_i): i=1,2,3\}$ of order 3. Let $v_1=\{(1,0,0),\ (0,1,0),\ (0,0,1)\}$, $v_2=\{(n_1,1,0),(n_1,0,1),(n_1-1,0,0,)\}$, $v_3=\{(1,n_2,0),\ (0,n_2-1,0),(0,n_2,0)\}$, $v_4=\{(n_1-1,n_2,0),\ (n_1,n_2-1,0),\ (n_1,n_2,1)\}$, $v_5=\{(0,0,n_3-1),(1,0,n_3),(0,1,n_3)\}$, $v_6=\{(n_1,0,n_3-1),(n_1,1,n_3),(n_1-1,0,n_3)\}$, $v_7=\{(n_1,n_2-1,n_3),(n_1,n_2,n_3-1),\ (n_1-1,n_2,n_3)\}$ and $v_8=\{(0,n_2-1,n_3),(1,n_2,n_3),(0,n_2,n_3-1)\}$.

Then $G(y)=y\times v_i=\{(y,a_i): a_i\in v_j$ for $i=1,2,3\}$ $(j=1,2,\cdots,8)$ is a simple closed surface graph.

Let $G(y)$ be a simple closed surface graph. Then $G(y)$ divides the go-board B into two separated regions $R(G(y))$ and $\bar{R}(G(y))$ such that $R(G(y))\cup\bar{R}(G(y))\cup c(G(y))=B$ and $R(G(y))\cap\bar{R}(G(y))\cap c(G(y))=\phi$. Two graphs G_1 and G_2 form a graph $G_1\cup G_2$ if $c(G_1)\cap c(G_2)=\phi$. (G_i is a (b,w)-graph, in general.) Let $G(x)$ be a graph and $G(y)$ be a simple closed surface graph. $G(x)$ is completely surrounded by $G(y)$ if $c(G(x))=R(G(y))$ or $c(G(x))=\bar{R}(G)y))$. We use $G(y)(G(x))$ to denote that $G(x)$ is completely surrounded by $G(y)$. (We note that $x\neq y$.)

NOTE. If $|R(G(y))| \leqq |\bar{R}(G(y))|$ for a simple closed surface graph $G(y)$, then we say that $R(G(y))$ is the *inner region of* $G(y)$. $G(x)$ is completely surrounded by $G(y)$ means that, in general, $c(G(x))=R(G(y))$.

NOTE. In example 1, we have that $R(G(y))=R(y\times v_1)=(0,0,0)$, $R(y\times v_2)=(n_1,0,0)$, $R(y\times v_3)=(0,n_2,0)$, $R(y\times v_4)=(n_1,n_2,0)$, $R(y\times v_5)=(0,0,n_3)$, $R(y\times v_6)=(n_1,0,n_3)$, $R(y\times v_7)=(n_1,n_2,n_3)$ and $R(y\times v_8)=(0,n_2,n_3)$.

EXAMPLE 2. There is a simple closed surface graph $G(y)=\{(y,a_i):i=1,2,\cdots,6\}$ of order 6 with one point inner region $R(G(y))=(i+1,\ j+1,\ k+1)$, where $a_1=(i+1,\ j,\ k+1)$, $a_2=(i+2,\ j+1,\ k+1)$, $a_3=(i+1,\ j+2,\ k+1)$, $a_4=(i,\ j+1,\ k+1)$, $a_5=(i+1,\ j+1,\ k)$ and $a_6=(i+1,\ j+1,\ k+1)$.

DEFINITION 2. A region.

Let $G_i(y)$ be a simple closed surface graph for $i=1,2,\cdots,m$. Suppose that $c(G_i(y))\subset R(G_1(y))$ $(i\neq 1)$ and $c(G_1(y))\subset \bar{R}(G_i(y))$ $(i\neq 1)$. The $R(G_1(y))\setminus \bigcup_{i\neq 1} c(G_i(y)) \bigcup_{i\neq 1} R(G_i(y))$ is called the *region of the union graph* $G(y)=\bigcup_{i=1}^{m} G_i(y)$ *of graphs* $G_i(y)$ and we denote the region by $R(\bigcup_{i=1}^{m} G_i(y))$. If $G(x)$ is a graph with $c(G(x))=R(\bigcup_{i=1}^{m} G_i(y))$, then we say that $G(x)$ is *completely surrounded by the graph* $G(y)=\bigcup_{i=1}^{m} G_i(y)$ and we denote this by $G(y)(G(x))$. If $a\in R(\bigcup_{i=1}^{m} G_i(y))$, then $R(\bigcup_{i=1}^{m} G_i(y)\cup (y,a))=R(\bigcup_{i=1}^{m} G_i(y))\setminus \{a\}$ is also called the *region* of $G'(y)=G(y)\cup (y,a)$.

DEFINITION 3. Ko. Let $G(y)$ be a simple closed surface graph with the one point region $R(G(y))=(i,j,k)$ and $3\leqq |G(y)|\leqq 6$. Let $x\in S\setminus \{y\}$. Let $G(x)$ be a graph such that $2\leqq |G(x)|\leqq 5$, $c(G(x))\cap c(G(y))=\phi$, $G(x)\cup (x,(i,j,k))$ forms a simple closed surface graph and $G(x)$ is not a simple closed surface graph. If there is a vertex (y,a) of $G(y)$ such that the inner region $R(G(x))\cup (x,(i,j,k))$ in the graph $G(x)\cup (x,(i,j,k))\cup G(y)\setminus (y,a)$ is equal to a, then we say that $G(y)$ and $G(x)$ form a Ko and we denote this Ko by $Ko(G(y),G(x),(i,j,k),(y,a))$. We may say that the player p_y initiated the Ko.

We remark $G(y)\cup G(x)$ is an (x,y)-graph when $c(G(x)\cap c(G(y))=\phi$. We now define two functions.

DEFINITION 4. A move function f and a capture function g. Let G_i be a sequence of (b, w)-graphs. Let $S^0 = S \cup \phi$. We define a move function $f: \boldsymbol{Z}_+ \to \boldsymbol{Z}_+ \times S^0 \times B$ as a mapping satisfying the following conditions:

(1) $f(1) = (1, G_1)$, where G_1 is a subset of $b \times B$ with $|G_1| = |f(1)| \geqq 1$.

(2) For $n \in \boldsymbol{Z}_+$,

$$f(2n-1) = \begin{cases} (2n-1, b, v), & \text{a black move by } p_b, \\ (2n-1, \phi, v), & \text{a pass move by } p_b, \end{cases}$$

$$f(2n) = \begin{cases} (2n, w, v), & \text{a white move by } p_w, \\ (2n, \phi, v), & \text{a pass move by } p_w, \end{cases}$$

where $v \in B$. We often write $f(i) = \phi$ when $f(i) = (i, \phi, v)$ is a pass move. We also use $|f(i)| = 0$ for a pass move $f(i) = \phi$. We have that $0 \leqq |f(i)| \leqq 1$.

(3) $f(n+1) = (n+1, x, v)$ and $x \neq \phi$, then $v \in B \setminus c(G_n)$.

We now define a capture function $f: \boldsymbol{Z}_+ \to G(b \times B) \cup G(w \times B) \cup \phi$ by the following:

(4) Let $f(n) = (n, y, v)$ $(n \geqq 2)$.

Suppose that there exists a set $\{G_j(y) : j = 1, 2, \cdots, m\}$ $(m \geqq 1)$ of m graphs and a graph $G(x)$ $(x \neq y)$ in G_{n-1} such that $\bigcup_{j=1}^{m} G_j(y) \cup (y, v)$ forms a union graph of m simple closed surface graphs and the region $R(\bigcup_{j=1}^{m} G_j(y) \cup (y, v))$ is the region defined in Definition 2. If $G(x)$ is completely surrounded by the union graph $\bigcup_{j=1}^{m} G_j(y) \cup (y, v)$, then $g(n) = G(x)$.

(5) (Suicide is illegal.)

Suppose that there exists a set $\{G_j(x) : j = 1, 2, \cdots, m\}$ $(m \geqq 1)$ of simple closed surface graphs in G_{n-1} such that $c(G_j(x)) \subset R(G_1(x))$ and $c(G_1(x)) \subset R(G_j))$ $(j \neq 1)$. Suppose that there exists a graph $G(y)$ in G_{n-1} such that $G(y) \cup (y, v)$ is completely surrounded by the union graph $\bigcup_{j=1}^{m} G_j(x)$. (See Definition 2 for a union graph and its region.) Then $g(n) = G(y) \cup (y, v)$.

(6) If (4) and (5) are not applicable, then $g(n) = \phi$.

Now we can define a seqence G_i inductively, where G_i are (b, w)-graphs mentioned in Definition 4.

Define G_1 by $f(1) = (1, G_1)$, $G_1 \subset b \times B$ and $|G_1| \geqq 1$. For $f(i+1) = (i+1, x, v)$, G_{i+1} is defined by

$$G_{i+1} = \begin{cases} G_i \cup (x, v) \setminus g(i+1) & \text{if } x \neq \phi, \\ G_i & \text{if } x = \phi. \end{cases}$$

We impose, in general, Ko-rule on a move function f.

Ko-rule is one of important concepts on go-games.

Ko-rule.

(1) If a move $f(n)=(n, y, v)$ forms the first Ko$=Ko(G(y), G(x), (i, j, k), (y, a))$, where (y, a) is a vertex of $G(y)$. Then the player p_x can take a move of the form $f(n+1)=(n+1, x, (i, j, k))$ and consequently $g(n+1)=(y, a)$. If $f(n+1)=(n+1, x, (i, j, k))$ and $g(n+1)=(y, a)$, then we say that p_x moves to the Ko and captures a y-stone of (y, a). Now we have $G_{n+1}=G_n \cup (x, (i, j, k)) \setminus (y, a)$. Ko-rule is that p_y can not take a move of the form $f(n+2)=(n+2, y, a)$, that is, a move $f(n+2)=(n+2, y, a)$ is illegal. REMARK. p_y can take a move $f(n+4)=(n+4, y, (i, j, k))$ when $f(n+3) \neq (n+3, x, (i, j, k))$.

(2) $k(G_n)$ denotes the set of all Kos in the (b, w)-graph G_n. Suppose that $k(G_n)=Ko(G(y), G(x), (i, j, k), (y, a))$, $f(n)=(n, y, v)$ and $g(n) \neq (x, (i, j, k))$. If $f(n+1)=(n+1, x, (i, j, k))$ and $g(n+1)\,(y, a)$, then we say that p_x moves to the Ko and captures a y-stone of (y, a). Suppose that p_x moves to the Ko. Then Ko-rule is that a move $f(n+2)=(n+2, y, a)$ with $g(n+2)=(x, (i, j, k))$ by p_y to the Ko of $k(G_{n+1})$ is illegal. If $f(n)=(n, y, v)$ with $g(n)=(x, (i, j, k))$, then, a move $f(n+1)=(n+1, x, (i, j, k))$ with $g(n+1)=(y, a)$ is illegal. Let $m \geqq 1$.

(3) Suppose that $k(G_n)=\{Ko(G_t(y), G_t(x), (i_t, j_t, k_t), (y, a_t)): t=1, 2, \cdots, m_1\} \cup \{Ko(G_t(x), G_t(y), (i_t, j_t, k_t)): t=m_1+1, m_1+2, \cdots, m_1+m_2\}$ $(m_2 \geqq 1)$. If p_x takes a move to Ko of $k(G_n)$, then a move by p_y to any Ko in $k(G_{n+1})$ is illegal. If $f(n+1)=(n+2, y(i_{t_0}, j_{t_0}, k_{t_0}))$ with $g(n+1)=(y, a_{t_0})$, then a move $f(n+2)=(n+2, y, (i_t, j_t, k_t))$ for $t=t_0$ or $t \in \{m_1+1, m_1+2, \cdots, m_1+m_2\}$, is illegal.

DEFINITION 5. The end of the game.

The game ends with the final graph G_n if $f(n+1)$ and $f(n+2)$ are first two consecutive pass moves. We shall say that the game ends at the $n+2$-th move.

SCORING.

Let $\Pi=\{G_i(y): i \in I\}$ be the set of all simple closed surface y-graphs in $G(y)$. Let $R=\{R_j: j \in J\}$ be the set of all regions determined by Π. We see that $R(G_i(y))$ and $\bar{R}(G_i(y))$ are members of R. (1) Consider $R(G_i(y)) \in R$. Suppose $|R(G_i(y)) \cap c(G(x))|=m_1$ and $|R(G_i(y)) \cap c(G(y))|=m_2$. If $m_1=0$, then we define $\|R(G_i(y))\|=|R(G_i(y))|-m_2$. If $m_1 \neq 0$, then there is a graph $G_t(x)$ such that $c(G_t(x)) \subset R(G_i(y))$. There are two cases. (2) There exists a positive integer k such that by k moves $f(n+2+i)$ $(i=1, 2, \cdots, k)$, it is possible to obtain $g(n+2+k)$ which contains $G_t(x)$. This

is the case, we set $\|R(G_i(y))\|=|R(G_i(y))|-m_2+m_1$.
(3) If it is not possible to obtain such $g(n+2+k)$ containing $G_t(x)$ by a finite number (k) of moves, then we set $\|R(G_i(y))\|=0$ (4) Let $R_j\in R$. Suppose that there is a subset K of I ($|K|\geqq 2$) so that the union graph $\bigcup_{i\in K} G_i(y)$ makes the region $R(\bigcup_{i\in K} G_i(y))=R_j$ as defined in Definition 2. Then we take $|R_j\cap c(G(x))|=m_1$ and $|R_j\cap c(G(y))|=m_2$. If $m_1=0$, then we define $\|R_j\|$ as $\|R_j\|=|R_j|-m_2$. If $m_1\neq 0$. Then there exists a graph $G_t(x)$ such that $c(G_t(x))\subset R_j$. We now follow (2) and (3) for R_j. We define c_y by $c_y=\sum_{i\in J}\|R_i\|+\sum_{g(j)\in G(x\times B)}|g(j)|$, including $j=2n+1$ when $y=b$ and $j=2n$ when $y=w$, as the final score of p_y. If $c_x>c_y$, then we say p_x with x-stones win the game by (c_x-c_y).

DEFINITION 6. A two person go-game on a 3-dimensional go-board with Ko-rule is a set $\{f, g, S, B\}$ of a move function f which obey the Ko-rule, a capture function g, a set $S=\{b, w\}$ and a three dimensional go-board $B=I_1\times I_2\times I_3$.

3. We now discuss problems of the game.

In a two person go-game on a 2-dimensional go-board, there is a method of capturing, which is called Shicho or ladder. Shicho is one of the fundamental techniques in the go-game. Any person who says that 'I can play Go' knows Shicho. One might ask this question: Is there Shicho in a 3-dimensional go-game? First we give an example of Shicho. Let $\{f, g, S, I_1\times I_2\}$ be a two-person go-game on a 2-dimensional go-board $I_1\times I_2$. (b means black and w means white.) Let $f(1)=(1, b, (4,4))$, $f(2)=(2, w, (4,5))$, $f(3)=(3, b, (5,5))$, $f(4)=(4, w, (4,6))$, $f(5)=(5, b, (6,5))$ and $f(6)=(6, w, (5,4))$. Then p_b (the person with black stones) can capture $(w, (5,4))$-stone by Shicho method.

See the moves: $f(7)=(7, b, (5,3))$, $f(8)=(8, w, (6,4))$, $f(9)=(9, b, (7,4))$, $f(10)=(10, w, (6,3))$, $f(11)=(11, b, (6,2))$, $f(12)=(12, w, (7,3))$, $f(13)=(13, b, (8,3))$, $f(14)=(14, w, (7,2))$, $f(15)=(15, b, (7,1))$, $f(16)=(16, w, (8,2))$, $f(17)=(17, b, (9,2))$, $f(18)=(18, w, (8,1))$, $f(19)=(19, b, (9,1))$, $g(i)=\phi$ $(i=1, 2, \cdots, 18)$ and $g(19)=\{(w, v_i): v_1=(5,4),\ v_2=(6,3),\ v_3=(6,4),\ v_4=(7,3),\ v_5=(7,2),\ v_6=(8,2)$ and $v_7=(8,1)\}$.

In our definition of the two-person go-game on a 3-dimensional board, the answer to the question is no. But, if we define a move function f as $f(2n-1)=(2n-1, b, V)$ with $|V|=3$ for all odd number $2n-1$ and $f(2n)=(2n, w, v)$ with $|v|=1$ for all even number $2n$, then we will have Shicho in a two-person go-game on a 3-dimensional board. We shall call this f a 3-1 move function.

EXERCISE. Let $I_1 \times I_2 \times \cdots \times I_n$ $(n \geqq 4)$ be a board B.

Let $\{f, g, S, B\}$ be a two-person go-graph on an n-dimensional board B. Let f be an $m-1$ move function. Find the number m for f to have Shicho in the game $\{f, g, S, B\}$.

We now discuss Life and Death.

One of the fundamental concepts on the go-game is determining when a group of stones is impossible to capture by the opponent. Benson [2] defined the term 'Unconditional Life' and established an important theorem.

We shall consider that a group of x-stones has a situation of the unconditional life in a 3-dimensional and 2-person go-game defined in 2.

We need more definitions. Let $\{f, g, S, B\}$ be a 2-person go-game on a 3-dimensional board B. (We may say that $\{f, g, S, B\}$ is a 2-3 go-game.) Let G_n be the final (b, w)-graph after first two consecutive pass moves $f(n+1)=\phi$ and $f(n+2)=\phi$. For $k<n$, G_k will be called a partial (b, w)-graph at the kth move $t=k$ or $f(k)$. Any subset $G(b, w)$ of G_k is called a local go-graph at the move $t=k$ or $f(k)$. Let $G_k(b, w)$ be a local go-graph at the move $t=k$ and let $G_k(b, w)=G(b) \cup G(w)$. Let $G(x) \in \{G(b), G(w)\}$.

DEFINITION 7. A strongly connected graph.

Any subset $B(x)$ of $G(x)$ is called a *strongly connected graph* if for any two distinct vertices (x, v_1) and (x, v_s) of $B(x)$, there exists a sequence (x, v_i) $(i=2, 3, \cdots, s-1)$ of vertices of $B(x)$ such that (x, v_i) and (x, v_{i+1}) $(i=1, 2, \cdots, s-1)$ form an edge. $SC(G_0(x))$ denotes the set of all strongly connected x-graphs in the x-graph $G_0(x)$.

NOTE. The strongly connectedness is an equivalence relation π on $G_0(x)$ and hence $G_0(x)/\pi = SC(G_0(x))$.

If p_x with x-stones captures a group of y-stones of a graph $G_0(y)$ which is a subgraph of G_k, then $G_0(y)$ is either a strongly connected graph or a union of strongly connected graphs and $G_0(y)$ can not be a part of a strongly connected graph. We follow the paper [2, pp. 22-26] by Benson.

DEFINITION 8. Safe, almost always safe and healthy regions. If any element $B_0(y)$ of $SC(G_0(y))$ remains in the final graph G_n, then we say that $B_0(y)$ is safe. $B_0(y)$ is almost always safe if in any sequence of moves $f(k+i)$ $(i=1, 2, \cdots, t_0)$, p_y must have $n(\geqq 2)$ non-pass moves (after $f(k)$) in order to make the y-stones of $B_0(y)$ safe. A region R_0 is healthy for a strongly connected x-graph $B_0(x)$, denoting by $H(R_0, B_0(x))$, if $R_0 = R(G_0(x))$ formed by a simple closed surface graph $G_0(x)$, $c(G(x)) \cap R_0 = \phi$ and for every point u in R_0, say $u=(i,j,k)$, there exists a vertex (x,v) in $B_0(x)$ so that v takes one of $\{(1),(2),(3),(4),(5),(6)\}$ the set of six coordinates in the Definition of u and v are adjacent.

DEFINITION 9. A graph is unconditionally alive and an eye.

Let G_k be a partial go-graph $t=k$ in the go-game $\{f,g,S,B\}$ and $G_k=G(b)\cup G(w)$. Let $G(x)\in\{G(b),G(w)\}$. Let $X\subset SC(G(x))$ and R_0 be a region. Recall that for R_0 there exists a set $\{G_i(x):i=1,2,\cdots,m\}$ of m simple closed surface graphs $G_i(x)$ so that $R_0=R(\bigcup_{i=1}^{m}G_i(x))$ as defined in Definition 2. We denote $\bigcup_{i=1}^{m}G_i(x)$ *by* $H(x)$. Suppose that $R_0=R(H(x))$ is an healthy region for an element $B_0(x)$ of X. $R_0=R(H(x))$ is said to be *vital to* $B_0(x)$ *in* X if $\{B(x)\in SC(G(x)):c(B(x))\cap c(H(x))\neq\phi\}\subset X$. X is *unconditionally alive* if for all $B(x)\in X$ have two distinct regions R_1 and R_2 which are vital to $B(x)$ in X. Let $B(x)\in SC(G(x))$ and $H(R_0,B(x))$. If $R_0=R(B(x))$, then R_0 is an eye. Otherwise, R_0 is a joiner. We can now state the following theorem.

THEOREM 1. *Let $\{f,g,S,B\}$ be a 2-person go-game on a 3-dimensional board B. Let G_k be a partial (b,w) go-graph at $t=k$. Then*

(1) A subset X of $SC(G(x))$ is unconditionally alive, then every element of X is safe.

(2) If $X\subset SC(G(x))$ is the set of all safe x-graphs, then X is unconditionally alive.

The proof of this theorem is technically the same as the proofs of Theorems 1 and 2[2] by Benson and we shall omit the proof of this theorem.

NOTE. Theorem 1 is true for any go-game $\{f,g,S,B\}$ on an n-dimensional board $B(n\geqq 3)$ and for a 1-1 move function f. But if f is a $m-1$ $(m\geqq 2)$ move function, Theorem 1 is no longer true.

One might ask this question: Suppose $G(x)$ be an unconditionally alive x-graph in a 2-person and 2-dimensional go-game. Is $G(x)$ unconditionally alive in a 2-person and 3-dimensional go-game?

For this we have:

THEOREM 2. *In a 2-person go-game on a 3-dimensional board, every two dimensional x-graph is not safe.*

PROBLEM. Prove or disprove: In an n-dimensional go-game $\{f,g,S,B\}$ with 1-1 move function, every k-dimensional go-graph $G(x)$ is not safe, where k is a positive integer with $2\leqq k<n$.

Proof of Theorem 2. Let $\{f, g, S, B\}$ be a go-game on a 3-dimensional board B. Let $G(b)$ be a 2-dimensional b-graph formed by moves $f(2n-1)$ $(n=1, 2, \cdots, k)$ and $f(2n)=\phi$ $(n=1, 2, \cdots, k-1)$. Suppose that there exists a simple closed surface graph $G_0(b)$ which is a subgraph of $G(b)$ such that $R(G_0(b))\neq\phi$. We let $f(2n-1)=\phi$ for $n>k$. Then p_w can move $f(2n)=(2n, w, v)$ $(n\geqq k)$ to any point $v\in R(G_0(b))$. This proves the theorem. We give an example of a go-graph $G(b)$ in a 3-dimensional board.

EXAMPLE 3. Define $G(b)$ by $c(G(b))=\{(n_1, 1, n_3-1),\ (n_1, 2, n_3-1),\ (n_1, 3, n_3-1),\ (n_1, 4, n_3-1),\ (n_1, 2, n_3),\ (n_1, 4, n_3),\ (n_1-1, 1, n_3),\ (n_1-1, 2, n_3),\ (n_1-1, 3, n_3)\}$. We see that $R(G(b))=\{(n_1, 1, n_3),\ (n_1, 3, n_3)\}$. We can see that $G(b)$ is a strongly connected, unconditionally alive and safe closed surface graph.

References

[1] David B. Benson, *Life in the game of go*, Computer Science Department, Washington State University, Pullman, WA. (1974?), 1-21.
[2] David B. Benson, *Life in the game of go*, Information Sciences **10** (1976), 17-29.
[3] H. Renus, *Simulation of a learning machine for playing go*, Information precessing 1962 (Proc. IFIP Congress, Munich, 1962), North Holland, Amsterdam, (1963), 192-194.
[4] E. Thorp and W. Walden, *A partial analysis of go*, Computer J. **7**, No. 3 (1964), 203-207.
[5] E. Thorp and W. Walden, *A computer assisted study of go on $m\times N$ boards*, Information Sciences **4**, No. 1 (1972), 1-33.

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