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A Study on the Method of Nonlinearity Correction in a GM Counter

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Abstract

A method is presented here in order to determine a nonlinearity correction of observed counter rates in a GM counter. An expression, which is developed under the assumption of dead time dependence on counting rates, gives good agreement with the results obtained from the experimental work.

The variation of pulse voltages from a GM counter with counting rates was measured with the aid of pulse height analyzer. This method gives accurate values for the dead time over a wide range of counting. The technique as outlined allows the exact correction of the nonlinearity caused by dead time loss. It was observed that the dead time decreases as the counting rate increases.

요 약

계측장치에서 관측한 계수율에 대한 비직선성 교정을 결정하는 한 방법을 제시하였다. 불감시간은 계수율의 의존성을 가지고 있다는 가정하에서 발전시킨 한 표현식은 실험결과와 잘 일치하였다.

계측장치의 펄스전압의 계수율에 따른 변화량을 파고분석장치를 이용하여 측정하였다. 이 방법은 광범한 계수율 범위에 걸치어 불감시간에 대한 정확한 값을 주었다.

개요한 이 측정기술로 불감손실로 인한 비직선성의 정확한 교정할 수 있었다. 불감시간은 계수율이 증가함에 따라 감소하는 것이 관측되었다.

1. Introduction

A radiation detector exhibits a continually decreasing sensitivity with increasing intensity of radiation. The response curve is thus not linear, as it would be for a hypothetical perfect instrument. With ion chambers the effect is due to the volume recombination of ions and mechanical sluggishness. With counters, it is due to the finite resolving time or interval that must separate two events if they are to be separatered. This "dead time" is due to space-charge effects in GM counters and to ion movilities or electrical circuit limitations in ionization and proportional counters.

Often in pulse counting experiments, nonlinearities between the observed rate and the true rate exist. Such nonlinearities may take the form of dead time loss due to the finite response time of the detector and associated electronics. To arrive at the true rate it is necessary to have a knowledge both of what this dead time is and of the mathematical relationship between the loss in a count rate and the dead time. A theory of the dead time phenomenon was well established by many investigators. Two simple theories involving a constant dead time have been discussed in detail.[1]

The theoretical expressions considered are:

$$N^*=\frac{N}{1-\tau N} \quad (1)$$

$$N^*=Ne^{\tau N} \quad (2)$$

In Eq. (1) (nonparalyzable), the every recorded event causes the counter to be inoperative for a assumed constant dead time, τ. Subsequent pulses which enter the system during τ, neither extend the dead time nor counted. In Eq. (2) (paralyzable), the subsequent pulse which arrive during the time interval, τ, continually extend the dead time by the same amount, τ, and are not counted as long as the system is dead. It should be noted from Eq. (2) that for large values of $N\tau^*$, the expression for N approached zero. This implies that at infinite true rate, one count will be recorded.

In practice, the behaviour of most counting systems is intermediate between the two extremes, and careful characterization is necessary to determine the actual system response. In intermediate, the relationship between the true and observed rates has been expressed as a truncated power series in N with variable coefficients[2]. Expansions of Eq. (1) or (2) give similar series.

This paper will describe a method of the exact correction of the nonlinearity caused by dead time loss, which is based on the mathematical relation between the dead time and the pulse voltage. The dead time is assumed to be a function of the counting rate. The variations of pulse voltage with the counting rate measured in multiscaling mode with a multichannel analyzer. Using this relation and Eq. (1) and (2) for processing the experimental data which arise from a decay curve measurement of ^{56}Mn, one can determine more precisely the dead time of a GM detector than with commonly used procedures[3-6]. The application of a multichannel analyzer to dead time mesurements has not been shown so far in the published literatures even though the procedure is a logical extension of the method described by Muehlihause and Friedman[7].

The purpose of this paper is to test the feasibilty of this method and to demonstrate that the dead time of a GM detection system is a function of the counting rate which can be approximated by the results provided by this procedures.

2. Dead Time Theory

The operation of self-quenched counters filled with a mixture of a noble gas and an organic or inorganic vapor has been treated by Stever[5], Trost[8], Ramsy[9], Montgomery[10], and others. The threshold for GM counting is characterized by the transition from localized single Townsend avalanches, to discharge which spread down the entire length of the tube. It characterized the pulse formation in the GM counting range.

When the counter is once triggered into breakdown, the avalanch electrons rush to the wire in a fraction of microsecond leaving behind a relatively stationary sheath of positive ions. The positive ion space charge reduces the field at the wire below the multiplication threshold. The time for the space charge to move a critical distance, R, at which the operating field in the counter recovers to GM counting threshold, is the dead time.

To a good approximation, the critical distance, R, defining the dead time was given by[5]

$$R=b\exp\left[-(V_o-V_t)/2q\right] \quad (3)$$

where b is radius of the cylinder, the quantity, V_o-V_t, is the voltage above the threshold at which the counter is operated q is the linear charge density in the positive ion space charge at the distance r_o from the center. On the other hand, an expression for the dead time in terms of R and other constants of the counter is needed. For this, an understanding of gas ion movilities is necessary. Due to the great complexity of the field of gas ion movilities, it is possible, for the purpose of this work, to examine only these factors directly affecting the ion movilities in a GM counter.

The movility constant, K, is the ratio of the velocity of the ion in question to the field strength:

$$K=\frac{\frac{dr}{dt}}{E} \quad (4)$$

In Eq. (4), E is the field acting on the positive ion sheath and can be expressed in the form of:

$$E=\frac{V}{r\log(b/a)} \quad (5)$$

where a is the wire radius, b is the cylinder radius and r is the variable distance. V is a potential across the counter at a time. Eq. (4) becomes

$$dt=\frac{r\log(b/a)\,dr}{KV} \quad (6)$$

If the movility constant is correctly assumed constant or if an expression of its variation with field strength is know, the dead time can also be calculated by integrating Eq. (6) from a to R. Assuming the former, we have

$$\tau=K_1\frac{R^2}{V}=\frac{K_2}{V}e^{-(V_o-V_t)/q}$$

$$=\frac{K_2}{V}e^{-V_u/q} \quad (7)$$

where K_1 and K_2 are constants for a given counter tube, and V_u is now termed as the overvoltage. The total charge which flows in a pulse single is given by

$$q_t=q\cdot l=C_t\cdot\Delta V \quad (8)$$

where l is the length of the counter wire, C_t is the total capacity of the counter system and ΔV is the pulse. If this is substituted in Eq. voltage.

(7), we find that the dead time, τ, is written as follows:

$$\tau=\frac{K_2}{V}\exp\left(-\frac{l\cdot V_u}{C_t\cdot\Delta V}\right) \quad (9)$$

All the parameters except for pulse voltage ΔV in Eq. (9) are the constants for a given counter.

The experimantal results on the dead time obtained by Baldinger and Hubber[11], and Muehlhause and Friedman[7] showed that the dead time is proportional to pulse voltage. If the dead time is a function of the counting rate, the pulse voltage is consequently also a function of the counting rate. With the assumption that the dead time is a function of the counting rate, the expression related to the dead time and the votage may be rewritten as follows:

$$\tau(\Delta V)=\frac{K_2}{V}\exp-\frac{l\cdot V_u}{C_t\cdot\Delta V(N)} \quad (10)$$

where N is a counting rate.

If the dead time is correctly assumed to be a function of counting rate, the dead time can be calculated by measuring the variations of pulse voltage with the counting rate. Therefore, this method may allow the correction of the nonlinearity caused by dead time loss.

3. Experimental Technique

The main problem in the determination of the dead time in this experiment is the measurement of variation of pulse voltage with counting rates. A pulse height analyzer is employed to measure the variation of pulse voltage with counting rates. Figure 1 shows the block diagram of the counting system which have been used in this work. The radioactive source used

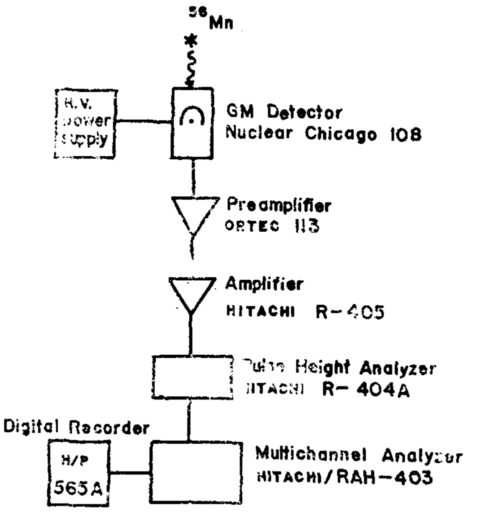

Fig. 1. Block diagram of the counting system used.

in the experiment is ^{56}Mn ($T_{\frac{1}{2}}$ =2.576 hrs), produced in the TRIGA Mark-II reactor. Sincee the neutral isotopic abundance of manganess-55 is almost 100%, there is no problem of contamination due to foreign radioactive materials when this manganese sample is bombarded with neutrons. The activity of the irradiated sample was measured by means of a shielded GM detector (Nuclear chicago, Model 108). The output pulses are amplified and counted in the multiscaling mode with a Hitachi RAH-403 400 channel pulse height analyzer.

The experiment was carried out in order to determine the dead time by sequential measurements of the counting rates $N_i(t_i)$ within the disintegration process of radioactive ^{56}Mn nuclide. It is not necessary to determine the absolute disintegration rate of the sample, but to measure some quantity proportional to the disintegration rate. The sample was thus placed with fixed solid angle at an appropriate distance from the GM detector, and the measurements of its activity were carried out at a 30 minute interval during 4.5 half-lives of ^{56}Mn. The average count rate in cpm was taken from the counting of the sample for two minutes in each

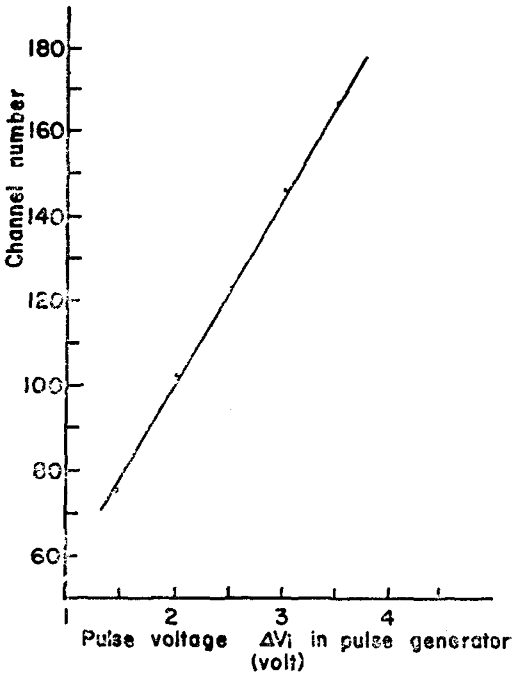

Fig. 2. Relationship between the channel number, T_i, and the pulse voltage ΔV_i

run. During the run, the counting system has been found quite stable. Stability of the whole counting system was controlled by a long-lived ^{137}Cs standard.

For accurate measurement of pulse voltage, it is required to check linearity between pulse voltage and channed number, of the pulse height analyzer. Measurement of the linearity was done with varying the pulse height from the pulse generator (Nuclear Chicago, Model 276 76). It was easy to verify that there existed a proportional relationship between variation of pulse voltage from the pulse generator and the corresponding variation of channel numbers in the pulse height analyzer. As the result show, the calculation of pulse voltage can be done from the channel numbers varying with the observed counting rate resulting from the decay of ^{56}Mn.

4. Results and Discussions

Table 1. The corrected dead time and pulse voltage as a function of counting rates resulting from the decay of ^{56}Mn.

N_i(cpm)	ΔV_i(Volt)	τ(μsec)*
127,640±357	1.495±0.0041	272.2±0.76
125,630±354	1.502±0.0042	273.9±0.77
116,310±341	1.517±0.0044	278.1±0.81
107,920±328	1.549±0.0047	286.4±0.87
96,840±311	1.575±0.0050	293.2±0.94
85,450±292	1.606±0.0054	301.1±1.02
75,000±274	1.650±0.0060	312.5±1.19
61,770±248	1.712±0.0068	327.9±1.31
51,040±226	1.770±0.0078	342.0±1.51
41,030±202	1.842±0.0090	358.8±1.76
33,450±183	1,907±0.0104	373.8±2.04
23,880±154	2.033±0.0131	401.4±2.58
22,470±147	2.050±0.0135	405.0±2.68

*: Calculated by Eq. (10) in which the constants are as follows:
V_u=100 Volts (V=852±10 Volts, V_t=752±20 Volts)
C_t=91×10^{-12} farads, K=0.4 μsec. Volt/cm^2
r_c=1.5875 cm, l=9.6cm

The variation of channel numbers with varying the pulse voltage by pulse generator was measured by means of the 700-channel pulse height analyzer. The observed measurements are shown in Fig. 2. The pulse voltage per channel was calculated from the least square fitting of the experimental data. A mathematical expression for the channel number-to-pulse voltage relationship is given by

$$T_i=44\Delta V_i+13.2 \qquad (11)$$

where T_i is channel number of the 400 channel pulse height analyzer and ΔV_i is the pulse voltage from the pulse generator.

On the other hand, a variation of the measured counting rate, $N_i(t_i)$ as a function of time during the disintegration process was also determined with the aid of this pulse height analyzer and the presented in Fig. 3. Together with experimental values, the extrapolated line drawn from the points for $N_i \geqq 2\times10^4$ counts

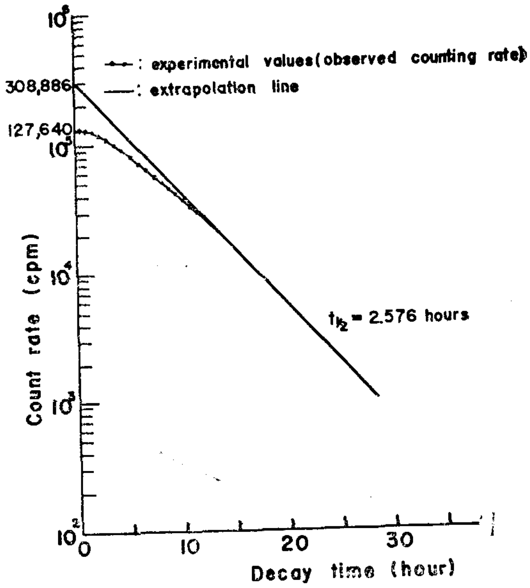

Fig. 3. The observed counting rate of ^{56}Mn as a function of time.

per minute is indicated in the figure as well. The deviation of experimental values from the extrapolated line are known due to counting loss caused by the dead time. Thus Fig. 3. shows a deviation from the linear behavior corresponding to an increase of the dead time at counting rates above 2×10^4 counts per minute.

A variation of the channel number varying with the observed counting rate, N_i as function of time was measured from the spectra resulting from the decay of ^{56}Mn and are shown in Fig. 4 and 5. On the basis of the results observed here, it has been found that the average channel number is increased with decreasing the measured counting rate. The average channel numbers were obtained from the observed results and then the corresponding pulse voltage were calculated by Eq. (11).

Once knowing the pulse voltage, the respective dead time, τ, for various pulse voltages can be explicitly calculated from Eq. (10). Some of these results are presented in Table 1.

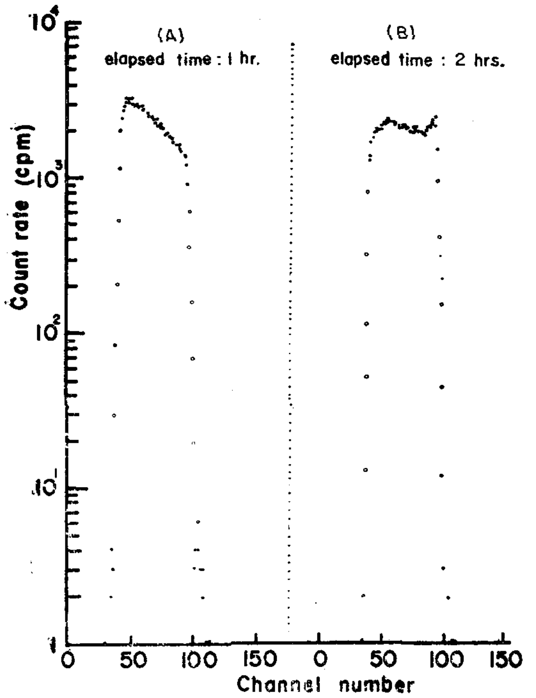

Fig. 4-A and B, The observed counting rate of ^{56}Mn versus the channel.

Table 2. Comparison of the counting rate corrected by the currnet method with those corrected by the commonly used formulae.

N_i(cpm)	τ(μsec)	N_i*	N_i**
127,640±357	271.4±0.7	302,055	222,172
125,630±354	272.7±0.7	292,772	216,144
116,310±341	278,6±0.8	252,953	189,956
107,920±328	284.4±0.8	220,908	177,812
96.840±311	292.7±0.9	183,557	143,001
85,450±292	302.0±1.0	149,934	118,707
75,000±274	312.4±1.2	123,050	100,003
61,770±248	327.3±1.3	93.167	77.786
51,040±226	342.0±1.5	71,984	61,504
41,030±202	358.9±1.8	54,373	47,531
33,450±183	374.6±2.0	42,979	37,649
23,880±154	400.6±2.4	28,409	25,945
22,470±249	405.2±2.6	26,490	24,289

N_i*: the counting rate corrected by the current method and calculated by Eq. (10)
N_i**: the counting rate obtained from the commonly used formulae

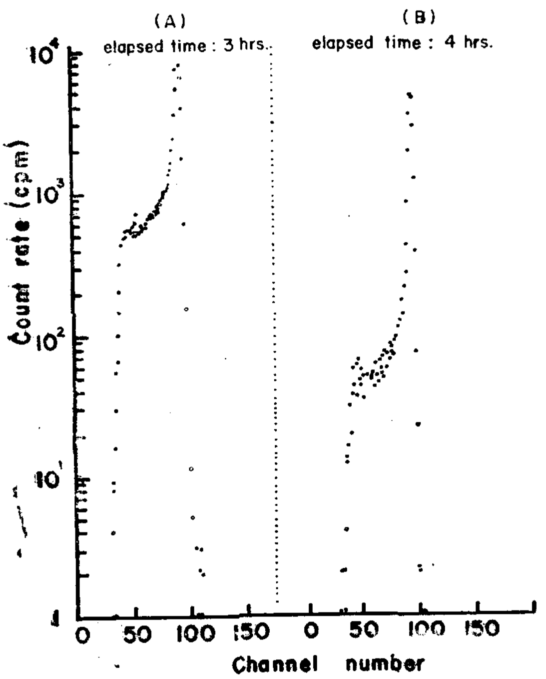

Fig. 5-A and B. number.

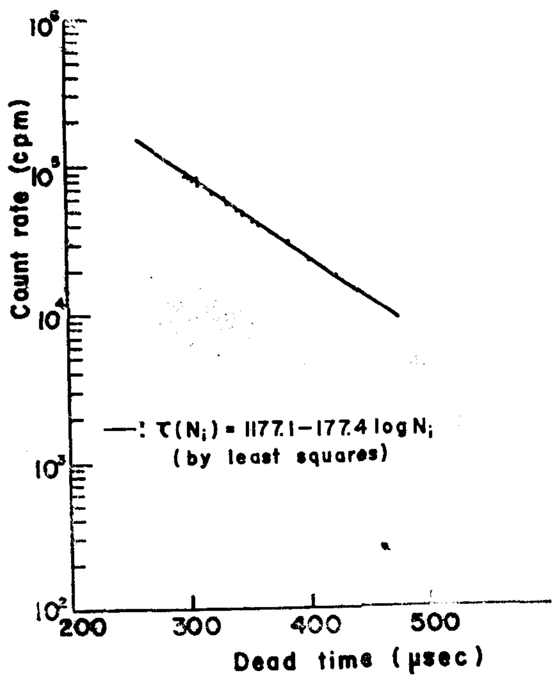

Fig. 6. The observed counting rate of ^{56}Mn versus the dead time.

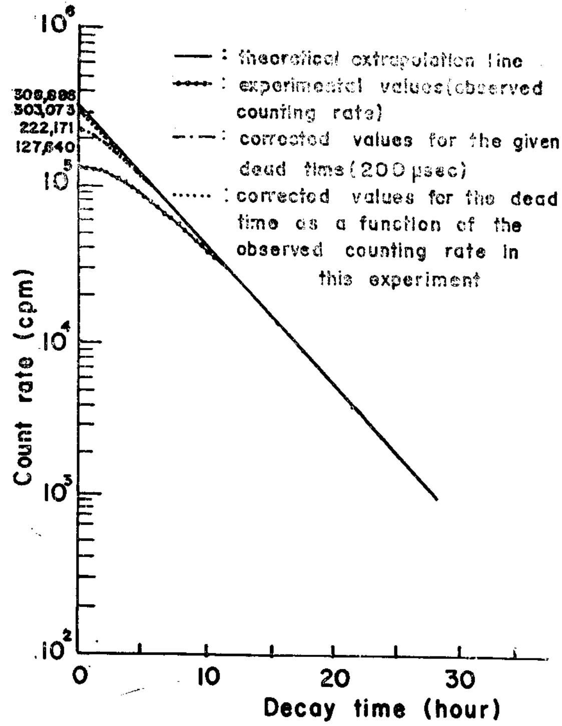

Fig. 7. Intercomparison of the two different correction values with the extrapolation line.

According to the experimental data as shown in Table 1, a relation between the dead time and the counting rate can be simply represented by the following function:

$$\tau(N_i) = 1171.1 - 177.4 \log N_i \qquad (12)$$

which is fitted by the least square method. The dead, $\tau(N_i)$ against the conting rate, N_i, is plotted in Fig. 6, where indicated the dependence of the dead time on the counting rate.

An intercomparison of the counting rates corrected by this method with those corrected by the commonly used fromula, $N_i^* = N_i/(1 - N_i\tau)$, for the various observed counting rates, is presented in tabular form as shown in Table 2, and also plotted in Fig. 7. It has been noticed from the Fig. 7 that these curves in Fig. 7 are practically identical in the range of $O \leq N_i \geq 2\times10^4$ counts per minute. There are large differences for $N_i^* \geq 2\times10^4$ counts per minute. It has been demonstrated, however, that there is a good agreement over a wide range of counting rates within 2% error between the theoretically expected values and the values corrected using the method developed in this work.

As a result of this experimental work, the assumption which the dead time of a of a GM counter is a function of counting rates, has been justified.

5. Conclusions

The technique as outlined in this work presents a method of determining nonlinearity correction in a GM counter. A basic advantage of the method is that the system dead time is determined over a wide range of counting rates.

The dead time calculated from the experimental data has been found to decrease with increasing counting rates. The dead time extracted from such data permits accurate correction. It is easy to obtain accurate values using the current method if due care is taken in making the measurements.

There is no intention to imply that the current method is the only or most desirable solution to the dead time problem. The effects due to a finite lifetime of the source (which would change the interval densities, the counting statistics) and variable dead times in a multichannel analyzer are also factors to be taken into accout in further study.

References

1) P.S. Stevenson, National Research Council Publication NAS-NS 3109, Nation Academy of Sciences, 112 (1965).
2) T.P. Kohman, Anal. Chem., **21** 352 (1949).
3) W.J. Price, Nuclear Radiation Detection, McGraw-Hill, New York (1964).

4) R. D. Evens, The Atomic Nucleus, McGraw-Hill, New York (1955).
5) H. G. Stever, Phys. Rev., **61**, 38 (1942).
6) E. Bleuer and G. J. Goldsmith, Experimental Nucleonics, Rinehart, New York (1952).
7) C. O. Muehlhause and J. Eriedman, Rev. Sic., Inst., **17** (11), 506 (1946).
8) A. Trost. Zeits. f. Physik, **105**, 339 (1937).
9) W. E. Ramsey, Phys. Rev., **57**, 1002 (1940).
10) C. G. Montgmery and D. D. Montgomery, Phys, Rev., **57**, 103 (1940).
11) E. Baldinger and H. Huber, Helv. Phys. Acta., **20**, 470 (1947).
12) T. P. Kohman, Anal. Chem., **21** (3), 352 (1949).