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Technical Paper

Optimal Design of Automatic Steering System of Ships at Sea

Cheol-Yeong Lee*

Abstract

本 論文은 船舶의 計算機制御問題의 一環으로서 一定한 速力으로 針路를 따라 航海하는 船舶을 對象으로 操舵系를 設計하는 問題를 取扱한다.

設計의 指針으로서는 船舶이 大洋을 航海함에 있어서 豫想되는 外亂으로 因하여 增加되는 航路 및 外亂의 影響을 除去하기 위해 취하는 操舵運動으로 因한 速力의 損失을 航路의 延長 또는 航海時間의 延長이라는 觀点에서 2次形式의 評價函數를 選定하고, 이 評價函數를 最小로하는 意味에서 最適操舵系를 設計한다.

實際로 最適操舵系를 實現하기 위해서 適合觀測器를 導入하였으며, 計算機의 시뮤레이션에 依해 本論文에서 提案한 最適操舵系의 有効性을 立證하였다.

實船實驗에 依해 操舵運動과 速力減少의 關係를 確認하였으며, 位置誤差로 因해 야기되는 問題点에 對해서도 若干의 考察을 行하였다.

1. INTRODUCTION

In the past, several autopilots for the ships have been developed. Generally they are specially designed for keeping a constant track. The design of a stable steering loop for the track-keeping ship by a conventional compensation method is introduced by J. Goclowski and A. Gelb(1966).

In recent, with the developement of control theory, optimal control methods have been applied to the design of better controllers which have improved the dynamic performance of steering system of ships.

T. Koyama(1967) defined the cost function and showed parameter optimization procedure in the sense of minimizing the increase of resistance under the disturbance of wind or waves in the ocean. Similar result is derived by P. Dagnion(1973).

More recently, K. J. Åström and C. G. Kallström(1973, 1976) have studied the method which determines ship's parameters using the maximum liklihood method.

Also, adaptive apilot for very large ships is introduced by J. Van Ameronger and A J. Udink ten Cate(1973, 1976)

* Regular member, Korea Merchant Marine College

In this paper, the author surveys design problem of optimal steering system of medium size ships and progression has been made towards more practical performance index and more realistic design procedure.

Section 2 reviews ship's equation as to speed reduction and determines the performance index in the sense of minimizing the increase of travel time by full scale experiment and theoretical computation.

Section 3 presents the optimal design procedure by using Luenberger type observer and simulated results.

List of symbols

M : mass of ship
M_x, M_y : added mass on X, Y-axis respectively
θ, $\dot{\theta}$: ship's heading angle, the rate of turn
ß : drift angle
δ : rudder angle or rudder deflection
u, v : ship's velocity on X, Y-axis respectively
U, V_n : total ship's velocity, navigational speed
N, I : moment, moment of inertia
X_f, Y_f : hydrodynamic force on X, Y-axis respectively
T_p, t_p : propeller thrust, thrust deduction
$T_{3\beta}$, T_3 : time constant of sway and yaw motion of ship respectively
K_β, K : gain of sway and yaw motion of ship respectively
L_p, R : ship's length, turning of radius
σ, I_c : speed reduction ratio, increasement ratio of travel distance
A : $n \times n$ system matrix
B : $n \times r$ input matrix
C : $r \times n$ output matrix
O_n : $n \times n$ null matrix
D, E, F : $p \times p$, $p \times r$, $p \times r$ observer matrices respectively
P_0, V, T : $r \times p$, $r \times r$, $p \times n$ observer matrices respectively

2. APPRAISEMENT OF STEERING SYSTEM AT SEA

When consider the ship as means of marine transportation, the first consideration is, quite clearly, the safety of the ship.

Minimum travel time and damage free cargo and hull are obvious cost factors, as are minimum fuel consumption and maximum passenger and crew comfort. Of course, minimum rolling or pitching are to be considered. But it is difficult to treat above mentioned factors altogether.

The particular goals and their relative priority will vary with the mission of ship.

The mathematical model under consideration is presently based on the minimum travel time when ship moves on the prescribed track.

Sailing along the prescribed track in a seaway, the ship make a detour due to external disturbance and rudder action, and such a detour will increase the travel time. If rudder action is taken to return the ship to the prescribed track, considerable speed loss will be induced.

When ship sails along the prescribed track with constant thrust power, speed loss also can be considered as increase of travel time or travel distance.

In this paper, two losses only, due to course deviation and rudder action, will be considered in the appraisement of steering system at sea. All other losses can be neglected compared with those two.

SPEED REDUCTION

A ship speed function defines a relationship between mean ship forward speed and a number of variables that take on specific average values during a given time interval of ship's operation.

In the case of track keeping, variables due to sea and wind can be neglected because those can not be removed by rudder action.

Considering ship's dynamics concern on the speed loss, ship's equation on X-axis becomes important.

To describe the equation of ship's equation, use the coordinate system fixed to the ship as shown in Fig. 1.

Let u, v be the ship's veloctity on X, Y-axis respectively, and $\dot{\theta}$ the rate of turn.

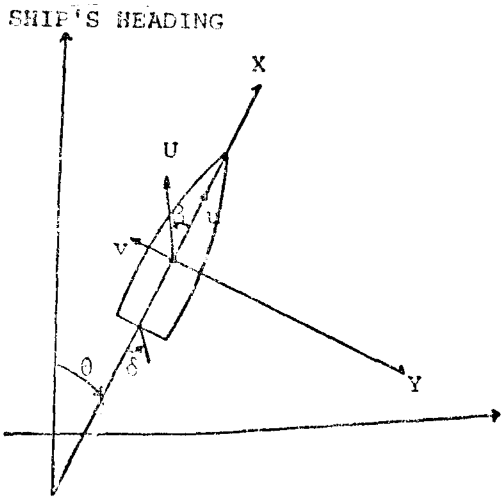

Fig. 1. Definition of Coordinates fixed to the ship.

The equations of ship are;

$$\left.\begin{aligned} M(\dot{u}-\dot{\theta}v) &= X_f \\ M(\dot{v}+\dot{\theta}u) &= Y_f \\ I\ddot{\theta} &= N \end{aligned}\right\} \left(\cdot = \frac{d}{dt}\right) \cdots\cdots (1)$$

where M is the mass of ship, I its moment of inertia, N the moment, and X_f, Y_f the component of the hydrodynamic forces on the X, Y-axis respectively.

The hydrodynamic force X_f is a complicated function of the motion, *i. e.*,

$$X_f = X_f(\dot{u},\ u,\ T_p,\ t_p,\ \dot{\theta},\ \delta,\cdots\cdots) \cdots\cdots (2)$$

and analogous for Y_f and N. Where T_p is the propeller thrust, t_p the thrust deduction, and δ rudder angle or rudder deflection.

These hydrodynamic force and moment expression are reduced to polynomials of reasonable length, if terms believed to be unimportant on the basis of available experimental and theoretical results are omitted.

Assuming that second order term takes precedence to first order term in eq. (2), ship's equation on X-axis as to speed reduction can be expressed as follows(Appendix).

$$X_u u + (M+M_x)\dot{u} = T_p(1-t_p) + (M+M_y)U\beta\dot{\theta} + X_{\theta 2}\dot{\theta}^2 + X_{\delta 2}\delta^2 \cdots\cdots (3)$$

where M_x, M_y are added mass of ship on X, Y-axis respectively.

As it is well known that the phase difference between rate of turn and drift angle is not too large, β can be expressed as function of $\dot{\theta}$(K. Nomoto, 1964).

$$\beta \simeq \sin\beta = L_p/R \simeq (K_\beta/K)(T_{3\beta}/T_3)\dot{\theta} \cdots\cdots (4)$$

where $T_{3\beta}$, T_3 and K_β, K are time constant and gain of sway and yaw motion of ship respectively, L_p the ship's length, R the turning radius.

The quantities X_u, $X_{\theta 2}$, and $X_{\delta 2}$ denote the hydrodynamic derivatives.

The derivative X_u is defined by $\partial x/\partial u$ and the other derivatives are defined analogously.

Rudder action whick is taken for the purpose of preventing the course deviation and rate of turn inevitably increases not only rudder angle itself but propeller load, and increase of propeller load will induce increase of thrust deduction.

Assuming that thrust deduction in eq. (3) is consisted of two components which are proportional to rudder angle and rate of turn respectively, and dividing this equation by X_u, and using eq. (4), then, next relation is obtained.

$$T_0\dot{u} + u = F_0(T_p) - F_1(\dot{\theta}^2) - F_2(\delta^2) \cdots\cdots (5)$$

As steady stiaight forward speed of ship u_0 is $F_0(T_p)$, eq. (5) is,

$$T_0\dot{u} + u = u_0(1 - K_1\dot{\theta}^2 - K_2\delta^2) \cdots\cdots (6)$$

where $F_0(T_p)$, $F_1(\dot{\theta}^2)$, and $F_2(\delta^2)$ are items as to the steady strait forward thrust, rate of turn, and rudder angle and T_0 the time constant as to speed reduction..

Full scale experiments are carried out with the ship M/V FUJI to obtaint the speed reduction ratio. The specification and experimental condition are as Table 1.

Table 1

Specification		Experimental Condition	
Length over all	125.0m	Displacement	4,500K/T
Displacement	6,000K/T	Weather condition	B
Power of main Engine	5,000H.P.	Wind condition	Calm
Service speed	15Kt	Draft: fore	5.0m
		aft	5.5m

From eq. (6), it is obvious that speed reduction is mainly caused by the centrifugal force proportional to rate of turn. Rate of turn is caused by wind, wave, and rudder action, but here only the rate of turn due to rudder action is considered because the other can not be removed on ship board.

The rate of turn due to rudder in the case of the ship M/V FUJI is measured and the result is shown in Fig. 2.

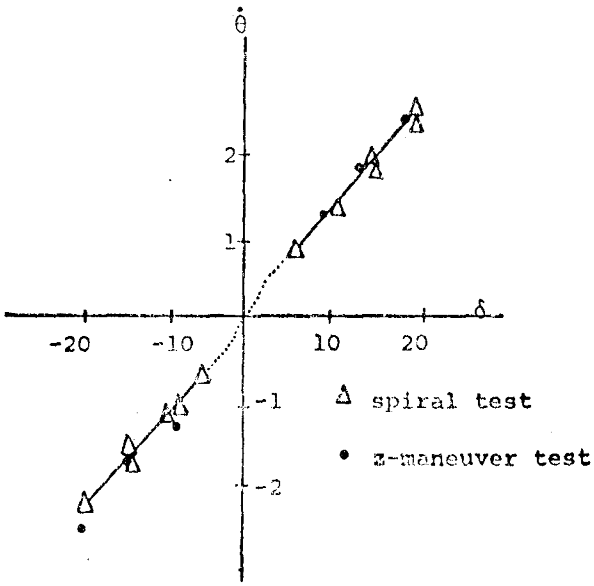

Fig. 2. Rate of turn vs. Rudder angle

From Fig. 2, it is seen that rate of turn is nearly proportional to rudder angle except the part of dotted line. It is assumed that the part of dotted line is due to nonlinearity of ship and/or residuals of rudder angle.

Considering that rudder action is the only way to keep track, to express the speed reduction as function of rudder angle will be reasonable. Hence, simplifying eq. (6),

$$T_0\dot{u}+u=u_0(1-K_\delta\delta^2) \quad \cdots\cdots (7)$$

To obtain the speed reduction ratio, spiral and Z-maneuver tests are carried out.

In the case of spiral test, as rudder is deflected to constant degree angle, the speed variation is to be zero. Then eq. (7) is,

$$u_{spiral}=u_0(1-K_\delta\delta^2) \quad \cdots\cdots(8)$$

Next, Z-maneuver test is excuted in the presettled time interval, and ship's mean speed is obtained as:

$$u_{mean}=\frac{\text{Travel distance by log}}{\text{Travel time}} \quad \cdots\cdots(9)$$

while excute this test, rudder angle is recorded and mean of rudder is computated numerically.

These test are carried out, also, in calm sea condition to reduce the influence of external disturbance.

The result is shown in Fig. 3.

By using the least squares method from Fig. 3, speed reduction ratio σ is obtained as,

$$\sigma=1-u_{mean}/u_0\simeq 2.0\delta^2 \cdots\cdots(10)$$

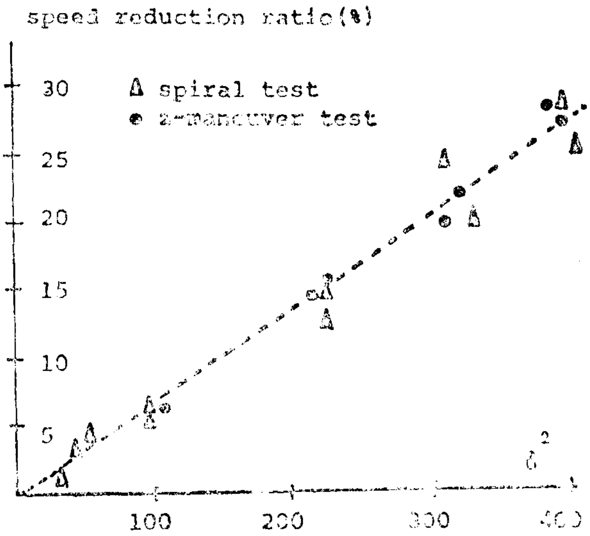

Fig. 3. Speed reduction ratio vs. Rudder angle

INCREASE OF TRAVEL DISTANCE

Course deviation is caused by wave, wind, and rudder action, but, those factors except rudder action are difficult to measure.

Here, leaving the cause of deviation out of account, assume that ship make a detour as Fig. 4 due to rudder action and external disturbance.

Designate this track as $d=x(t)$.

Fig. 4. Ship's Sailing Track

As deviation is not too large in the case of track keeping, total travel distance D_a is,

$$D_a=\int_0^a(1+\dot{d}^2)^{1/2}\,dx\simeq\int_0^a\left(1+\frac{1}{2}\dot{d}^2\right)dx \cdots\cdots(11)$$

Hence, increasement ratio of travel distance I_c is,

$$I_c=1/2\dot{d}^2=1/2\tan^2\theta\simeq 1/2\theta^2 \quad \cdots\cdots(12)$$

Evaluating increasement ratio of travel distance and speed reduction ratio on same unit, especially in time, increase of travel time ΔT is,

$$\Delta T=\int_0^a\left(\frac{1+I_c}{1-\sigma}-1\right)\frac{dx}{dV_n}\simeq\int_0^a(I_c+\sigma)dt \quad\cdots\cdots(13)$$

where V_n is the navigational speed.

As ship's time constants are very small comparing with the total travel time, performance index J can be expressed as,

$$J=\text{Min}\ \Delta T=\frac{1}{2}\int_0^\infty(\theta^2+\rho\delta^2)dt \quad\cdots\cdots(14)$$

where ρ is about 4.0 in case of the ship FUJI

3. OPTIMAL DESIGN OF AUTOMATIC STEERING SYSTEM

Since the yaw motion is sufficient to discuss steering and autopilot design, here will limit the following discussion to the yaw motion only.

As in the case of track keeping small rudder cccur, a simple linear model of ship's motion can be used to describe the behaviour of the ship.

Such a model is as follows (K. Nomoto, 1964),

$$T_1T_2\,\dddot{\theta}+(T_1+T_2)\ddot{\theta}+\theta=K(\delta+T_3\dot{\delta}) \quad\cdots\cdots(15)$$

or as in state space from:

$$\left.\begin{aligned}X&=AX+B\delta\\ \dot{Y}&=CX\end{aligned}\right\} \quad\cdots\cdots(16)$$

where,

$$A=\begin{pmatrix}0 & 1 & 0\\ 0 & -(T_1+T_2)/T_1T_2 & 1\\ 0 & -1/T_1T^2 & 0\end{pmatrix} \qquad B=\begin{pmatrix}0\\ KT_3/T_1T_2\\ K/T_1T_2\end{pmatrix}$$

$$X'=(\theta,\ \dot{\theta},\ x_3)\ \ ('=\text{transpose}) \qquad C=(1,\ 0,\ 0)$$

and T_1, T_2 and T_3 are time constant of ship.

A comprise design will be achieved by minimizing the performance index of eq. (14).

The problem posed here is to find an optimal control δ_o which minimizes the performance index.

The existence of the optimal control law requires that the system be controllable, and can be obtained as follows.

$$\left.\begin{aligned}\delta_0&=-WX\\ W&=R^{-1}B'P\end{aligned}\right\}\quad\cdots\cdots(17)$$

where P is positive definite solution of the Riccati equation

$$PA+A'P-W'RW+Q=O_n\cdots\cdots(18)$$

The optimally controlled system is than given by

$$\dot{X}=(A-BW)X\cdots\cdots(19)$$

Thus, the entire state vector X must be available for the implementation of the feed back control law given by eq. (17), but, as the basic measurment of system performance under cosideration is the course angle only, a compatible observer has to be designed.

A. Inoue (1974) has shown the condition of linear function for the existence of an observer of a given order in the case that the observer has only a specified pole configuration.

The observer is defined as:

$$\left.\begin{aligned}\dot{Z}&=DZ+EY+F\delta\\ W&=P_0Z+VY\end{aligned}\right\}\cdots\cdots(20)$$

for a choice of

$$F=TB\quad\cdots\cdots(21)$$

where T is the solution of

$$TA-DT=EC$$

and D is defined as,

$$D=\begin{pmatrix}0 & & -\lambda_p\\ \ddots & & \vdots\\ 1 & 0 & -\lambda_2\\ & \ddots & \\ & 1 & -\lambda_1\end{pmatrix}\quad\cdots\cdots(22)$$

also, V and P_0 are matrices obtained as

$$\begin{aligned}V&=V_p\\ P_0&=(P_1,\ P_2,\cdots\cdots P_p)\end{aligned}\quad\cdots\cdots(23)$$

where,

$$\begin{aligned}P_1&=V_{p-1}-\lambda_1V_p\\ P_2&=V_{p-2}-\lambda_2V_p-\lambda_1P_1\\ &\vdots\\ P_p&=V_0-\lambda_pV_p-\lambda_{p-1}P_1-\cdots\cdots-\lambda_1P_{p-1}\end{aligned}$$

and, $V_p\cdots\cdots V_1$ are row vector satisfying the following relation,

$$W'F(A)=V_0C+V_1CA+\cdots\cdots+V_pCA^p\quad\cdots\cdots(24)$$

where,

$$W'F(A)=W'A^p+\lambda_1 W'A^{p-1}+\cdots\cdots+\lambda_p W'$$

and p is the dimension of observer.

For the system under consideration, the second order observer with poles at -0.9 and -0.1 is to be constructed from the the coures angle measured by gyro compass.

The system matrices are as follows.

$$P_0=(0.08688,\ -0.03928)\quad V=(-0.58591)$$

$$D=\begin{pmatrix}0 & -0.02\\ 1 & -0.5\end{pmatrix}\qquad E=\begin{pmatrix}1\\ 0\end{pmatrix}\qquad F=\begin{pmatrix}0.7599\\ -1.4451\end{pmatrix}$$

Having obtained these matrices, the observer dynamics are completely determined. The time constants and gain in half loading condition of the ship FUJI are $T_1=10.0$, $T_2=4.0$, $T_3=6.0$, and $K=0.16$.

DISCUSSION ON OFF-SET ERROR

In a seaway, ship's track is slight different from the ship's heading due to the existence of off-set error.

In order to remove off-set error, it seems reasonable that the control law should make use of not only the nomial state X, but also of μ the integral of deviation.

As is well known, off-set error is mainly connected with the yaw and sway motion of ship, i. e. :

$$\mu=\int_0^t U\sin(\theta-\beta)dt+\mu_0\simeq\int_0^t U(\theta-\beta)dt \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(25)$$

$$\dot{\mu}/U=\theta-\beta$$

or neglecting drift angle,

$$\dot{\mu}/U\simeq\theta$$

where μ_0 is initial value.

When off-set error exists, it is assumed that increase of travel distance occured as shown in Fig. 5.

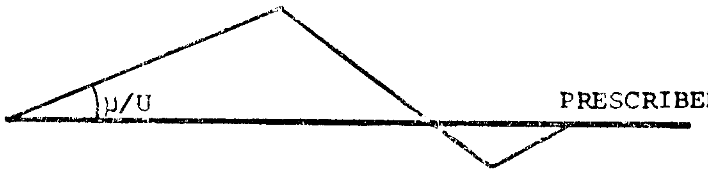

Fig. 5. Increase of Travel distance due to off-set error

Hence, the increasement ratio of travel distance O_I due to off-set error is,

$$O_I = \frac{1}{\cos(\mu/U)} - 1 \simeq \frac{1}{2}(\mu/U)^2 \quad \cdots\cdots (26)$$

Thus performance index of eq. (14) can be,

$$J = \frac{1}{2}\int_0^\infty ((\mu - U)^2 + \theta^2 + \rho\delta^2)dt \quad \cdots\cdots (27)$$

From eqs. (16), (25), the nomial state equation are augmented to include μ in the following manner,

$$\begin{pmatrix} \dot{X} \\ \mu/U \end{pmatrix} = \begin{pmatrix} A-BW & -BW_I \\ -C & O \end{pmatrix} \begin{pmatrix} X \\ \mu/U \end{pmatrix} \quad \cdots\cdots (28)$$

and a linear control low can then be written as,

$$\delta_{0+I} = -WX - W_I\mu/U \quad \cdots\cdots (29)$$

where W_I is so-called the integral feed back gain.

This system is shown in Fig. 6.

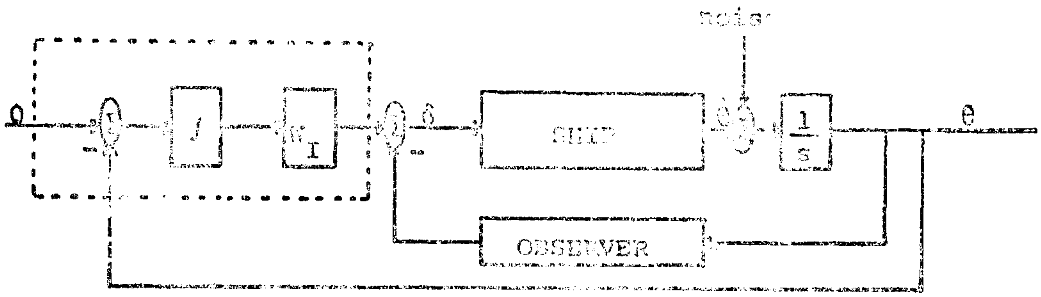

Fig. 6. Automatic Steering System of Ship

It is clear from eq. (25) that provided an off-set error exists and is approached asymptotically by the use of the control law of eq. (29), then the desired zero off-set error will be obtained when $(\mu/U) \to O$.

But, as in the case of ocean going vessel such a off-set error can be simply corrected by using of precise position information which is provided by Decca and/or Omega navigation system, it will be sufficient to treat this as another problem closely related to safety of ship in coast or narrow channel sailing.

Simulation

Using linear function $\omega \simeq WX(t)$ estimated by observer instead of optimal control δ_o, the increase in performance index due to observer error results in general as,

$$W(t)=W(t)+e^{Dt}(Z(0)-TX(0))\rightarrow W(t)(t\rightarrow\infty)$$

or

$$\Delta J=(Z(0)-TX(0))'\times\int_0^\infty e^{D't}P'RPe^{Dt}\,dt(Z(0)-Tx(0))$$

where

$$D'Y+YD=-P'RP$$

That is;

$$\Delta J=\int_0^\infty(\omega(t)-\delta_0(t))R(\omega(t))-\delta_0(t))dt \quad \cdots\cdots (30)$$

Here, the performance of the steering system is investigated by simulation in both case of using optimal control law and estimated linear function by observer.

Also, this simulation is carried out, firstly, in the case that initial value in the heading angle is present, and secondly, in the case that initial value in the heading angle and measurement noise which is represented by a Gaussian noise signal with standard deviation of 0.3 degree in the rate of turn are simultaneously present.

In Figs. 7a and 7b, responses are shown of the heading angle $\theta(t)$, the actual rudder angle $\delta(t)$ and the desired heading is zero.

It is seen that the ship's responses are very stisfactory by indroducing observer, but more practical experiments will be necessary for the purpose of application the above mentioned design method on the ship.

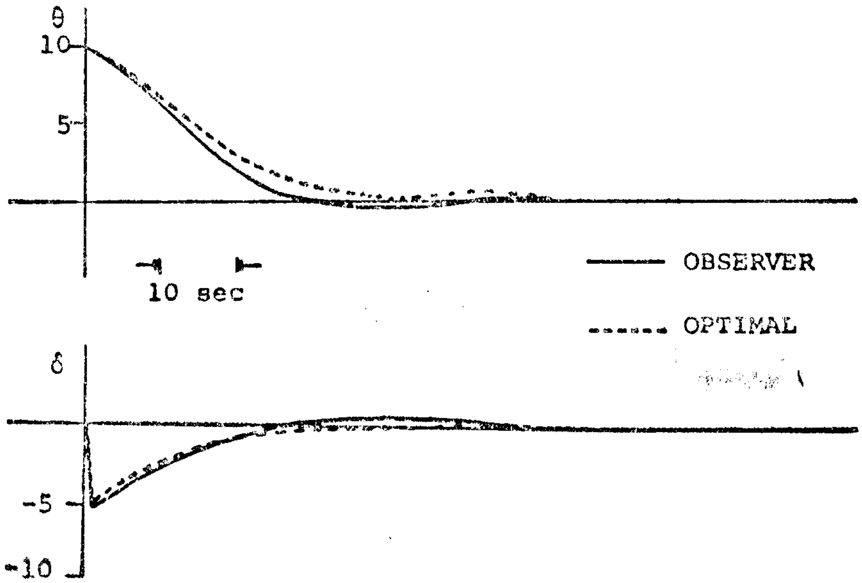

Fig. 7a. Response (Devitation only pressent)

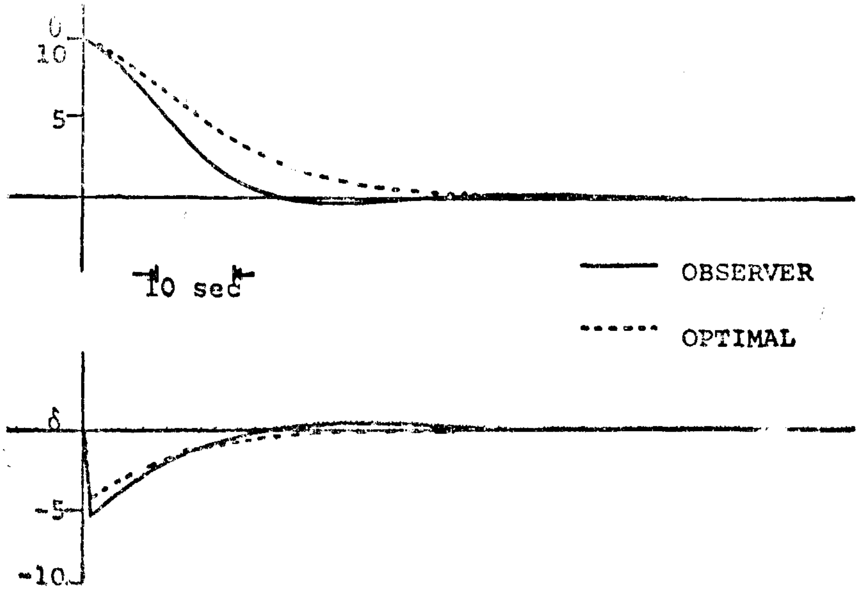

Fig. 7b. Response (Deviation and measurement noise present)

4. CONCLLUSION

In this paper, the optimal design for the ship sailing along the prescribed track at constant speed is considered.

The cost function is obtained in the view point of minimizing the increase of travel time, and full scale experiment is carried out by the ship M/V FUJI to obtain the speed reduction ratio.

For the implementation of the optimal feedback control law, Luenburger type of observer is introduced, and simulation is carried out to investigate performance of automatic steering system of ship.

It can be said that design procedure proposed in this paper is very effective and stisfactory.

APPENDIX

Assuming that coupling effects of roll, pitch and heave motion into the horizontal motion are negligibly small, from eq. (2) the hydrodynamic force term X_f can be expressed in the form under calm sea condition (E. Eda and C. L. Crane, Jr., 1976, J. Goclowski and A. Gelb, 1966):

$$X_f = X_{hull} + X_{rudder} + X_{propeller} + X_{wave}$$
$$\simeq X_u u + X_{\dot{u}} u + X_{1\dot{\theta}} \dot{\theta} + X_{\dot{\delta}} \delta + X_\delta \delta + T_p (1 - t_p) \quad \cdots\cdots\cdots (A1)$$

Considering that rudder angular rate is from eq. (1), eq. (A1) can be expressed as:

$$X_u u + (M + M_x) u \simeq T_p (1 - t_p) + (M + M_y) U\beta\dot{\theta} + X_{\theta^2} \dot{\theta}^2 + X_\delta \delta + X_{\delta^2} \delta^2 \quad \cdots\cdots\cdots (A2)$$

where,

$$M_x = -X_{\dot{u}},$$
$$X_{1\dot{\theta}} = M_y v + X_{\dot{\theta}}$$
$$v = U \sin\beta \simeq U_\beta$$

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