論文 78-15-3-5

Analysis and Design of an Improved UHF Amplifier

康 昌 彥* B. Ho**
(Kang, Chang Eon, and B. Ho**)

要 約

사이크로트론波와 電子束波의 結合에 依한 新型모드의 動作이 관찰되었다. 電波 形成 과정을 分析하는데 電子動力學과 結合모드에 依한 方法이 적용되었으며 誘導된 理論的 利得式에서 新型超高調波 증폭기가 設計 製作 되었으며 實驗結果雜音이 없이 高電力을 얻을 수 있는 증폭기임이 보여진다.

Abstract

A new mode of operation is observed in this paper which involves the active wave interaction between a cyclotron wave and a synchronous electron beam. The wave interaction is analyzed by using both electron dynamics and coupled-mode methods. The gain expression derived from the analysis proposes a new type of improved uhf amplifier with low noise and high power. Based on the theoretical analysis, experimental tubes have been designed, constructed and experimented. As a result, a low noise amplifier is anticipated well into millimeter wave region.

1. Introduction

Microwave beam devices are mostly designed on the principle of longitudinal interaction between slow space charge wave on the electron beam and a propagating electiomagnetic wave circuit[1,2,3] The noise figure of such devices has a typical lower limit of several *db*. The reason is that the beam noise of the slow space charge wave, which carries negative kinetic power, cannot be removed from the beam by an appropriate circuit. The interaction described here involves an active coupling between a fast cyclotron wave and a positive synchronous wave of a beam, both of which carry positive kinetic power and thus presents a low noise possiblity.

The device consists of five principle parts: a multianode gun to provide a small-diameter beam, input and output quadrifilar couplers, a quadrifiiar helix pump structure, and a collector.

2. Coupled Mode Analysis of the Wave Interaction

Consider a filamentary electron beam in spatially varying electrostatic field and constant axial magnetic field. Small signal analysis and $e^{j\omega t}$ type of variation are assumed. The equation of electron motion is:

$$\frac{d\vec{u}}{dt} = -\eta(\vec{E}+\vec{u}\times\vec{B}) \tag{1}$$

where u, η, E, and B are the velocity of electron beam, the charge to mass ratio, electric field, and magnetic field respectively, Let all quantities involved contain *dc* and *ac* components in the form

$$\vec{A} = \vec{A}_0 + \vec{A}_1 \tag{2}$$

and assume that $A_1 \ll A_0$ where subscription 1 and 0 represent *ac* and *dc* quantities respectively.

The *ac* parts of Eq. 1 are decomposed into the X, Y, and Z-components as:

$$\frac{du_{1x}}{dt} = -\eta(E_{1x} + u_0 y B_{1z} + u_{1y} B_{0z} - u_{0z} B_{1y} - u_{1z} B_{0y}) \tag{3}$$

---

*正會員,
Northern Illinois Univ. 敎授
**Dr Ho is Professor of the Electrical Engineering Department at Michigan State University.
接受日字 : 1978年 4月 22日

$$\frac{du_{1y}}{dt}=-\eta(E_{1y}-u_{0x}B_{1z}-u_{1x}B_{0z}+u_{0z}B_{1x}+u_{1z}B_{0x}) \quad (4)$$

$$\frac{du_{1z}}{dt}=-\eta(E_{1z}+u_{1x}B_{0y}+u_{0x}B_{1y}-u_{0y}B_{1x}-u_{1y}B_{0x}) \quad (5)$$

where u_0 and u_1 are the *dc* and *ac* beam velocities. The transverse displacements and velocities are related by

$$\frac{dx}{dt}=u_x \quad (6)$$

$$\frac{dy}{dt}=u_y \quad (7)$$

Conveniently, we define

$$u_{\pm}=u_{1x}\pm ju_{1y}$$
$$E_{\pm}=E_x\pm jE_y$$
$$B_{\pm}=B_x\pm jB_y$$
$$F_{\pm}=E_{\pm}\pm ju_0B_{\pm}$$

To simplify the analysis, assume that the *ac* magnetic fields in the transverse direction are negligible, i.e., $B=0$, that the cyclotron frequency ω_c is a constant, $u_0=u_0Z$, and $B_0=B_0Z$. The coupled mode equations from Eqs. 3 through 7, become

$$\left(\frac{d}{dt}-j\omega_c\right)a_1=-\eta kE_+ \quad (8)$$

$$\left(\frac{d}{dt}+j\omega_c\right)a_2=-\eta kE_- \quad (9)$$

$$\frac{da_3}{dt}=-\eta kE_+ \quad (10)$$

$$\frac{da_4}{dt}=-\eta kE_- \quad (11)$$

where $a_{1,2}$ = fast and slow cyclotron modes $=A_{1,2}e^{-j(\beta_e\mp\beta_c)z}$

$a_{3,4}$ = negative and positial synchronous modes $=A_{3,4}e^{-j\beta_e z}$

k = beam impedance $=\frac{1}{4}\sqrt{\frac{I_0\omega}{\eta\omega_c}}$

ω_c = cyclotron frequency $=\eta B_0$, $\beta_c=\frac{\omega_c}{u_0}$, $\beta_e=\frac{\omega}{u_0}$

Since u_0 is assumed to be constant, $d/dt=u_0(d/dz)$. Eqs. 8 through 11 are rewritten as:

$$\left(\frac{d}{dz}-j\beta_c\right)a_1=-\frac{\eta k}{u_0}E_+ \quad (12)$$

$$\left(\frac{d}{dz}+j\beta_c\right)a_2=-\frac{\eta k}{u_0}E_- \quad (13)$$

$$\frac{da_3}{dz}=-\frac{\eta k}{u_0}E_+ \quad (14)$$

$$\frac{da_4}{dz}=-\frac{\eta k}{u_0}E_- \quad (15)$$

Differentiating Eqs. 12 and 14 with respect to Z. and combining them together yield:

$$\frac{d^2A_1}{dz^2}+j\beta_c\frac{dA_1}{dz}e^{j\beta_c z}-\frac{d^2A_3}{dz_2}+j\beta_c\frac{dA_3}{dz}=-j\beta_cME_+ \quad (16)$$

$$\frac{d^2A_2}{dz^2}-j\beta_c\frac{dA_2}{dz}e^{-j\beta_c z}-\frac{d^2A_4}{dz^2}-j\beta_c\frac{dA_4}{dz}=j\beta_cME \quad (17)$$

where $M=\frac{\eta k}{u_0}$

For a quadrifilar pumpfield, the transverse electric fields $E_\pm$ are

$$E_{\pm}=-\frac{2V_p}{k\omega_c a^2}(A_4-A_2)e^{\pm j2\beta_q z} \quad (18)$$

where β_q = pump field phase constant $=\frac{2\pi}{p}$, p = pitch of the helices.

The coupling between the fast cyclotron mode and positive synchronous mode can be found by using $\beta_q=\frac{1}{2}\beta_c$ in Eqs. 16 and 17.

The coupling equations are

$$\frac{d^2A_1}{dz^2}+j\beta_c\frac{dA_1}{dz}=jNA_4 \quad (19)$$

$$\frac{d^2A_4}{dz^2}+j\beta_c\frac{dA_4}{dz}=-jNA_1 \quad (20)$$

where N is the pump parameter defined as

$$N=\frac{2\eta\beta_cV_p}{u_0\omega_c a^2}$$

and a is the radius of the quadrifilar helixstructure, V_p is the pump voltage.

The propagation constant Γ^2 of the modes has positive real part when the pump parameter $N>\frac{\beta_c}{4}$ The gain expression is found to be

$$G=10\log\left[\cosh\Gamma z+\frac{2\eta V_p}{u_0\omega_c a^2\Gamma}\frac{A_4(0)}{A_1(0)}\sinh\Gamma z\right]db \quad (21)$$

The derivation of Eq. 21 is given in Appendix *A*.

3. Electron Dynamic Solution in the pump region

The equation of electron motion in the pump region can be better described by using cylindrical coordinate system due to the geometrical configuration of the structure. The equation in cylindrical coordinates can be written in the three component forms.

$$\frac{\partial^2 r}{\partial t^2}-r\left(\frac{\partial\theta}{\partial t}\right)^2=-\eta\left(-\frac{\partial V}{\partial r}+B_z r\frac{\partial\theta}{\partial t}\right) \quad (22)$$

$$r\frac{\partial^2\theta}{\partial t^2}+2\frac{\partial r}{\partial t}\frac{\partial\theta}{\partial t}=-\eta\left(-\frac{1}{r}\frac{\partial V}{\partial\theta}-B_z\frac{\partial r}{\partial t}\right) \quad (23)$$

$$\frac{\partial^2 z}{\partial t^2}=\eta\frac{\partial V}{\partial z} \qquad (24)$$

where $V(r, \theta, z)$ is the pump potential distribution which is given by

$$V(r, \theta, z)=\frac{V_p}{J_2({}_q2\beta a)}J_2(2\beta_q r)\sin(2\theta-2\beta_q z) \quad (25)$$

where J_2 is the Bessel function of order two.

The computer solution of the energy variations is shown in Fig. 1. It is interesting to note that the axial energy remains almost constant during the amplification process, while the rotational energy increases rapidly when the pump strength is above the critical value. The critical pumping voltage obtained from the computer solution checks closely with that from the coupled-mode analysis.

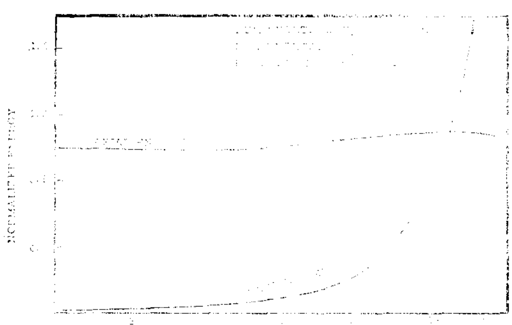


Fig. 1. Energy variations of the cyclotron-synchronous mode interaction.

4. Electron Dynamic Solution in the Transition Region

The fields in this region consist of an axial $d-c$ magnetic field and the fringe electrostatic pump field at the end of the pump section. The fringe field is rather complex and important to the operating principle of the device. We first obtain the potential distribution in the transition region and then solve for the energy variations in that region. The potential distribution is found to be

$$V(r, \theta, z)=\frac{z}{L}V_2+b(V_0+V_p)\left(\frac{1}{\sqrt{U+S}}+\frac{1}{\sqrt{U-S}}-\frac{1}{\sqrt{V+S}}-\frac{1}{\sqrt{V-S}}\right)+(V_0-V_p)\left(\frac{1}{\sqrt{U+T}}+\frac{1}{\sqrt{U-T}}-\frac{1}{\sqrt{V+T}}-\frac{1}{\sqrt{V-T}}\right) \quad (26)$$

where $U=a^2+r^2+z^2$, $V=a^2+r^2+(2L-z)^2$, $S=2ar\cos\theta$, $T=2ar\sin\theta$ and b is the radius of the helix wire, L is the length of the transition region, and V_2 is the $d-c$ potential on the output coupler.

The computer solution of the rotational and axial energies as a function of distance along the transition region is shown in shown in Fig. 2. The rotational energy is practically constant as expected, while the axial energy depends on the relative potential between the output coupler and the point at which the beam leaves the quadrifilar helix. This result confirms the belief that the rotational energy, which is the amplified signal, will carry on into the output coupler.

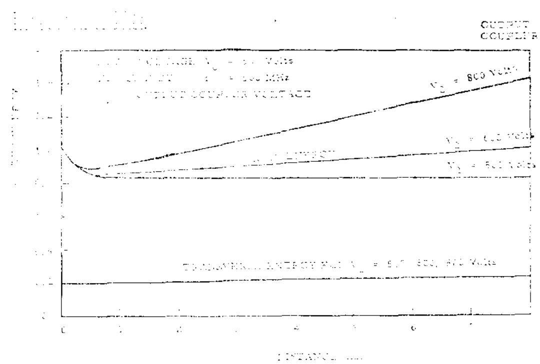
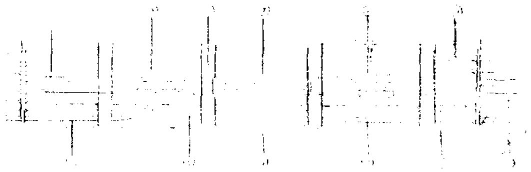


Fig. 2. Energy variations in the transition region between the pump section and the output coupler. (The beam leaves the pump section at a potential of approximately 600 volts.)

5. Results and Discussions

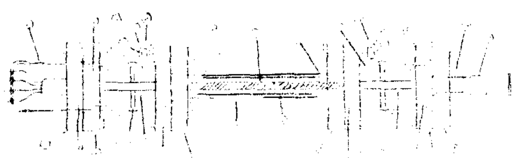
The schematic diagrams of the experimental tubes are presented in Figs. 3 and 4. The electronic gain as a function of pump voltage is shown in Figs. 5 and 6.

There are several attractive characteristics of this interaction. Frst, an extreme low noise amplifier is possible, since this interaction involves two positive kinetic power beam waves whose beam noise can be stripped before the amplifying process. A possible arrangement is shown in Fig. 7. Here the beam noise from the positive synchronous mode is removed by the matched synchronous



1. Electron Gun
2. Signal Lead-In
3. Input Cuccia Coupler
4. Supporting Ring
5. Aligning Rod
6. Glass Enevelope
7. Quadrifilar Helix
8. Output Cuccia Coupler
9. Collector
10. D-C Pump Leads
11. Signal Lead-Out
12. Ion Pump **Figure**

Fig. 3. Schematic diagram of the experimemental UHF cyclotron-synchronous wave ampifier.



1. Coupler Cavity
2. Cavity Flange
3. Helix Housing Flange
4. Cavity End Plate
5. Cavity End Plate
6. Helix Assembly
7. Joint for Q.H. Housing
8. Quadrifilar Helix
9. Helix Support Rod.
10. Coupler Plate
11. Coupler Supporting Rod
12. Feed Through
13. Collector
14. Kovar Tubing
15. Gun End Flange
16. Gun Supporting Ring
17. Cavity Plunger
18. Bellows
19. Tuning Assembly
20. Tuning Screws
21. Copper Glass Seal

Fig. 4. Schematic diagram of the S-band cyclotron-synchronous wave amplifier.

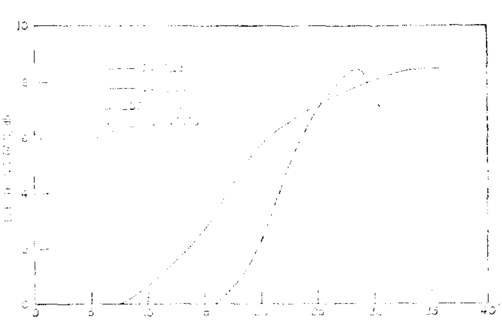


Fig. 5. UHF amplifier gain vs. pump voltage.

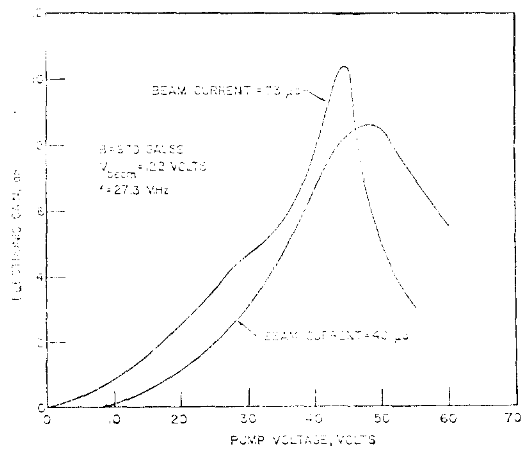


Fig. 6. S-band amplifier gain vs. pump voltage for a cyclotron-synchronous wave interaction.

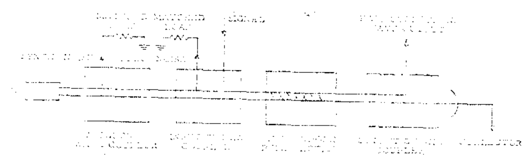


Fig. 7. A proposed low-noise scheme.

wave coupler, while the beam noise of the cyclotron wave is removed by the input coupler. Secondly, a broadband, high frequency operation is anticipated this comes from the fact that the structure is frequency independent, and wide-band cyclotron wave couple is available[4] Finally, the interaction presents a possibility of high power operation. This can be explained as follows.

From the computer solution of the energy variations in both the quadrifilar helix and the transition region, it is clear that the rotational energy gained by the beam is equal to the additional amount of energy supplied by the collector for keeping the axial energy constant during the entire process. It is therefore the collector $d-c$ source which supplies the rotational energy of the beam. In view of this, together with the fact that the $r-f$ energy on the beam is proportional to the square of the radius and the $d-c$ beam current, the proposed device will have a high power handling capacity if a high $d-c$ pump field and consequently a high collector $d-c$ supply and a high beam current are used.

This mode of interaction can be obtained by adjusting the beam voltage of a conventional $d-c$ pumped quadrapole amplifier. Several investigators[5,6] have reported unexplained anomalous gain phenomena under certain operating conditions, which can now be identified from the above analysis as the proposed mode of interaction.

References

1. Kang, Chang E. and Ho, B., "Instability characteristics of Solid State Carrier Waves in Microwave Circuits, I. Academy of Science, Transaction on Applied Science and Technology, Vol. 69, No. 4, pp. 369-384, September 1977.
2. Kang, Chang E. "Transmission Line Analogue of Transferred Electron Amplifiers and Oscillators", The annual symposium of ISAS and IMAS at University of Missouri, April 30, 1977.
3. Solymer L. and Ash E., "Some Traveling-Wave Interaction in Semiconductors", Int. J. of Electronics, Vol. 20, No. 2, pp. 127-148, February 1966.
4. Hayes, R.E., "The Theory of Twisted Transverse-Wave Couplers," IEEE Trans. on ElectronDevices, Vol. ED-11, No. 9, pp. 434-440, September 1964.
5. Saito, S., et al., "D-C Pumped Cyclatron-Beam Tebus Using Quadrifilar Helix," Proc. of International Congress on Microwave Tubes, Bergen, Norway, pp. 244-248, 1962.
6. Mao, S. and Siegman, A.E., "Cyclotron Wave Amplifier Using Simultaneous R.F. …Coupling and D-C Pumping, Proc. of International Congress on Microwave Tubes, Bergen, Norway, pp. 268-276, 1962.

APPENDIX A. GAIN EXPRESSION

The second-order, coupled-mode equations for this coupling are written as

$$\frac{d^2A_1}{dz^2}+j\beta_c\frac{dA_1}{dz}=jMA_4, \qquad (A.\ 1)$$

$$\frac{d^2A_2}{dz^2}-j\beta_c\frac{dA_2}{dz}=-jMA_3, \qquad (A.\ 2)$$

$$\frac{d^2A_3}{dz^2}-j\beta_c\frac{dA_3}{dz}=jMA_2 \qquad (A.\ 3)$$

and

$$\frac{d^2A_4}{dz^2}+j\beta_c\frac{dA_4}{dz}=-jMA_1 \qquad (A.\ 4)$$

The propagation constants are found to be

$$\Gamma_1=-j\frac{\beta_c}{2}+\sqrt{M-\frac{\beta_c^2}{4}}$$

$$\Gamma_2=-j\frac{\beta_c}{2}-\sqrt{M-\frac{\beta_c^2}{4}}$$

$$\Gamma_3=-j\frac{\beta_c}{2}+j\sqrt{M+\frac{\beta_c^2}{4}}$$

$$\Gamma_4=-j\frac{\beta_c}{2}-j\sqrt{M+\frac{\beta_c^2}{4}} \qquad (A.\ 5)$$

Since Γ_3 and Γ_4 are purely imaginary, they can be neglected in calculating the gain expression. The coupled-mode amplitudes for the fast cyclotron wave and the positive synchronous wave can be written as

$$A_1=B_{11}e^{\Gamma z}+B_{12}e^{-\Gamma z} \qquad (A.\ 6)$$

and

$$A_4=B_{41}e^{\Gamma z}+B_{42}e^{-\Gamma z} \qquad (A.\ 7)$$

Now substituting Eqs. A. 6 and A. 7 into Eq. A. 1 yields

$$\Gamma^2B_{11}e^{\Gamma z}+\Gamma^2B_{12}e^{-\Gamma z}+j\beta(\Gamma B_{11}e^{\Gamma z}-\Gamma\beta_{12}e^{-\Gamma z})$$
$$=jM(B_{41}e^{\Gamma z}+B_{42}e^{-\Gamma z}). \qquad (A.\ 8)$$

When terms with the same exponential factor are equated,

$$B_{41}=\frac{1}{jM}(\Gamma^2+j\beta_c\Gamma)B_{11}$$

$$B_{42}=\frac{1}{jM}(\Gamma^2-j\beta_c\Gamma)B_{12} \qquad (A.\ 9)$$

Applying initial conditions gives

$$A_1(0)=B_{11}+B_{12}$$

$$A_4(0)=\frac{1}{jM}(\Gamma^2+j\beta_c\Gamma)B_{11}+\frac{1}{jM}(\Gamma^2-j\beta_c\Gamma)B_{12} \qquad (A.\ 10)$$

The solutions for B_{11} and B_{12} are

$$B_{11}=\frac{1}{2}A_1(0)+j\frac{\Gamma}{2\beta_c}A_1(0)+\frac{M}{2\beta_c\Gamma}A_4(0) \qquad (A.\ 11)$$

and

$$B_{12}=-\frac{M}{2\beta_c\Gamma}A_4(0)+\frac{1}{2}A_1(0)-j\frac{\Gamma}{2\beta_c}A1(0) \qquad (A.\ 12)$$

The fast cyclotron wave coupled-mode amplitude becomes

$$A_1(z)=\left(\frac{1}{2}A_1(0)+\frac{M}{2\beta_c\Gamma}A_4(0)\right)e\Gamma^z+j\frac{2\beta_c}{\Gamma}A_1(0)e\Gamma^z$$

$$+\left(\frac{1}{2}A_1(0)-\frac{M}{2\beta_c\Gamma}A_4(0)\right)e^-\Gamma^z-j\frac{2\beta_c}{\Gamma}A_1(0)e^-\Gamma^z \quad \text{(A. 13)}$$

and the gain for the fast cyclotron wave is then

$$\text{Gain}=\frac{R_eA_1(z)}{A_1(0)}$$

$$=\frac{\left(\frac{1}{2}A_1(0)+\frac{M}{2\beta_c\Gamma}A_4(0)\right)e\Gamma^z+\left(\frac{1}{2}A_1(0)-\frac{M}{2\beta_c\Gamma}\right.}{A_1(0)}$$

$$\frac{\left.A_4(0)\right)e^-\Gamma^z}{}=\cosh\Gamma z+\frac{MA_4(0)}{\beta_c\Gamma A_1(0)}\sinh\Gamma z \quad \text{(A. 14)}$$

For zero synchronous wave input, $[A_4(0)=0]$,

$$\text{Gain}=\cosh\Gamma z$$

$$=\cosh\sqrt{M-\frac{\beta_c^2}{4}}z \quad \text{(A. 15)}$$

By taking log Eq. A. 14, the gain is obtained in decibel as shown in Eq. 21.