原子爐의 最適 運轉停止 制御方法의 數値解

論 文
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Optimal Shutdown Control of Nuclear Reactor: A Numerical Solution

姜 英 圭* 卞 增 男**
(Young Kyu Kang, Zeun Nam Bien)

Abstract

The problem of optimal shutdown control of nuclear reactor having nonlinear dynamics is considered. Since the problem, being a bounded state space problem, is difficult to solve by conventional analytic methods such as Pontryagin's maximum principle, it is approached directly by the quasilinearization technique, and solved numerically. The solution obtained in this manner proves to be an improvement over the previous results.

I. INTRODUCTION

In a typical power nuclear reactor, it is required to maintain the neutron population at some constant level for its steady state operation. During the operation, however, fission fragments such as Xenon-135 and Samarium-149 and their decay products are produced, which absorb neutrons, and adversely affect the equilibrium of neutron concentration in the reactor. These fission products are often called "poison". If the reactor is abruptly shut down, an extremely large amount of poison accumulates in the reactor, and makes it impossible to restart the reactor. This inability of restarting after shutdown, which may last 2 days or more for a typical reactor, is not tolerable in many situations, and it is desirable to make by some means as short as possible the duration during which the reactor cannot restart. For this, it has been suggested in [3] [4] [5] and [6] that the reactor should be shut down, not abruptly, but, according to a given shutdown program so that the poison build-up thereafter be effectively controlled.

The process of poison build-up and its control can be approximately modelled by a second-order nonlinear differential equation [1] [2]. For the problem of controlling the process while minimizing the maximum poison concentration after shutdown, Rosztoczry and Weaver in [5] applied Pontryagin's maximum principle and reduced the optimal control problem to a nonlinear two point boundary value problem. But the difficulties involved for an analytic solution were so severe that the authors suggested a subotimal controllers of bang-bang type such as one-pulse or two-pulse controllers. In [3], Ash employed the method of dynamic programming to obtain a digital computer algorithm as an optimal shutdown program. This result, however, was obtained under the assumption that the controller can take only zero or its maximum value. As a consequence, when the state is on the boundary of a Xenon override-

* 正會員：原子力硏究所연구원
** 正會員：韓國科學院 電氣・電子工學科 敎授・工博
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constraint, the controller needs to be of a multi-pulse type and does not allow the state to stay on the boundary.

In this paper, the quasilinearization technique is employed in reformulating the problem as a mathematical programming problem, and then the problem is solved using gradient projection method. The result obtained in this manner shows that the optimal control policy is different from that of [3], not of bang-bang type and the cost is less.

II. Problem Statement

Since, among others, Xe-135 has a relatively large cross-section for thermal neutron absorption, the effect of Xe-135 only is usually accounted as a poison in the mathematical model.

Let [O,T] be a given time interval for the shutdown operation. The process of poison build up during the period of [O,T] is given by the balance equation ([1], [3], [4] [5])

$$\frac{dX(t)}{dt}=-\lambda_X X(t)-\sigma_X\phi(t)X(t)+\lambda_I I(t)+\gamma_X\Sigma_f\phi(t)$$

$$\frac{dI(t)}{dt}=-\lambda_I I(t)+\gamma_I\Sigma_f\phi(t) \qquad (1)$$

where $\phi(t)$ is the thermal neutron flux, Σ_f is the macroscopic fission crosssection and, $X(t)$ and $Y(t)$ are xenon and iodine concentration at t. Others are parameters of nuclear reactor.

For simplicity, the model Eq.(1) may be given in a normalized dimensionless form as in the following;

$$\frac{dx(t)}{\cdot}=-(w+r_0u(t))x(t)+g_0y(t)+g_2u(t),$$

$$\frac{dy(t)}{dt}=u(t)-y(t) \qquad (1)$$

where x and y indicate X/X_0 and I/I_0 with X_0 and I_0 denoting the equilibrium concentrations of Xenon and Iodine, u denotes the normalized thermal neutron flux, and w, r_0, g_0 and g_0 are nonnegative constant parameters. It is well established in [3] and [5] that the variables involved in the proess must be constrained due to physical restrictions. Specifically, it is required that $0\leq u(t)\leq 1$, $0\leq y(t)\leq 1$ and $0\leq x(t)\leq x_c$, $0\leq t\leq T$, where $x_c\geq 1$ is a constant determined by the amount of positive reactivity for partial xenon override. The objective of control is to make as short as possible the period during which the reactor cannot restart, which justifies as the cost functional the following expression:

$$C(u(\cdot))=G\ y(T)(1+F\frac{x(T)}{y(T)})^{r} \qquad (2)$$

where G,F and γ are positive constants expressed in terms of w, r_0, g, and g_2. The expression $C(u)$ in Eq. (2) actually denotes the peak value of xenon concentration after the complete shutdown, i. e., $C(u(\cdot))=\max x(t)t\geq T$ where $x(t),t\geq T$, is the solution of (1) with $u(t)=0$.

Therefore the optimal control problem is to find a measurable control $u(t)$, $0\leq t\leq T$ with $0\leq u(t)\leq 1$ such that its response of (1) at the initial state $x(0)=1$, $y(0)=1$ satisfies the constraints $0\leq x(t)\leq x_c$ and $0\leq y(t)\leq 1$ while minimizing the cost functional $C(u(\cdot))$.

III. Numerical Solution

Let N be a given positive integer and let $\Delta t=\frac{T}{N}$. Then the equation (1) can be rewritten in the discrete-time form as:

$$x_{i+1}=x_i+\Delta t[-(w+r_0u_i)x_i+g_0y_i+g_2u_i]$$

$$y_{i+1}=y_i+\Delta t[u_i-y_i],\quad i=0,1,\ldots N-1, \qquad (3)$$

where x_i and y_i denote the normalized xenon and iodine concentrations at the i-th time, respectively. Applying the quasilinearization method to Eq.(3), one obtains

$$x_{i+1}^{j+1}=\left[1-\Delta t\left(w+r_0u_i^j\right)\right]x_i^{j+1}+\Delta tg_0y_i^{j+1}+\Delta t\left(g_2-r_0x_i^j\right)u_i^{j+1}-\Delta tr_0x_i^ju_i^j$$

$$y_{i+1}^{j+1}=(1-\Delta t)y_i^{j+1}+\Delta tu_i^{j+1} \qquad (4)$$

where x_i^j (or y_i^j) is the xenon (or iodine) concentration at the i-th time and at j-th iteration.

This system is solved using the specified initial condition $x(0)=y(0)=1$ so that x_i^{j+1} and y_i^{j+1} are given explicitly as some linear functions of u_k^{j+1}, $k=0,1,\ldots,i_{-1}$, i.e.,

$$x_i^{j+1}=\alpha_i^{j+1}\left(u_0^{j+1},\ u_1^{j+1},\ \ldots,\ u_{i-1}^{j+1}\right)$$

$$y_i^{j+1}=\beta_i^{j+1}\left(u_0^{j+1},\ u_1^{j+1},\ \ldots,\ u_{i-1}^{j+1}\right),\quad i=1,2\ldots,N.$$

Then the following mathematical programming problem is solved at each iteration to give the new optimal control u_i^{j+1}:

$$\text{Minimize} \quad G\beta_N^{j+1}\left(u_0^{j+1}, \ldots, u_{N-1}^{j+1}\right)$$

$$\times\left[1+F\frac{\alpha_N^{j+1}\left(u_0^{j+1}, \ldots, u_{N-1}^{j+1}\right)}{\beta_N^{j+1}\left(u_0^{j+1}, \ldots, u_{N-1}^{j+1}\right)}\right]^r$$

subject to

(i) $0 \le u_i^j \le 1, \quad i=1,\ldots,N$

(ii) $0 \le \alpha_i^{j+1}\left(u_0^{j+1}, \ldots, u_{i-1}^{j+1}\right) \le x_c, \quad i=1,\ldots,N$

(iii) $0 \le \beta_i^{j+1}\left(u_0^{j+1}, \ldots, u_{i-1}^{j+1}\right) \le 1, \quad i=1,\ldots,N$

Optimum shutdown programs were computed for the reactor having the following cases of constants.

Case 1. $\phi_0=2x10^{14}$ neutrons/cm²sec (Equilibrium thermal neutron flux)

$r_x=0.003, r_I=0.056$

$\lambda_I=2.9x10^{-5}sec^{-1}$ $\lambda_x=2.1x10^{-5}sec^{-1}$

$\sigma_x=3.5x10^{-18}cm^2$

Case 2. $\phi_0=1x10^{14}$neutrons/cm²sec

$r_x=0.002, \ r_I=0.061$

$\lambda_I=2.9x10^{-5}sec^{-1}$ $\lambda_x=2.1x10^{-5}sec^{-1}$

$\sigma_x=3.21x10^{-18}cm^2$

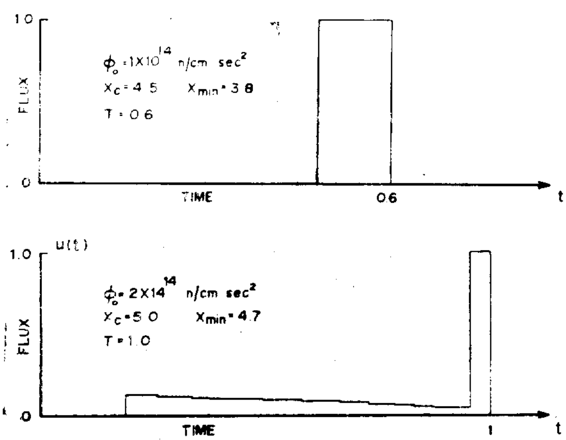

Fig. 1. Optimal Shutdown Programs

Fig. 1 shows the computed optimal shutdown program for the above two eases.

Fig.2 indicates corresponding trajectories on the Xe-I state space. Fig.3 shows the convergence of the cost. In the figures, x_{min} denotesthe minimized peak value of x while x is not on the boundary x_e.

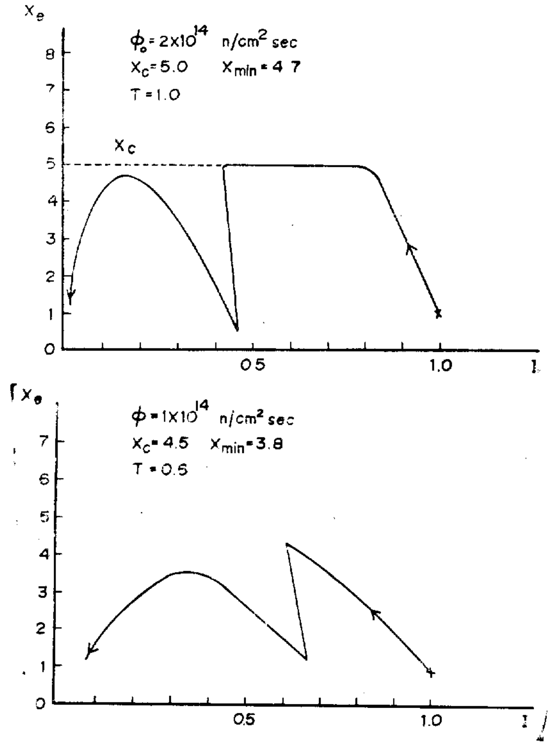

Fig. 2 Optimal trajectories on the Xe-I state space.

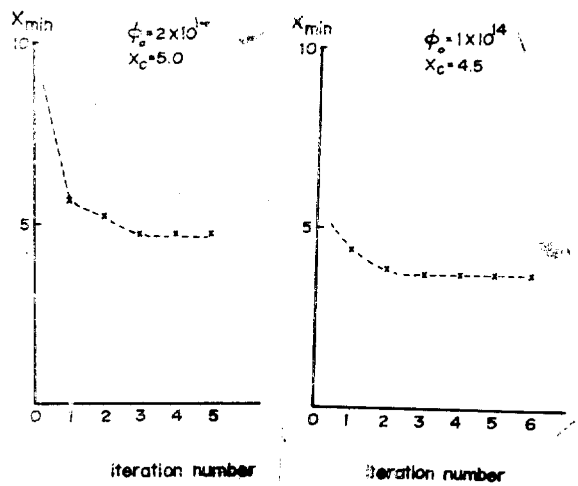

Fig. 3. Convergence of the cost.

Observe that the numerical solutions obtained are either single pulse trajectories or trajectories that follow the Xenon boundary. A single pulse control occurs when the Xenon trajectory meets the override constraint x_c at the end of the allowed control time $T$and the strongly limited Xenon override.

Fig. 4 shows the reduction in the Xenon peak as a function of the control time parameter , while Xenon override constraint x_c remains fixed. Fig.5 shows the reduction in the Xenon peak as a function of the Xenon override constraint x_c, while the allowed control time T remains fixed.

Finally, the results obtained here are compared with the pulse type shutdown program previously obtained in [4]. The comparison for the normalized values is given in the Table 1 and 2 below for the parameters of Case 1. In the figures, x_p denote the normalized peak value of x after the abrupt shutdown.

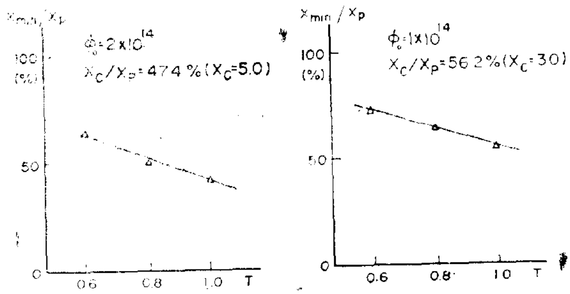

Fig. 4. Reduction of the peak value of Xenon with x_c fixed.

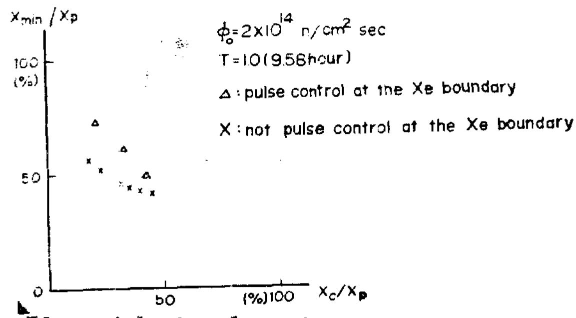

Fig. 5. Reduction of the Xenon peak value with T fixed.

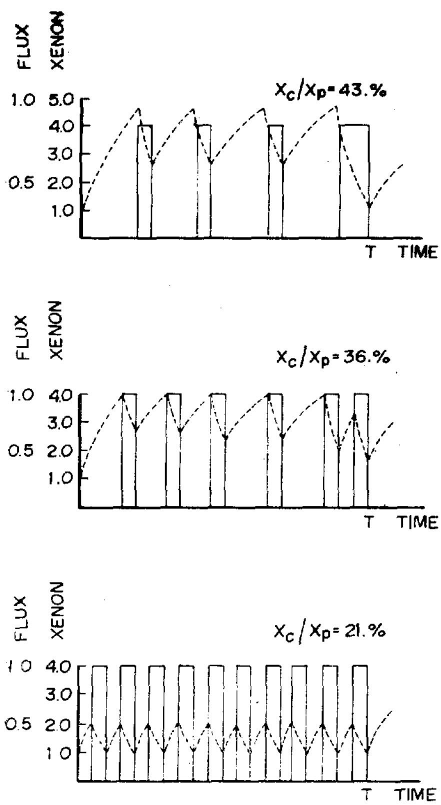

Fig. 6. Shutdown programs (pulse type) and corresponding Xenon concentration

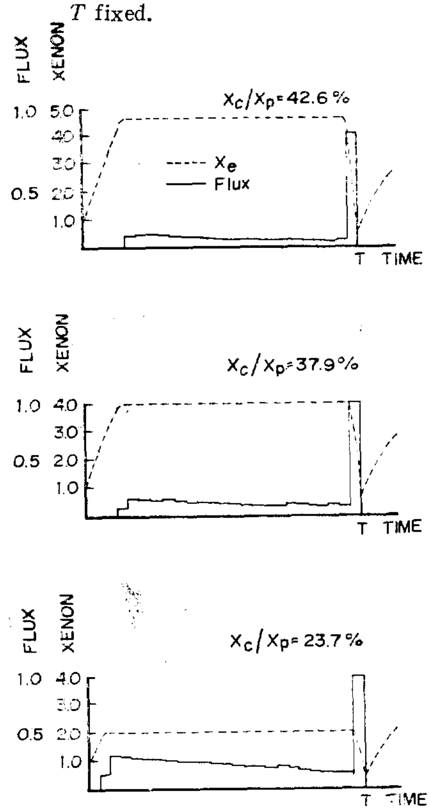

Fig. 7. Shutdown programs (non pulse type) and corresponding Xenon concentration

Table 1 : Pulse Control

x_c/x_p(%)	x_{min}/x_p(%)
43.	50.
36.	61.
21.	71.

Table 2 : Optimal control on the Xe boundary

x_c/x_p(%)	x_{min}/x_p(%)
47.4	44.55
46.6	45.88
37.9	47.0
33.2	48.5
23.7	54.2
18.96	59.8

(Same parameters as Table 1)

These tables 1 and 2 are illustrated inFig.5. **Fig.6** and Fig.7 show the difference between the pulse type control (shutdown program) and our optimal control for the strongly limited Xenon override.

IV. Concluding Remarks

The problem of optimally controlling Xenon poison in the reactor has been presented. The computational results show that the optimal shutdown programs need not be bang-bang type for the strongly limited Xenon override constraints, and the fractional reduction in the Xenon poisoning is greater than that of the pulse type in [3] This is a definite improvement. Also, we can observe that the amount of reduction in Xenon concentration depends on the given control duration and the Xenon overide constraint. It is remarked that other problems involving nuclear plant reactors are discussed in a recent survey article in [12].

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