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Contact Stress Analysis and Contact Surface Design : An Application to Spot Welder Electrode Design†

by

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탄성체들의 접촉 응력 해석 및 접촉면 형상의 설계 : Spot welder 용접봉에의 응용

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초 록

스폿 용접 기구를 탄성체로 생각하여 접촉 응력과 용접봉의 최적 형상을 구하였다.

접촉 응력은 2차 계획법 문제로 바꾸어 해석하였고, 이에 필요한 용접봉과 피용접판의 Flexibility matrix 들은 유한 요소법의 프로그램인 SAP IV를 사용하였다. 최대 접촉 응력을 최소로 하는 용접봉의 형상 설계를 위해 축차적인 선형 계획법을 적용하고 특정한 경우의 해를 도시하였다. 최적의 용접봉 모양은 중심부가 오목하고 가장자리 부분이 볼록한 형태로 나타났다.

Introduction

Contact problems are often very important in mechanical systems, because of the unusually high stresses in the contacting region, Contact problems are non-classical and nonlinear. Difficulty lies in that one does not initially know the contact rigion or the contact stress.

Contact problems between two elastic bodies were first treated in the classical paper by Hertz(1). The analysis technique was limited to very simple geometry of contact such as sphere, cylinder and plane. Recently considerable research has been pursued to develop constructive methods(2). In this method, the contact problem is transformed to an equivalent problem of quadratic programming.

In this paper, a resistance spot welding mechanism is treated as a multibody contact problem. It is desired to minimize the peak contact stress at the electrode tip, thus reducing any abnormally high current density which may accelerate the wear of the tip. Therefore, the problem posed here is to find an optimal shape of the electrode tip for a desirable contact stress distribution.

Although the friction force in contacting surface can be important in many cases, it is negligibly small for a lubricated or a nearly flat contacting surface. Therefore, this effect is not considered in this study. Similarly, thermal effect will not be considered. This assumption will make the mathematical formulation much easier.

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Analytical Formulation of the Contact Problem as a Quadratic Programming

Analytical formulations of two-body and multibody elastic contact problems are well described in the references(2). Here, only a brief summary of the method will be presented.

Consider that two or many elastic bodies are in contact with each other under applied loads. The approach is an approximation using discretization. The contacting areas in a given problem are first estimated and a region is defined so that it can at least cover these areas. This will be called a potential contact region. This potential contact area is then divided into a set of nodal points and the nodal contact forces are used as an approximation to the contact stress distribution. By choosing a minimal potential contact region, the size of the problem can be minimal as can be seen later in the development. From a geometrical consideration, the final gap vector ε for the potential contact nodes is given by

$$\varepsilon = Aq + Bs + a, \qquad (1)$$

where A is an affine transformation matrix such that Aq denotes the displacements due to the rigid body degrees of freedom q; B is a matrix formed from the flexibility coefficients of bodies such that Bs gives the elastic displacements due to the contact force distribution s only; and a combines both the displacements due to external loads and any initial gap in the potential contact region.

Equilibrium equation is obtained through direct application of the principle of virtual work, which results in the linear equation,

$$A^T s = c \qquad (2)$$

where the vector c depends on the externally applied load.

Geometric compatibility between the contacting bodies requires that the product of each gap variable and contact force be zero. Analytically, this is,

$$\varepsilon_i s_i = 0, \quad i = 1, 2, \cdots, n, \qquad (3)$$

where n is the number of potential contact points. This condition may be interpreted as stating that either the gap or the contact force must be zero at each point on the contacting bodies. Further, the gap and contact force must be nonnegative. That is,

$$s_i \geq 0, \quad \varepsilon_i \geq 0, \quad i = 1, 2, \cdots, n. \qquad (4)$$

It is recognized that Eqs. (1) to(4) are same as the Kuhn-Tucker necessary conditions for a convex quadratic programming problem (3) given by,

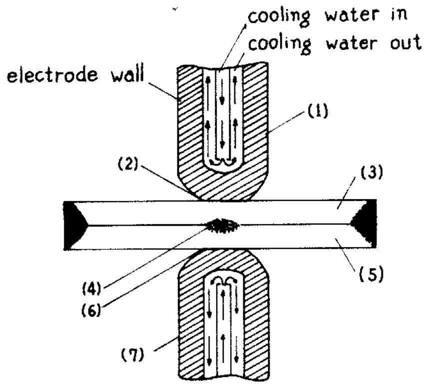
$$\text{Minimize} \quad a^T s + \frac{1}{2} s^T B s$$
$$\text{subject to} \quad A^T s = c \qquad (5)$$
$$s \geq 0.$$

The cost function here represents the total potential energy. Hence it is positive definite for a well formulated problem. Any quadratic programming code can be utilized to solve the elastic contact problem as described here.

Spot Welding Mechanism as a Multibody Elastic Contact Problem

Spot welding is a basic type of solid resistance welding, and discussed in detail in texts(4). The fundamental variables of spot welding is squeeze time, weld time, hold time, and off time. The squeeze time brings the two workpieces together in intimate contact prior to the current flow. The weld time is the time at the interface of the two metal pieces. The hold time is basically a cooling period and it is the interval from the end of the flow until the electrodes are apart before the cycle repeats for the next weld.

A schematic diagram of spot welding is shown in Fig. 1. The passage way for cooling water is necessory when high currents and high duty cycles are required. The amount of overlapping of base metals and spot spacing are important factors in spot welding. They are determined by the thickness of the plates.



(1) Upper electrode
(2) Point of contact between upper electrode and base metal
(3) base metal
(4) Point of contact of base metals in area of nugget
(5) base metal
(6) Point of contact between lower electrode and base metal
(7) lower electrode

Fig. 1. Resistance spot welding

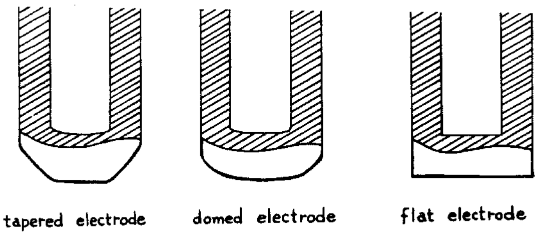


Fig. 2. Types of electrode

Several types of electrode are available; tapered electrodes, domed electrodes, and flat electrodes. They are shown in Fig. 2. Tapered tips are used, since they wear uniformly. Domed tips are useful by their ability to withstand heavy pressure and severe heating without mushrooming. A flat tip is used when a minimum weld indentation is desired. It is in general difficult to maintain the original contour of electordes, due to the high normal contact stress, near edge of tip when electrode force is applied during welding spueeze time. Thus, it is desirable to reduce the peak contact stress to reduce the rate of wear.

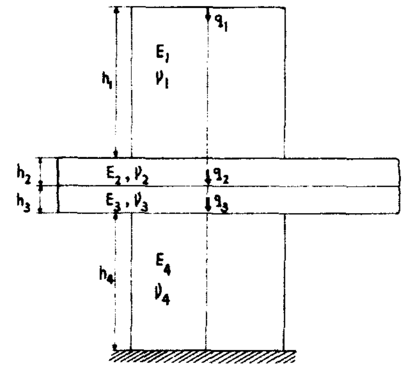
As shown in Fig. 1. the spot welding mechanism consists of four main elastic bodies: the top and bottom electrodes and two base metal plates to be joined. Usually electrodes are made of copper and base metal plates are commonly mild steel or stainless steel or aluminum alloys. The data used in this model formulation is based on(4). It is assumed that both electrodes have same size and properties and two base metals also have same size and properties. It is also assumed that the electrodes have circular cross section and the welding spot spacing is relatively large so that the base metal plates can be treated as infinite plates. Under this circumstance, the problem becomes a rotationally symmetric problem. The cooling water passage, which is an empty space in electrodes, is ignored in calculating flexibility of electrode. The global shape of electrode tip is considered flat in modeling and calculating contact stresses.

A schematic sketch of the model is shown in Fig. 3. The following data are used.

$E_1 (=E_4) = 15 \times 10^6$ psi
$E_2 (=E_3) = 30 \times 10^5$ psi
$\nu_1 (=\nu_4) = 0.355$
$\nu_2 (=\nu_3) = 0.3$
$h_1 = 1$ inch
$h_2 (=h_3) = 0.1$ inch

radius of the electorde : 0.28 inch
radius of the plate : 0.6 inch
externaly applied load : 1500 pounds

where E_1, E_2, E_3 and E_4 denote Young's Modulus, ν_1, ν_2, ν_3 and ν_4 denote Poisson's ratio, and h_1, h_2, h_3 denote heights of each body as described in Fig. 3.

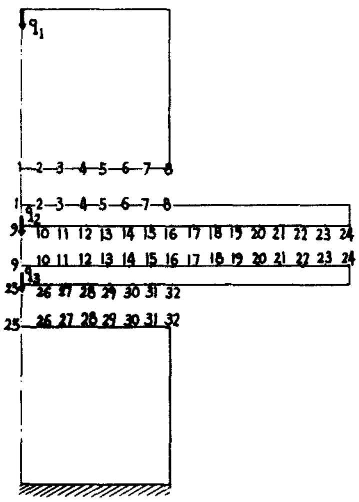


q_1, q_2, q_3 ; generlized coordinates
h_1, h_4 ; heright of the electrode
h_2, h_3 ; thickness of the plate
E_1, E_2, E_3, E_4 ; Young's Modulus of body 1, 2, 3, 4,
$\nu_1, \nu_2, \nu_3, \nu_4$; Poisson's ratio of body 1, 2, 3, 4
Body 4 is assumed to be fixed

Fig. 3. Sketch of the model

Contact Stress Analysis

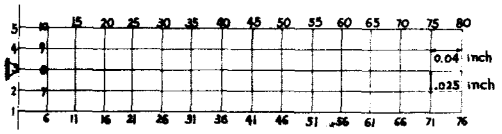
Eight potential contact points are chosen along the radius of the electrode and sixteen potential contact points are chosen along the radius of the base metal plates with a uniform interval size of 0.04 inches. The numbering of potential contact points is shown in Fig.4.



q_1 is located at the center of electrode top surface
q_2, q_3, are located at the center of the plates

Fig. 4. Potential contact points

Flexbility coefficients for the electrode and the base metal plate are obtained using SAP IV program(5). Both the electordes and plates are axi-symmetrical, so only one plane along the axis is considered. Quadrilateral finite elements are chosen for both electrode and plate. The finite element model for the plate is illustrated in Fig. 5.



height = 0.1 inch
radius = 0.6 inch
rigid body displacement is the displacement of point 3

Fig. 5. Nodal numbering for SAP IV of the base metal plate

Flexibility coefficients for the potential contact points are obtained by solving the displacements for a unit load applied at each node, one by one. Classical elasticity analysis for the case with a unit load at node 1 is a singular problem, leading to an infinite displacement there, although a finite displacement is expected in reality. Finite element analysis leads to a finite displacement, which is, however, much dependent on the finite element shape around node 1, and hence the solution at node 1 is more or less meaningless. A better solution is to use an equivalent load distributed uniformly over a disc around node 1. Solutions for both of these methods (with a concentrated load and with an equivalent distributed load over a disc of 0.01 inch radius) are obtained and compared.

It is noted that the present problem has another symmetry about the center contacting surface. In this study this feature is not used so that the program can be used for similar problems without this symmetry. Due to the symmetry about the axis, only vertical displacement is sufficient to represent the rigid body displacement.

Rigid body displacement of the top electrode is defined by q_2 which is the downward displacement of the center of top surface. Similarly, rigid body displacements of base metal plates are defined by q_2 and q_3 as shown in Fig. 4. The bottom electrode is assumed fixed. Since q_1, q_2 and q_3 are taken in downward direction, matrix A in Eq.(2) is obtained as,

$$A^T = \begin{bmatrix} -1 & -1 & \cdots & -1 & \vdots & 0 & 0 & \cdots & 0 & \vdots & 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 & \vdots & -1 & -1 & \cdots & -1 & \vdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \vdots & 1 & 1 & \cdots & 1 & \vdots & -1 & -1 & \cdots & -1 \end{bmatrix}$$

such that Aq gives the displacements of the potential contact points due to the rigid body displacements q_1, q_2 and q_3:

Flexibility matrix B in Eq(1) is composed of

$$\begin{bmatrix} F_1^{21} + F_2^{12} & F_3^{21} & O \\ F_1^{23} & F_2^{32} + F_3^{23} & F_4^{32} \\ O & F_2^{34} & F_3^{43} + F_4^{34} \end{bmatrix}$$

where F_k^{ij} is a matrix of influence coefficients providing displacements of points on surface Γ^{ij} due to load s^{ik} acting on surface Γ^{ik}. Here Γ^{ij} denotes the contact surface of body i with body j. It is noted (2) that $F_k^{ij} = F_j^{ik}$

The vector a in equation(1) is zero, because initial gap is zero and elastic deformation due to external load is zero. The vector c in equilibrium equation(2) is calculated by equilibrium of body 1,2and 3. So, it is $[-t,0,0]^T$, where t denotes the externally applied force.

All informations necessary for the contact problem (5) are obtained. Any quadratic programming code can be used to solve equation(5). In this study, a program developed in(6) is used. The contact stresses are calculated by dividing contact force by the effective area over which the forces are distributed, that is;

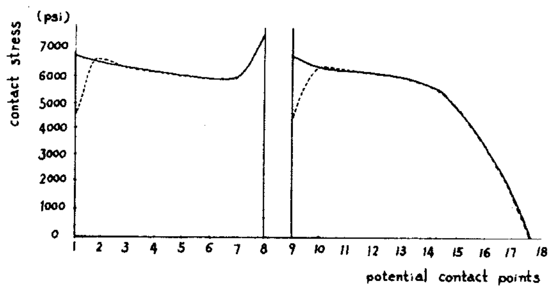
$$\sigma_i = s_i/W_i, \quad i = 1, \cdots, 32, \tag{6}$$

where σ_i denotes contact stress and s_i denotes contact force and W_i is the effective area of i-th potential contact point. The result shows that the whole area of electrode surface is in contact and nine potential contact points of plates are in contact. The numerical result is presented in Table 1 and the stress distribution is shown in Fig. 6. In this figure, dotted line

Table 1. Contact Forces and Stresses

points	contact force (lbs)	contact stress (psi)	points	contact force (lbs)	contact stress (psi)
1	. 84505 E01	6724.69	17	. 86184 E02	1071.61
2	. 61190 E02	6086.68	18	0 .	0.
3	. 12317 E03	6125.97	19	0 .	0.
4	. 18003 E03	5969.30	20	0 .	0.
5	. 23573 E03	5862.12	21	0 .	0.
6	. 28994 E03	5768.17	22	0 .	0.
7	. 34624 E03	5740.19	23	0 .	0.
8	. 25525 E03	7254.34	24	0 .	0.
9	. 80328 E01	6392.30	25	. 84505 E01	6724.69
10	. 63050 E02	6271.70	26	. 61190 E02	6086.68
11	. 12747 E03	6339.84	27	. 12317 E03	6125.97
12	. 18847 E03	6249.15	28	. 18003 E03	5969.30
13	. 24702 E03	6142.88	29	. 23573 E03	5862.12
14	. 28703 E03	5710.28	30	. 28994 E03	5768.17
15	. 28169 E03	4670.04	31	. 34624 E03	5740.19
16	. 21106 E03	2999.22	32	. 25525 E03	7254.34

denotes the stress distribution of plate and electrode when flexibility coefficients of each body are calculated without modification of distributing the center load. Both of the results with or without this modification are shown in Fig. 6. The solution with the modification seems more reasonable, and the optimal contour will be given for this case.

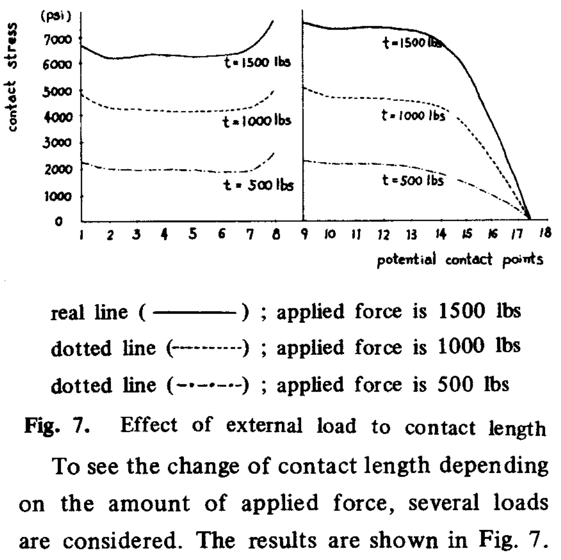


(a) stress of electrode (b) stress of plate

real line (————) ; with modification distributing center load

dotted line (---------) ; without modification

Fig. 6. Graph of stress distribution for t=1500 lbs



real line (————) ; applied force is 1500 lbs

dotted line (---------) ; applied force is 1000 lbs

dotted line (-·-·--) ; applied force is 500 lbs

Fig. 7. Effect of external load to contact length

To see the change of contact length depending on the amount of applied force, several loads are considered. The results are shown in Fig. 7. They indicate that contact length is independent of the amount of applied force. It means that although the electrode force is increased, the contact length between the base metals remains same. Since the contact resistance is inversely related to the contact pressure(7), the electrode force can be determined in such a way that a necessary electric current density is obtained.

Optimal Design of Electrode Tip

When two bodies or multibodies come into contact, the contact stress distribution is generally irregular.

Excessive stress may occur some parts of the contact region. This is undesirable, since high normal forces may accelerate wear of machine parts. In spot welding the peak contact stress occurs at the edge of the electrode as shown in previous section. This is a place where cooling is not effective and the local electric current density can be excessively high, causing fusion. Thus one way preventing wear of the electrode is to minimize the peak contact stress.

The basic idea of contour design problem is that a small change in gap variable will change the contact stress distribution. Thus, the contour design problem is same as a contact problem formulated earlier except a small change in gap variable a in equation(1). The gap change vector b which is to be designed is assumed small so that it is not necessary to recalculate the flexibility matrices each time for the changed configuration of the bodies. A method is presented in(6) for adjusting the contact surface in order to minimize the peak contact stress. Here, the algorithm of the iterative contour design process is summarized.

Step 1. Estimate the design variable(normally $b_i=0$) where b_i is the vector of contour modification.

Step 2. Solve the contact problem for s and ε by quadratic programming.

Step 3. Define index sets $\hat{I}$ and $\tilde{I}$ for points of contact ($s_i \geq 0, \varepsilon_i=0$) and for points not in contact ($\varepsilon_j>0$, $s_j=0$) respectively.

Step 4. Construct $\hat{B}$, $\hat{A}$, $\hat{a}$ and $\hat{b}$ for points in $\hat{I}$ and $\tilde{A}$, $\tilde{B}$, $\tilde{a}$ and $\tilde{b}$ for points in $\tilde{I}$, where $\hat{B}$

is the matrix B with rows and columns corresponding to points not in contact removed, $\hat{A}$ is the matrix A with rows corresponding to points not in contact removed and the vectors $\hat{a}$ and $\hat{b}$ are the vectors a and b with rows corresponding to points not in contact removed. $\tilde{B}$ is the matrix B with columns corresponding to points not in contact and rows corresponding to points in contact removed, $\tilde{A}$ is the matrix A with rows corresponding to points in contact removed, and the vectors $\tilde{a}$ and $\tilde{b}$ are the vectors a and b with rows corresponding to points in contact removed.

Step 5. Solve the linear programming problem defined by

$$\min \quad b_{n+1}$$

$$\text{subject to} \quad s_i/W_i - b_{n+1} \leq 0, \quad i = 1, \cdots, n$$
$$\hat{B}\hat{s} + \hat{A}q + \hat{b} = 0$$
$$\hat{A}^T\hat{s} = c$$
$$-\tilde{B}\hat{s} - \tilde{A}q \leq \tilde{b}$$
$$b^\circ - b_i \leq 0, \quad i = 1, \cdots, n$$
$$b_i - b^1 \leq 0, \quad i = 1, \cdots, n$$
$$\hat{s} \geq 0,$$

where b_{n+1} is the upper bound (denoting peak contact stress) on contact stress. It is often necessary to put lower and upper bound on the amount of contour modification. b^0 and b^1 denotes these bounds. To use a linear programming code, it may be necessary that q, which can be negative, be replaced by positive and negative parts as

$$q = q^+ - q^-$$

where both q^+ and q^- are positive.

Step 6. Evaluate $\tilde{\varepsilon}$ from equation

$$\tilde{\varepsilon} = \tilde{B}\hat{s} + \tilde{A}q + \tilde{a} + \tilde{b} \geq 0$$

where $\tilde{\varepsilon}$ is the gap variable not in contact.

Step 7. Form $\hat{I}$ to include points for which $\tilde{\varepsilon}_j=0$ and deleting points for which $s_j=0$ and form $\tilde{I}$ to include points for which $s_j=0$ and deleting points for which $\tilde{\varepsilon}_j=0$

Sept 8. If $\hat{I}$ is unchanged, terminate; otherwise return to Step 4.

For the present problem, the region of contour modification is the tips of both electrodes. Lower and upper bounds on design are chosen as

$$b^0 = 0$$

Table 2. Contact Stress and Designed Gap

points	contact stress (psi)	design gap (inch)	points	contact stress (psi)	design gap (inch)
1	.60592 E04	.31924 E-04	17	.20162 E04	0.
2	.60592 E04	.16405 E-04	18	0.	0.
3	.60592 E04	.13448 E-04	19	0.	0.
4	.60592 E04	.47942 E-05	20	0.	0.
5	.60592 E04	.13552 E-19	21	0.	0.
6	.60592 E04	.21923 E-05	22	0.	0.
7	.60592 E04	.20716 E-04	23	0.	0.
8	.60592 E04	.94399 E-04	24	0.	0.
9	.61942 E04	0.	25	.60592 E04	.31924 E-04
10	.63266 E04	0.	26	.60592 E04	.16405 E-04
11	.65060 E04	0.	27	.60592 E04	.13448 E-04
12	.65378 E04	0.	28	.60592 E04	.47942 E-04
13	.65217 E04	0.	29	.60592 E04	.13552 E-04
14	.59608 E04	0.	30	.60592 E04	.21923 E-04
15	.44762 E04	0.	31	.60592 E04	.20716 E-04
16	.15149 E04	0.	32	.60592 E04	.94399 E-04

$b^1 = 0.01\ (1+(R_i/0.28)^2),\ i=1, \ldots, 8, 25, \ldots 32,$
where R_i is the distance from the axis to the i-th potential contact points.

Solutions obtained are shown in Table 2 and Fig. 8.

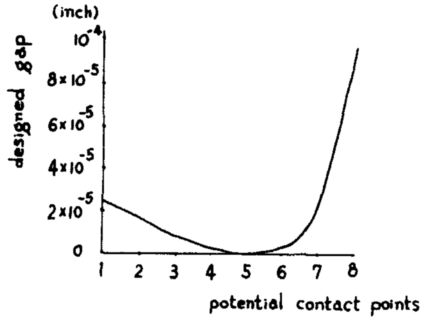


Fig. 8. Optimally designed contour of electrode to minimize peak contact stress

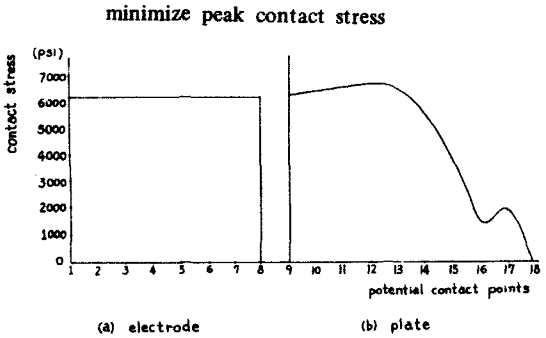


Fig. 9. Contact stress distributions of electrode and plate for optimally designed gap

It took one iteration for convergence. The contact stress distribution for this design is shown in Fig. 9. As shown in Fig. 8, the optimal shape of the electrode is not a shape of commonly used electrode in Fig. 2. It has a shape which is concave at the center and convex around the edge of the electrode. It is noted that the designed gap is rather small as shown in table 2. This means that a small change in the electrode tip can give a significant change in the quality of stress distribution. In Fig. 9, the contact stress of point 16 is smaller than that of point 17. It is conjectured that this is due to the particular configuation of the deformed plate under the uniform load.

Conclusions

In this paper, a resistence spot welding mechanism is modeled as a multibody elastic contact problem considering the welding stage of the welding process.

Contact stress is analyzed without considering friction effect and thermal effect. An optimal surface contour of electrode to minimize the peak contact stress is obtained.

The following conclusions are obtained.

(1) The peak contact stress in flat electrode appears at the edge of the electrode as expected.

(2) The contact area between the base metal plates does not depend on the amount of electrode force.

(3) The optimal shape of the electrode is found such that the shape of the edge is convex and the electrode center is concave.

(4) Contact problem with friction and thermal effect is a future topic to be studied. Although the welding stage of the welding process was studied here, for a complete understanding and a better design, the whole process with time must be considered, with appropriate experiments.

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