◀ 論 文 ▶

온돌 난방에서의 지면을 통한 열손실

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Analysis of Ground Heat Loss in Ondol Heating Systems

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요 약

온돌 난방계통에서 지면을 통한 열손실을 수학적으로 등각사상과 변수변환방법을 사용하여 해석하였다. 2차원 및 3차원 해석 결과를 얻었다. 난방되는 주택은 직사각형이라고 가정하였다.

그림 2를 사용하여 평균 지면 열손실을 구할 수 있다. 열 특성치가 열손실에 미치는 효과에 대하여 설명되었다. 열손실을 구하는 수치적 보기를 주었다.

Abstract

Heat loss to the ground in ondol heating system is analyzed mathematically using conformal mapping and variable transformations. Both two-and three-dimensional systems are analyzed. Heated house is assumed to be rectangular shape. Mean value of heat loss to the ground can be obtained using Figure 2. Effects of thermal parameters to heat loss are discussed. Numerical examples are also given.

I. Introduction

Ondol is a house heating method used in Korea for long time, and is still used in many houses. In ondol heating method, hot combustion gas, which flows through channels('gorae') below floor, heats the floor above it. The hot gas also loses heat to the ground below.

Engineering study on ondol is restricted in Korea and in Hokaido Japan [1-5]***. Korean government supported ondol study since 1960's.

It is important in ondol heating that hot gas transfers most of its energy to the floor, and that heat loss must be minimized. Heat loss to the ground amounts a large portion of its total heat loss.

The purpose of present study is to estimate heat loss to the ground in ondol heating system using mathematical analysis, and from the result to derive a method to decrease the heat loss.

II. Assumptions

Figure 1. shows schematic diagram of a house

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heated by ondol. For the present study following assumptions are used.

(1) Ground is a semi-infinite solid and its surface is flat.

(2) Thermal properties of soil are isotropic and constant.

(3) Temperature of hot gas in 'gorae' is constant and uniform.

(4) Atmospheric temperature is constant.

(5) Heat transfer coefficient between hot gas and the ground and that between atmosphere and the ground are constant and equal to each other.

(6) The house heated is a rectangular.

Above assumptions are necessary for the present study, however, they are rarely satisfied in reality.

Assumption (1) says that there are only one house on the ground. Composition of soil is not uniform, and thermal properties of soil depend on its porosity, water content, composition, etc. Uncertainty of thermal properties of soil is quite large. Assumption (2) is necessary for any theoretical study.

Hot gas loses its energy as it fows through 'gorae', so that temperature of the hot gas cannot be uniform. In ondol heating coal is used for fuel, and hot gas temperature varies [2]. Assumption (4) neglects daily and seasonal variation of atmospheric temperature.

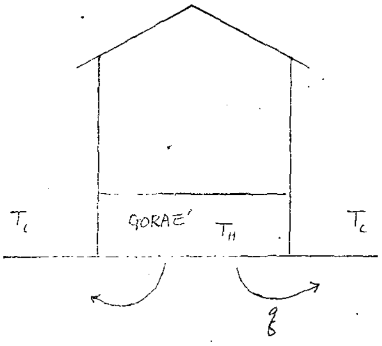

Fig. 1. Ondol Heating System

It is necessary to use mean temperature and mean values of thermal properties for heat loss calculation. Heat loss so obtained will give good mean value.

Experimental values of heat transfer coefficient between hot gas in 'gorae' and the ground have not been reported. The gas velocity in 'gorae' is small so that heat transfer coefficient can be approximated as that between atmospheric air and the ground.

Ⅱ. Two-Dimensional analysis

If one side of the house is very long relative to the other side, two-dimensional approximation can be used. Temperature distribution in the ground satisfies two-dimensional steady state heat conduction equation:

$$\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Z^2}=0 \qquad (1)$$

Boundary conditions at the surface are

$$\text{at } Z=0,\ k\frac{\partial T}{\partial Z}+h(T_H-T)=0 \quad \text{for } |X|<L \qquad (2)$$

$$\text{at } Z=0,\ k\frac{\partial T}{\partial Z}-h(T-T_C)=0 \quad \text{for } |X|>L \qquad (3)$$

Temperature must be finite for $|X|\to\infty$ or $Z\to\infty$. Symbols are given in Nomenclature.

Equation (2) describes that the ground exchanges heat with hot gas within the house. Equation (3) shows that the ground loses heat with atmosphere outside the house. Rate of heat loss to the ground per unit length of the house is obtained from.

$$q=2h\int_0^L (T_H-T)\Big|_{z=0}dX \qquad (4)$$

Let

$$u=\frac{T-T_C}{T_H-T_C}-\frac{k}{h(T_H-T_C)}\frac{\partial T}{\partial Z} \qquad (5)$$

$$x=\frac{X}{L} \tag{6}$$

$$z=\frac{Z}{L} \tag{7}$$

$$B=\frac{2hL}{k} \tag{8}$$

Then equations (1)—(3) are transformed to

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}=0 \tag{9}$$

at $z=0$, $u=1$, for $|x|<1$ (10)

at $z=0$, $u=0$, for $|x|>1$ (11)

Solution of equations (9)—(11) can be obtained by conformal mapping, and is given in standard text [6].

$$u=\frac{1}{\pi}\arctan\frac{2z}{x^2+z^2-1} \tag{12}$$

Here range of arc tangent function is defined as:

$0\leq\arctan x<\frac{\pi}{2}$ for $x>0$, and

$\frac{\pi}{2}<\arctan x<\pi$ for $x<0$. (13)

Substituting equation (12) into equation (5), and solving the first order differential equation, one obtains

$$\frac{T-T_C}{T_H-T_C}=\frac{B}{2\pi}e^{Bz/2}\int_z^{\infty}e^{-Bt/2}\arctan\frac{2t}{t^2+x^2-1}dx \tag{14}$$

Substituting equation (14) into equation (4), heat loss is obtained.

$$E=\frac{q}{2hL(T_H-T_C)}=1-\frac{B}{2\pi}\int_0^1\int_0^{\infty}e^{-Bt/2}\arctan\frac{2t}{t^2+x^2-1}dt\ dx \tag{15}$$

Performing the integration (Appendix A)

$$E=\frac{2}{\pi B}\Big[A+\ln B+\Big\{\frac{\pi}{2}-\mathrm{Si}(B)\Big\}\sin B-\mathrm{Ci}\ (B)\ \cos B\Big] \tag{16}$$

Here

$A=0.5772156649..$ (Euler constant)

$\mathrm{Si}(B)=\int_0^B\frac{\sin x}{x}dx$ (Sine integral)

$\mathrm{Ci}(B)=\int_B^{\infty}\frac{\cos x}{x}dx$ (Cosine integral)

are given in mathematical tables [7].

Ⅳ. Three-Dimensional Analysis

When ondol being heated is a rectangular with $2L_1\times 2L_2$, temperature distribution in the ground satisfies

$$\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}+\frac{\partial^2 T}{\partial Z^2}=0 \tag{17}$$

at $Z=0$, $k\frac{\partial T}{\partial Z}+h(T_H-T)=0$

for $|X|<L_1$ and $|Y|<L_2$ (18)

at $Z=0$, $k\frac{\partial T}{\partial Z}-h\ (T-T_C)=0$

for $|X|>L_1$ or $|Y|>L_2$ (19)

Without loss of generality, it is assumed that $L_1\leq L_2$.

Heat loss to the ground is obtained from

$$Q=4h\int_0^{L_1}\int_0^{L_2}(T_H-T)\Big|_{z=0}dXdY \tag{20}$$

Now let

$$u=\frac{T-T_C}{T_H-T_C}-\frac{k}{h(T_H-T_C)}\frac{\partial T}{\partial Z} \tag{21}$$

$$x=X/L_1 \tag{22}$$

$$y=Y/L_1 \tag{23}$$

$$z=Z/L_1 \tag{24}$$

$$B=2hL/k \tag{25}$$

$$s=L_2/L_1 \tag{26}$$

Then, equations (17)—(19) are transformed to

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0 \tag{27}$$

at $z=0$, u=1 for $|x|<1$ and $|y|<s$ (28)

at $z=0$, $u=0$ $|x|>1$ or $|x|>s$ (29)

Assume

$$u(x,y,z)=f(x,\ z)\ g(y,z) \tag{30}$$

then $f(x,z)$ and $g(y,z)$ are separated. That is, $f(x,z)$ satisfies

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial z^2}=0 \tag{31}$$

at $z=0$, $f=1$ for $|x|<1$ (32)

at $z=0$, $f=0$ for $|x|>1$ (33)

and $g(y,z)$ satisfies

$$\frac{\partial^2 g}{\partial y^2}+\frac{\partial^2 g}{\partial z^2}=0$$

at $z=0$, $g=1$ for $|y|<s$ (35)

at $z=0$, $g=0$ for $|y|>s$ (36)

Systems of equations (31)−(33) and equations (34)−(36) are similar to equations (9)−(11), and solutions are easily obtained.

$$u=\frac{1}{\pi^2}\left\{\arctan\frac{2z}{x^2+z^2-1}\right\}\left\{\arctan\frac{2sz}{y^2+z^2-s^2}\right\} \quad (37)$$

Range of arc tangent function is defined in equation (13).

Substituting equation (37) into equation (21), and solving the first order differential equation with respect to z, one obtains

$$\frac{T-T_C}{T_H-T_C}=\frac{B}{2\pi^2}e^{Bz/2}\int_z^\infty e^{-Bt/2}\left\{\arctan\frac{2t}{t^2+x^2-1}\right\}\left\{\arctan\frac{2st}{t^2+y^2-s^2}\right\}dt \quad (38)$$

Substituting equation (38) into equation (20), heat loss to the ground can be obtained.

$$E=\frac{Q}{4L_1L_2h(T_H-T_C)}=1-\frac{B}{2\pi^2 s}\int_0^1\int_0^s\int_0^\infty e^{-Bt/2}\left\{\arctan\frac{2t}{t^2+x^2-1}\right\}\left\{\arctan\frac{2st}{t^2+y^2-s^2}\right\}dt\,dy\,dx \quad (39)$$

Performing the integration (Appendix B)

$$E=1-\frac{B}{\pi^2}\int_0^\infty e^{-Bt}\left\{2\arctan\left(\frac{1}{t}\right)-t\ln\left(\frac{t^2+1}{t^2}\right)\right\}\left\{2\arctan\left(\frac{s}{t}\right)-\left(\frac{t}{s}\right)\ln\left(\frac{t^2+s^2}{t^2}\right)\right\}dt \quad (40)$$

Ⅴ. Discussion and Conclusions

Two-dimensional analysis can be otained from three-dimensional analysis for $s=L_2/L_1$

Numerical integration of equation (40) is performed using Laguerre integration [7], and the results are plotted in Figure 2. The accuracy of the numerical integration is checked by comparing results obtained from equation (40) for large values of s with results of equation (16).

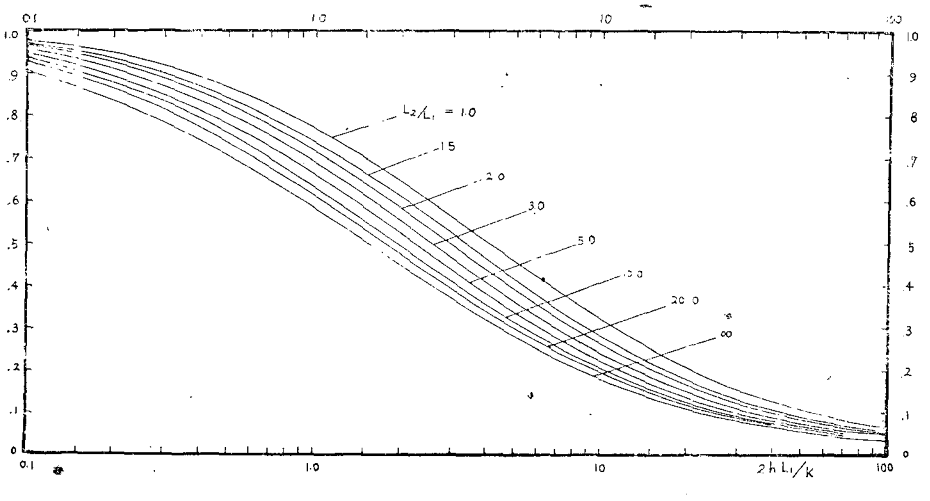

Fig. 2. Nondimensionalized heat loss to the ground

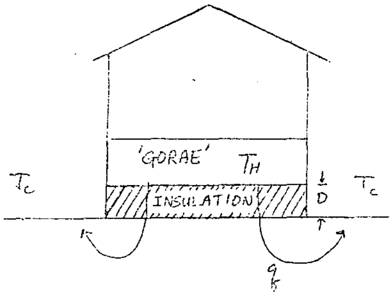

Fig 3. Insulation

Figure 2. shows that for constant values of s, E decreases with increasing B, However, heat loss, Q, increases as B increases, since for doubling B, E does not decread by half. This means that when the dimension of the room (L_1 and L_2) is constant, heat loss to the ground increases as heat transfer coefficient, h, increases. Also when thermal conductivity, k, and heat transfer coefficient, h, are fixed heat loss to the ground does not quadruple as each side of the house doubles.

The gradient of the curves in Figure 2. is great for $1<B<10$, therefore, the effect of heat transfer coefficient to the heat loss to the ground is relatively small in these regions.

In order to decrease the heat loss to the ground, the surface of the ground below 'gorae' can be covered with insulating material (Figure 3.). When the thickness of the insulation (thermal conductivity k_i) is D, effective heat transfer coeffient h_e must be used in equations (25) and (40) to obtain heat loss. Effective heat transfer coeffient h_e can be obtained approximately from one-dimensional steady state heat conduction as

$$\frac{1}{h_e}=\frac{1}{h}+\frac{D}{k} \qquad (41)$$

Numerical examples are given in Appendix C.

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NOMENCLATURE

$B=2hL_1/k$: Biot number;
D: thicknessof insulation, m;
E: nondimensionalized heat loss to the ground;
$f(x,z)$ and $g(y,z)$: functions defined by eqations (30);
h: heat transfer coefficient, W/m²°C;
h_e: effective heat transfer coefficient, W/m^2°C;
k: thermal conductivity of soil, W/m°C;
k_i: thermal conductivity of insulating layer, W/m°C;
L, L_1, L_2: half lengths of heated house, m;
Q: heat loss to the ground, W;
s: ratio of lengths of house;
T: temperature, °C;
T_H: temperature of the hot gas in 'gorae'; °C;
T_C: atmospheric temperature, °C;
t: dummy variable used in integration;
u: nondimensionalized function defined by equation (5) or (21);
X, Y, Z: space coordinates, m;
x, y, z: nondimensionalized space coordinates.

Appendix A

From equation(15)

$$E=1-\frac{B}{2\pi}\int_0^1\int_0^\infty e^{-Bt/2}\ \arctan\frac{2t}{t^2+x^2-1}dt\ dx \qquad (A1)$$

If the order of integration is changed,

$$E=1-\frac{B}{2\pi}\int_0^\infty e^{-Bt/2}\int_0^1\arctan\frac{2t}{x^2+t^2-1}dx\ dt \qquad (A2)$$

Inner integration can be obtained by integration by parts, that is,

$$\int_0^1\arctan\frac{2t}{t^2+x^2-1}\ dx=2\arctan\left(\frac{2}{t}\right)-\left(\frac{t}{2}\right)\ln\left(\frac{t^2+4}{t^2}\right) \qquad (A3)$$

$$\therefore E=1-\frac{B}{2\pi}\int_0^\infty e^{-Bt/2}\left[2\ \arctan\left(\frac{2}{t}\right)-\left(\frac{t}{2}\right)\ln\left(\frac{t^2+4}{t^2}\right)\right]dt \qquad (A4)$$

Let $t'=t/2$, then

$$E=1-\frac{B}{\pi}\int_0^\infty e^{-Bt'}\left[2\arctan\left(\frac{1}{t'}\right)-t'\ \ln\left(\frac{t'^2+1}{t'^2}\right)\right]dt' \qquad (A5)$$

But arctan $(1/t')=\frac{\pi}{2}-\arctan t'$.

$$E=1-B\int_0^\infty e^{-Bt'}\ dt'+\frac{2B}{\pi}\int_0^\infty e^{-Bt'}\ \arctan t'\ dt'+\frac{B}{\pi}\int_0^\infty e^{-Bt'}\ t'\ln(1+t'^2)\ dt'\ -\frac{2B}{\pi}\int_0^\infty e^{-Bt'}\ t'\ln t'\ dt' \qquad (A6)$$

Each integration can be obtained from theory of Laplace transformation.

$$E=\frac{2}{\pi B}\left[A+\ln B+\left\{\frac{\pi}{2}-\mathrm{Si}(B)\right\}\sin B-\mathrm{Ci}(B)\ \cos B\right] \qquad (A7)$$

Appendix B

From equation (39),

$$E=1-\frac{B}{2\pi^2 s}\int_0^1\int_0^s\int_0^\infty e^{-Bt/2}\left\{\arctan\frac{2t}{t^2+x^2-1}\right\}\left\{\arctan\frac{2st}{t^2+y^2-s^2}\right\}dt\ dy\ dx \qquad (B1)$$

Interchanging the order of integration gives

$$E=1-\frac{B}{2\pi^2 s}\int_0^\infty e^{-Bt/2}\left[\int_0^1\arctan\frac{2t}{t^2+x^2-1}dx\right]\left[\int_0^s\arctan\frac{2st}{t^2+y^2-s^2}dy\right]dt$$

$$=1-\frac{B}{2\pi^2}\int_0^\infty e^{-Bt/2}\left[2\ \arctan\left(\frac{2}{t}\right)-\left(\frac{t}{2}\right)\ln\left(\frac{t^2+4}{t^2}\right)\right]\left[2\ \arctan\left(\frac{2s}{t}\right)-\left(\frac{t}{2s}\right)\ln\left(\frac{t^2+4s^2}{t^2}\right)\right]dt \qquad (B2)$$

Let $t'=t/2$,

$$E=1-\frac{B}{\pi^2}\int_0^\infty e^{-Bt'}\left[2\ \arctan\left(\frac{1}{t'}\right)-t'\ \ln\left(\frac{t'^2+1}{t'^2}\right)\right]\left[2\ \arctan\left(\frac{s}{t'}\right)-\left(\frac{t'}{s}\right)\ln\left(\frac{t'^2+s^2}{t'^2}\right)\right]dt' \qquad (B3)$$

Since t' is a dummy variable, t can be used instead of t'.

$$E=1-\frac{B}{\pi^2}\int_0^\infty e^{-Bt}\left[2\ \arctan\left(\frac{1}{t}\right)-t\ \ln\left(\frac{t^2+1}{t^2}\right)\right]\left[2\ \arctan\left(\frac{s}{t}\right)-\left(\frac{t}{s}\right)\right]\ln\left(\frac{t^2+s^2}{t^2}\right)\Big]dt \qquad (B4)$$

Appendix C

Example 1

Calculate heat loss to the ground when following conditions are given:

Room size=4m×6m

mean hot gas temperature in 'gorae'=100°C

mean atmospheric temperature$=-10°C$
heat transfer coefficient$=25\ W/m^{2}°C$
thermal conductivity of soil$=2.0\ W/m°C$

Solution:

$s=L_2/L_1=3/2=1.5$

$B=2hL_1/k=2\times25\times2/2=50$

From Figure 2., $E=0.1$

$Q=4hL_1L_2\ (T_H-T_C)\ E$

$E=4\times25\times2\times3\times110\times0.1=6600\ W$
$(=5670 kcal/h)$

Example 2

Surface of the ground of the house given in Example 1 is covered with diatomaceous earth ($k_1=0.061\ W/m°C$). The thickness of the insulation is 50mm. Calculate the heat loss to the ground.

Solution:

Effective heat transfer coefficient is obtained from equation (41)

$$h_e=1/\left(\frac{1}{25}+\frac{0.050}{0.061}\right)=1.163\ W/m^{2}°C$$

$B=2h_eL_1/k=2\times1.163\times2/2=2.33$

$s=1.5$

From Figure 2., $E=0.58$

$Q=4h_eL_1L_2(T_H-T_C)\ E$

$E=4\times1.63\times3\times3\times110\times0.58=1780\ W$
$(=1530\ kcal/h)$