<Research paper>

Journal of the Korean Institute of Industrial Engineers
Vol. 3 No. 2 December 1977

Reliabilty of a System with Standbys and Spares

Kyung Soo Park*

Abstract

This paper investigates the reliability characteristics of a system consisting of a unit operating on-line and backed by n spares among which m units are kept "warm" as standbys ready to go on-line. The on-line unit has an arbitrary lifetime distribution, while the warm standbys have exponential failure time distributions. The failed units are repaired and brought back to service. The cold spares do not fail while in storage.

Solution of this extremely complicated queuing problem using a "renewal counting" approach is presented and extended to the situation where the warming-up takes non-negligible time. Finally, an approach to the economic system management is discussed, considering the long-run availability, cost of keeping spares and repair facility, and the associated cost of restarting the system after a system failure.

The model presented in this paper will have many applications including the determination of the spares inventory and the number of field spares to be "carried."

Introduction

One way of increasing a system's reliability is to provide sufficient spares as standys. When the on-line unit fails, it is replaced by one of the standbys. However, in order to minimize the delay during a switchover (until the replacing unit is properly "warmed up"), it may be necessary to keep a specified number of spares "warm" ready to go on-line.

There are at least two distinct situations where this concept of warm standbys is appropriate:

1) For example, an electronic gear may stand by with its vacuum tubes literally "warm."

2) As a second example, a standby unit may "accompany" the principal unit. In this case, the standby unit is not actually used on the job, but may fail due to the environmental stress and handling. The extra units at the base (or storage) may be regarded as the "cold" spares.

It may not be necessary to keep all the spares warm. However, the risk of keeping too few warm standbys is that the system is unavilable during the switchover to a cold spare. Furthermore, once the system shuts down, it may be necessary to restart the system at a great cost.

Natarajan, et al [1], have investigated the reliability characteristics of a single-unit system with spares when all associated distributions are exponential. Subramanian, et al [2], have considered the reliability of a repairable system with several standby redundant units where the failure

*Department of Industrial Science, Korea Advanced Institute of Science

distributions of the standby units are identical. In this paper, the results of Subramanian, et al, are extended to two types of standbys and a method of handling the warming-up delay is presented.

The on-line unit is subject to specified loading conditions and is assumed to have a general lifetime distribution. The warm standbys, however, are not actively loaded and are subject to chance failures caused by external random influences. Then the failure rate of a standby unit may be assumed to be a constant $(=\lambda)$. Then due to the memoryless property of the negative exponential distribution, the lifetime of an on-line unit is indepedent of the duration it served as a standy unit.

When the on-line unit fails, it is immediately replaced by one of the warm standbys, if one is available. If not, it is replaced by one of the cold spares and after a warm-up period the system restarts. The repair time is assumed to be exponential with parameter μ. The repairs ae are carried out on a first-in, first-out basis.

In the following presentation, analysis begins with a simple case where the warm-up time is negligible compared to the cumulative lifetime of all the warm standbys.

Reliability Analysis

When the system does not have the repair privilege, the reliability of the system, R(t), is a very important design criterion, which is defined as the probability that the system has been operating over the interval 0 to time t.

Let f(t) be the general probability density function (pdf) of the failure time of an on-line unit. Let $F_i(t)$ be the probability that the system is down at time t, given that at t=0 there were i operable units in the system including the on-line unit among which min (m,i-1) units were kept warm ready to go on-line. Then, assuming that the warm-up time is negligible, the reliability of the system is

$$R(t)=1-F_{n+1}(t).$$

Since the system can fail only at a renewal point in time when the on-line unit fails and there are no operable spares,

$$F_o(t)=1$$

$$F_i(t)=\sum_{j=0}^{i-1}\int_o^t \phi_{i-1j}(z)\ F_j(t-z)dz, \quad i=1,\cdots\cdots n+1. \qquad \text{(Eq. 1)}$$

where $\phi_{ij}(t)=p_{ij}(t)\cdot f(t)$,

$$p_{ij}(t)=\text{Prob.}\begin{Bmatrix}\text{j spares are} & i \text{ spares were} \\ \text{operable at t} & \text{operable at t=0}\end{Bmatrix}, \quad i>j.$$

Taking Laplace transforms of Eq. 1 and writing

$$F_i^*(s)=\mathcal{L}\{F_i(t)\}=\int_0^\infty e^{-st}F_i(t)dt$$

$$F_o^*(s)=\frac{1}{s}$$

$$F_i^*(s)=\sum_{j=0}^{i-1}\phi^*_{i-1j}(s)F^*_j(s), \quad i=1,,\cdots\cdots,n+1. \qquad \text{(Eq. 2)}$$

The $F_{n+1}^*(s)$ can be obtained by solving the Eq. 2 successively, and then inverted to obtain $F_{n+1}(t)$.

In order to obtain the expression for $p_{ij}(t)$, which are necessary for $\phi^*_{ij}(s)$, consider the following failure process associated with spares. The system starts working at time $t=0$ with i spares among which min(m,i) are warm standbys. Suppose the on-line unit fails at time t. Let the probability that the system is in state i, $p_i(t)$, be defined as

$$p_i(t) = \text{Prob. } \{\text{i spares are operable at time t}\}.$$

The transition probabilities λ_i of the transition from state i to state i-1 are given by

$$\lambda_i = \begin{cases} i\lambda & \text{if } 0 \leq i \leq m, \\ m\lambda & \text{if } m \leq i \leq n \\ 0 & \text{otherwise.} \end{cases}$$

The equations governing $p_i(t)$ are:

$$\begin{aligned} &p_0'(t) = \lambda p_1(t), \\ &p_i'(t) = -i\lambda p_i(t) + (i+1)\lambda p_{i+1}(t), \quad 1 \leq i \leq m-1, \\ &p_i'(t) = -m\lambda p_i(t) + m\lambda p_{i+1}(t), \qquad m \leq i \leq n-1, \\ &p_n'(t) = -m\lambda p_n(t) \end{aligned} \tag{Eq. 3}$$

Taking Laplace Transforms of Eq. 3 and writing $p_i^*(s) = \mathcal{L}\{p_i(t)\} = \int_0^\infty e^{-st} p_i(t)\,dt$, with the initial conditions, $p_i(0)$, $i=0, \ldots, n$, the following equations are obtained:

$$\begin{aligned} &sp_0^*(s) - \lambda p^*_1(s) = p_0(0) \\ &(s+i\lambda)p_i^*(s) - (i+1)\lambda p_{i+1}^*(s) = p_i(0); \; 1 \leq i \leq m-1 \\ &(s+m\lambda)p_i^*(s) - m\lambda p_{i+1}^*(s) = p_i(0); \qquad m \leq i \leq n-1 \\ &(s+m\lambda)p_n^*(s) = p_n(0) \end{aligned} \tag{Eq. 4}$$

Solving the above system of equations by Cramer's rule, we obtain

$$p_i(t) = \mathcal{L}^{-1}\{p_i^*(s)\} = \mathcal{L}^{-1}\{D_i(s)/D(s)\},$$

where

$$D(s) = \begin{vmatrix} s & -\lambda & 0 \cdots\cdots\cdots\cdots\cdots\cdots 0 & & & & \\ 0 & s+\lambda & -2\lambda \quad 0 \cdots\cdots\cdots\cdots\cdots 0 & & & & \\ & & \cdots\cdots\cdots & & & & \\ 0 \cdots\cdots 0 & & s+(m-1)\lambda & -m\lambda & 0 \cdots\cdots 0 & & \\ 0 \cdots\cdots\cdots\cdots 0 & & & s+m\lambda & -m\lambda & 0 \cdots 0 & \\ & \cdots\cdots\cdots & & & & & \\ 0 \cdots\cdots\cdots\cdots\cdots\cdots 0 & & & & s+m\lambda \cdots -m\lambda & & \\ 0 \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & & & & 0 \cdots\cdots s+m\lambda & & \end{vmatrix} \tag{Eq. 5}$$

The $D_i(s)$ is the determinant obtained by replacing the i^{th} column (counting from the zeroth) in $D(s)$ by the initial vector.

Then $p_{ij}(t) = p_j(t)$ with initial conditions $p_i(0)=1$, $p_j(0)=0$, j=i. That is,

$p_{ij}(t) = \mathcal{L}^{-1}\{p_{ij}^*(s)\} = \mathcal{L}^{-1}\{D_{ij}(s)/D(s)\}$; where $D_{ij}(s)$ is obtained by replacing the j^{th} colum (counting from the zeroth) of $D(s)$ in Eq. 5 by a unit vector i.

System Availability

When the system has the repair privilege, the pointwise availability of the system, $A(t)$, is a pertinent design criterion, which is defined as the probability that the system is operating at time t.

Let $f(t)$ be the pdf of the failure time of an on-line unit as before. Let $G_i(t)$ be the probability that the system is down at t, given that at $t=0$ there were i operable units in the system including the on-line unit among which $\min(m,i\text{-}1)$ units were kept warm.

When the warm-up time is negligible, the availability of the system is

$$A(t)=1-G_{n+1}(t).$$

Proceeding as earlier, noting that the number of operable spares may increase or decrease in time,

$$G_o(t)=e^{-\mu t}+\int_o^t \mu e^{-\mu z}G_1(t-z)\,dz,$$

$$G_i(t)=\sum_{j=0}^{n}\int_o^t \Psi_{i-1j}(z)\ G_j(t-z)\,dz,\quad i=1,\ldots,n+1, \qquad \text{(Eq. 6)}$$

where $\Psi_{ij}(t)=q_{ij}(t)\cdot f(t)$

$$q_{ij}(t)=\text{Prob.}\left\{\begin{matrix}j \text{ spares are}\\ \text{operable at } t\end{matrix}\ \middle|\ \begin{matrix}i \text{ spares were}\\ \text{operable at } t=0\end{matrix}\right\}$$

Taking Laplace transforms of Eq. 6,

$$G_o{}^*(s)=\frac{1}{s+\mu}+\frac{\mu}{s+\mu}G_1{}^*(s) \qquad \text{(Eq. 7)}$$

$$G_i{}^*(s)=\sum_{j=0}^{n}\Psi_{i-1j}{}^*(s)G_j{}^*(s),\ \ i=1\ldots,n+1.$$

In order to obtain the expression for $q_{ij}(t)$, consider the following birth-and-death process associated with the spares.

The system starts working at time $t=0$ with i spares among which $\min(m,i)$ units are warm standbys. Suppose the on-line unit fails at t. Let the probility that the system is in state $i, q_i(t)$, be defined as

$$q_i(t)=\text{Prob. }\{i \text{ spares are operable at time } t\}.$$

The transition probabilities λ_i and μ_i of the transition from the states i to $i-1$ and $i+1$ are given by

$$\lambda_i=\begin{cases} i\lambda & \text{if } 0\le i\le m,\\ m\lambda & \text{if } m\le i\le n,\\ 0 & \text{otherwise,}\end{cases} \qquad \text{and } \mu_i=\begin{cases}\mu & \text{if } 0\le i\le n,\\ 0 & \text{otherwise.}\end{cases}$$

The equations governing $q_i(t)$ are:

$$\begin{aligned}
&q_o{}'(t)=-\mu q_o(t)+\lambda q_1(t)\\
&q_i{}'(t)=\mu q_{i-1}(t)-(i\lambda+u)q_i(t)+(i+1)\lambda q_{i+1}(t),\ \ 1\le i\le m-1 \qquad \text{(Eq. 8)}\\
&q_i{}'(t)=\mu q_{i-1}(t)-(m\lambda+\mu)q_i(t)+m\lambda q_{i+1}(t),\ \ m\le i\le n-1\\
&q_n{}'(t)=\mu q_{n-1}(t)-m\lambda q_n(t).
\end{aligned}$$

Taking Laplace transforms of Eq. 8 and writing $q_i{}^*(s)=\mathcal{L}\{q_i(t)\}$, with the initial conditions, $q_i(0)$, $i=0,\ldots,n$, the following equations are obtained:

$$\begin{aligned}
&(s+\mu)q_o{}^*(s)-\lambda q_1{}^*(s)=q_o(0)\\
&-\mu q_{i-1}{}^*(s)+(s+i\lambda+\mu)q_i{}^*(s)-(i+1)\lambda q_{i+1}{}^*(s)=q_i(0),\ \ 1\le i\le m-1 \qquad \text{(Eq. 9)}\\
&-\mu q_{i-1}{}^*(s)+(s+m\lambda+\mu)q_i{}^*(s)-m\lambda q_i{}^*(s)=q_i(0),\ \ m\le i\le n-1,\\
&-\mu q_{n-1}{}^*(s)+(s+m\lambda)q_n{}^*(s)=q_n(0).
\end{aligned}$$

Solving above system of equations by Cramer's rule, we obtain

$$q_i(t)=\mathcal{L}^{-1}\{q_i{}^*(s)\}=\mathcal{L}^{-1}\{D_i(s)/D(s)\},$$

where

$$D(S)=\begin{vmatrix} s+\mu & -\lambda & 0 & 0\cdots\cdots & & & \cdots\cdots 0 \\ -\mu & s+\lambda+\mu & -2\lambda & 0\cdots\cdots & & & \cdots\cdots 0 \\ & & \cdots\cdots & & & & \\ 0 & 0 & -\mu & s+(m-1)\lambda+\mu & -m\lambda & 0\cdots\cdots & \cdots\cdots 0 \\ 0 & 0 & & -\mu & s+m\lambda+\mu & -m\lambda & 0\cdots\cdots 0 \\ & & \cdots\cdots & & & & \\ 0\cdots\cdots & & \cdots\cdots 0 & -\mu & s+m\mu+\mu & -m\lambda & \\ 0\cdots\cdots & & & \cdots\cdots 0 & & -\mu & s+m\lambda \end{vmatrix} \quad \text{(Eq.10)}$$

The $D_i(s)$ is the determinant obtained by replacing the i^{th} column in $D(s)$ by the initial vector. Then $q_{ij}(t)=\mathfrak{L}^{-1}\{q_{ij}^*(s)\}=\mathfrak{L}^{-1}\{D_{ij}(s)/D(s)\}$, where $D_{ij}(s)$ is obtained by replacing the j^{th} column of $D(s)$ in Eq. 10 by a unit vector $\underline{i}$.

The $G_{n+1}^*(s)$ can be obtained by solving the system of equations Eq. 7. Then the steady state availability of the System, A, is

$$A=\lim_{t\to\infty}A(t)=1-\lim_{t\to\infty}G_{n+1}(t)=1-\lim_{s\to 0} sG_{n+1}^*(0).$$

Illustrative Solution Procedure

Suppose the failure of an on-line unit is governed by the gamma distribution,

$$f(t)=4te^{-2t},$$

whose Laplace transform is,

$$f^*(s)=\left(\frac{2}{s+2}\right)^2.$$

Assume that there are 2 spares 1 of which is a warm standby. Without loss of generality, the failure rate of the warm standby is assumed to be $\lambda=1$, since the measurement of the time can be made on an arbitrary scale.

Then without the privilege of repairs, the reliability analysis may proceed as following:
From Eq. 5,

$$D(s)=s\begin{vmatrix} s & -1 & 0 \\ 0 & s+1 & -1 \\ 0 & 0 & s+1 \end{vmatrix}=s(s+1)^2$$

The $p_{ij}^*(s)$ are obtained by $D_{ij}(s)/D(s)$, then inverted, multiplied by $f(t)$, and retransformed into $\phi_{ij}^*(s)$. Then from Eq. 2,

$$F_3^*(s)=\frac{12s^2+128s+324}{s(s+2)^2(s+3)^4}.$$

Inverting $F_3^*(s)$ by partial fractions [3] or numerically [4] and subtracting from 1.0,

$$R(t)=(58t-163)e^{-2t}+(\frac{8}{3}t^3+28t^2+108t+164)e^{-3t}.$$

A plot of this reliability curve is given in Figure 1 along with the reliability characteristics of the on-line unit only,

$$R_o(t)=e^{-2t}(1+2t),$$

and the reliability of the system with 2 cold spares only,

$$R_{2c}(t)=e^{-2t}\sum_{k=0}^{5}(2t)^k/k!$$

If repairs are performed at the rate of $\mu=3$, the analysis of the steady state availability may proceed as following:

From Eq. 10,

$$D(s)=\begin{vmatrix} s+3 & -1 & 0 \\ -3 & s+4 & -1 \\ 0 & -3 & s+1 \end{vmatrix}=s(s+4-\sqrt{3})(s+4+\sqrt{3}).$$

The $q_{ij}^*(s)$ are obtained by $D_{ij}(s)/D(s)$, then inverted, multiplied by $f(t)$, and retransformed into $\Psi_{ij}^*(s)$. The matrix of Ψ_{ij} (0) and $\Psi_{ij}'(0)$ for the given problem are,

$$\Psi_{ij}(0)=\begin{pmatrix} .24 & .289 & .471 \\ .096 & .3 & .603 \\ .052 & .201 & .747 \end{pmatrix} \qquad \Psi_{ij}'(0)=-\begin{pmatrix} .15 & .268 & .582 \\ .089 & .254 & .656 \\ .065 & .219 & .717 \end{pmatrix}$$

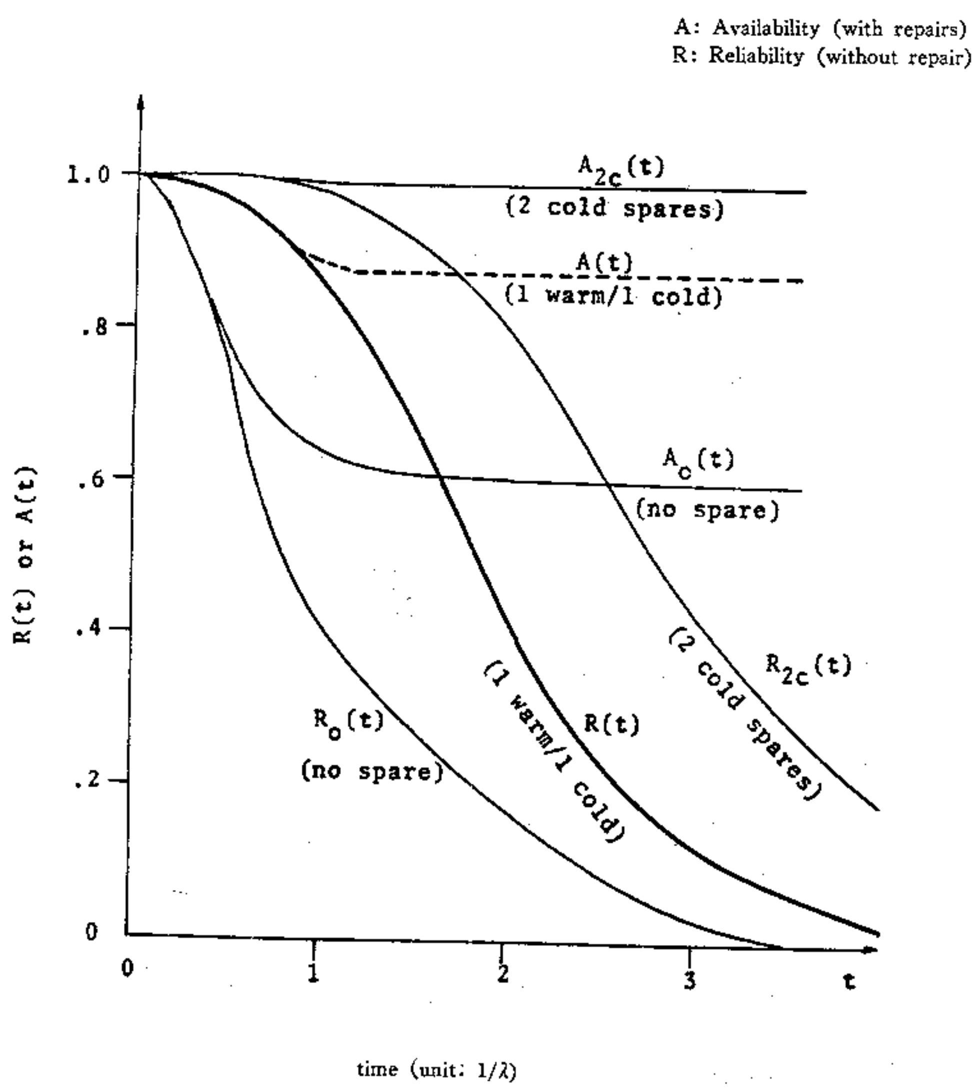

Fig. 1 Reliability/Availability of the System Given in the Example with Different Provisions

Specializing the system of equations Eq. 7 for $n=2$ and solving for $G_3^*(s)$

$$G_3^*(s)=D_1(s)/D(s),$$

where

$$D(s)=\begin{vmatrix} 1 & -\frac{3}{s+3} & 0 & 0 \\ -\Psi_{00}^*(s) & 1-\Psi_{01}^*(s) & -\Psi_{02}^*(s) & 0 \\ -\Psi_{10}^*(s) & -\Psi_{11}^*(s) & 1-\Psi_{12}^*(s) & 0 \\ -\Psi_{20}^*(s) & -\Psi_{21}^*(s) & -\Psi_{22}^*(s) & 1 \end{vmatrix}$$

$D_1(s)$ is obtained by replacing the last column in $D(s)$ by a column vector $\left[\frac{1}{s+3},\ 0,\ 0,0\right]$

Then $A(t)=1-\lim_{s\to 0} sG_3^*(s)=1-\left[\frac{D_1(s)}{D'(s)}\right]_{s=0}=.8788.$

It is interesting to compare this result with the steady state availability of the system with 2 cold spares only, $A_{2c}=.9897$, and the availability of the on-line unit without spares,

$$A_o(t)=.6+e^{-2.5t}\{.4\cos 1.9365t+.5164\sin 1.9365t\}$$

as are plotted in Figure 1.

Warmup Time

When the warming up of the spares takes considerable time, the availability of the cold spares can not guarantee the system's performance. For example, an "accompanying" standby unit replaces the failed principal (on-line) unit and another unit at the base (or storage) is called in (warming up) as a warm standby. But the current on-line unit without any warm standby may fail before the requested unit arrives.

Let $w(t)$ be the general pdf of the warmup time of a cold spare, and $W(t)$ be its cumulative distribution function. Then Eq.6 should be modified to take the warmup time into account as following:

$$G_0(t)=e^{-\mu t}+\int_o^t \mu e^{-\mu z}\{1-W(t-z)\}\,dz+\int_o^t\int_o^z \mu e^{-\mu y}w(z-y)G_1(t-z)\,dy\,dz$$

$$G_i(t)=\sum_{j=0}^{n}\int_o^t \Psi_{i-1j}(z)G_j(t-z)\,dz,\ \ i=1,\ldots,n+1. \qquad \text{(Eq. 11)}$$

The additional terms in $G_o(t)$ in the Eq. 11 reflect the fact that in order for the system to fail at t (system was down at $t=0$), one of the following should occur: (1) no units are repaired by time t, or (2), at least one unit is repaired some time before t but not warmed up by time t, or (3), at least one unit is repaired at $y<t$, warms up at $z(y<z<t)$ to fail at t.

Taking Laplace transforms of Eq. 11,

$$G_o^*(t)=\frac{1}{s}-\frac{\mu}{s+\mu}\{W^*(s)+w^*(s)G_1^*(s)\} \qquad \text{(Eq. 12)}$$

$$G_i^*(s)=\sum_{j=0}^{n}\Psi_{i-1j}^*(s)\ G_j^*(s), \qquad i=1,\ldots,n+1.$$

Note that if warmup time is ignored, that is if $w(t)$ is assumed to be an impulse function $\delta(0)$, Eq. 12 reduces to Eq. 7.

Distribution of the Time to System Failure (with Repair)

One of the design criteria frequently neglected in reliability analysis is the restart cost accompanying a system failure. Unless the failed on-line unit is immediately replaced by a warm standby, the system will be down until the cold spare warms up properly. Depending on the system, the restart cost may be substantial even if the warmup time (and the unavailability during the warmup) is of short duration. Generally, the penalty for restarting, in the long run, is inversely proportional to the expected duration of time to system failure.

Let $f(t)$ be the pdf of the failure time of an on-line unit. Let $f_i(t)$ be the pdf of the time to system failure, given that there are i operable spares at $t=0$. Since the system can fail only at a renewal point in time when the on-line unit fails and there are no operable warm standbys,

$$f_i(t)=\Psi_{io}(t)+\sum_{j=1}^{n}\int_{o}^{t}\Psi_{ij}(z)f_{j-1}(t-z)dz,\ i=0,1,\ldots,n. \qquad \text{(Eq. 13)}$$

where

$$\Psi_{io}(t)=q_{io}(t)f(t),$$

$$\Psi_{ij}(t)=\int_{o}^{t}f(y)q_{ij}(y)w(t-y)dy, \qquad \text{if } m=0,$$

$$\Psi_{ij}(t)=f(t)\int_{o}^{t}q_{ij}(y)w(t-y)dy, \qquad \text{if } 1\leq m\leq n,$$

$$q_{ji}(t)=\text{Prob.}\left\{\begin{matrix}j \text{ spares are} \\ \text{operable at } t\end{matrix}\ \middle|\ \begin{matrix}i \text{ spares were} \\ \text{operable at } t\end{matrix}\right\}.$$

Since all the spares are operable at $t=0$, the function $f_n(t)$ is the desired one.

The expected value of the system lifetime, given that there are i operable spares at $t=0$, Exp $[T_i]$ is

$$\text{Exp}\ [T_i]=-\frac{d}{ds}f_i^*(0) \quad \text{with} \quad f_i^*(0)=1.$$

Since from Eq. 13,

$$f_i^*(s)=\Psi_{io}^*(s)+\sum_{j=1}^{n}\Psi_{ij}^*(s)f_{j-1}^*(s)$$

$$\text{Exp}\ [T_i]=-\Psi_{io}^{*\prime}(0)-\sum_{j=1}^{n}\Psi_{ij}^{*\prime}(0)f_{j-1}^*(0)-\sum_{j=1}^{n}\Psi_{ii}^*(0)f_{j-1}^{*\prime}(0)$$

$$=-\sum_{j=0}^{n}\Psi_{ij}^{*\prime}(0)-\sum_{j=1}^{n}\Psi_{ij}^*(0)f_{j-i}^{*\prime}(0)$$

$$=\text{Exp}[T_{f+w}]+\sum_{j=1}^{n}\Psi_{ij}^*(0)\ \text{Exp}\ [T_{j-1}], \qquad \text{(Eq. 14)}$$

where

$$\text{Exp}\ [T_{f+w}]=-\sum_{j=0}^{n}\Psi_{ij}^{*\prime}(0)$$

$$=-\frac{d}{ds}\mathcal{L}\left\{\int_{o}^{t}f(y)w(t-y)dy\right\}\Big]_{s=o}$$

$=\text{Exp}\ [T_f]+\text{Exp}[T_w]$ when $m=0$.

$=$Mean lifetime of the on-line unit$+$mean warmup time

$$\text{Exp}\ [T_{f+w}]=-\frac{d}{ds}\mathcal{L}\{f(t)\cdot W(t)\}\Big]_{s=o}$$

$=\text{Exp}[X]$ where $pdf_X(t)=f(t)\cdot W(t)$ when $m\geq 1$.

The expected value of the system lifetime with repair privilege is $\text{Exp}[T]=\text{Exp}[T_n]$ which can be obtained by solving the system of equations Eq. 14.

System Management

In order to manage the system in an optimal fashion, the optimal number of warm standbys and cold spares may be determined for a given level of repair capability. Furthermore, if the cost of repair efficiency can be quantified, overall optimal policy may be obtained for the system design.

To quantify the cost of repair efficiency, consider the penalty of the system unavailability and the cost of repair facilities. It is reasonable to assume that the penalty is proportional ($\$_p$) to the fraction of time the system is unavilable, i.e., $1-A(n,m,\mu)$. The $A(n,m,\mu)$ is the long-run availability of a system when there are n spares, m of which are warm standbys, and serviced by a repair facility at the rate of μ. The cost per unit time of keeping the repair efficiency at the rate of μ is denoted by $\$_r(\mu)$. The cost per unit time of keeping n spares is denoted by $\$_s(n)$.

If the restart cost is of such a magnitude that cannot be neglected, the long-run cost of restarting/unit time, $\$_t(n,m,\mu)$, is

$$\$_t(n,m,\mu)=\frac{\text{cost of restarting system}}{\text{Exp}[T]}$$

Finally, we wish to minimize the total cost given by

$$\$(n,m,\mu)=\$_r(\mu)+\$_s(n)+\$_t(n,m,\mu)-\$_pA(n,m,\mu).$$

Summary

A mathematical theory for the solution of reliability/availability characteristics of a system with warm standbys and cold spares has been presented with an illustrative numerical example. No simplifying assumption was made regarding the lifetime distribution of the on-line unit. The result was further extended to the situation where slow warmingup of the cold spares causes the system unavailable during the switch-over.

The computations involved are generally cumbersome if there are more than several spares. For any realistic application of the theory, numerical analysis would be most desirable.

References

(1) Natarajan, R. and S. Rao, "Reliability of a Repairable System with Spares Deterioration in Storage," *CORS Journal*, Vol. 8, pp. 154-164 (1970).

(2) Subramanian, R.,K. Venkatakrishnan, and K. Kistner, "Reliability of a Repairable System with Standby Failure," *Operations Research*, Vol. 24, No. 1 (Jan-Feb 1976)

(3) Churchill, R.V., *Operational Mathematics*, McGraw-Hill Book Co., New York (1958)

(4) Dubner, H. and J. Abate, "Numberical Inversion of Laplace Transforms by Relating Them to the Finite Fourier Cosine Transform," *Journal of the Association for Computing Machinery*, Vol. 15, No. 1 (January 1968)