〈Research Paper〉

Journal of the Korean Institute of Industrial Engineers
Vol. 3, No. 2, November 1977

Mission Effectiveness Model Applicable For Military System's Evaluation and Test Design

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Abstract

Mission effectiveness, which is the probability of successfully completing the assigned mission, is introduced as an appropriate measure of effectiveness for a military system. The model of mission effectiveness is developed for a system which is required to carry out various types of a mission. Each mission type is characterized by the maximum allowable time that determines the success of a given mission type. A given type of a mission is successful if and only if (i) the system is available at the start of a mission and (ii) the system completes its mission within the maximum allowable duration of time that this given mission type specifies without any failure during this period. Both analytic and simulation approaches are employed. Difficulties involved in the anayticl approach are discussed. The model is proposed as a useful tool for consistent system evaluation and optimum test design.

1. Introduction

Mission effectiveness, which is the probability of successfully completing the assigned mission, has been used as an appropriate measure of effectiveness for some military systems. Some previous mathematical models of mission effectiveness can be seen in [2, 5] These models, however, do not consider the factors of environmental effect and operator performance which, in practice, are believed to have a siginficant impact on missión effectiveness. Simulation models of mission effect iveness which include these additional factors are developed in [7]. In [7], the mission effectiveness is determined by the product of the availability, the mission reliability, the system performance under various environmental conditions, and the operator performance as a function of the qualific ations of the operator and the retraining of the operator. The model in [7] is developed for a single mission type.

In this study, an attempt is made to develop a model for a system which is required to carry out various types of a mission. Here, each mission type is characterized by the maximum allowa ble time that determines the success of a given mission type. For a given type of a mission to be successful, the system is required to be available at the start of a mission and the system must complete its mission within the maximum allowable duration of time that this given mission type specifies without any failure during this period. Furthermore, the effects of the environment and

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the operator are reflected into the mission duration. In other words, poor environmental condition and poor operator performance are assumed to make the actual mission duration longer than the mission duration under ideal conditions. Thereby, adverse effects of the environment and the operator tend to reduce the probability of mission success.

Two approaches are employed in developing the model: analytic and simulation approach. Difficulties involved in the analytic approach are discussed. The model is illustrated by a numerical example based on the data taken from an actual test of a military weapon system [6]. This model can be used to determine the sensitivity of some data elements to the mission effectiveness. In this way, the model can be used to determine the critical data element early in the test. Thereby, it can provide the military agency in charge of operational test and evaluation with a direction and thoroughness to the test planning

The model, therefore, is proposed as a useful tool for consistent system evaluation and optimum test design.

2. Description of an Illustrative System and Assumptions

A system consists of a single military unit. The system is required to carry out various types of a mission. Each mission type is characterized by the maximum allowable time that determines the success of a given type of a mission. For a given type of a mission to be successful, the system should be available at the start of a mission and the system should accomplish its mission (e.g., hit a target) within the maximum allowable duration of time that this given mission type specifies without any failure during this period. If the system cannot accomplish a mission within this specified duration of time, the mission is terminated at this point and is considered to be failed even though the system is still operable. Failures of the system are induced by both the hardware itself and the operator. The environmental condition encountered during a mission affects the performance of the system. The operator performance is affected by the qualification level of the operator and the retraining of the operator. Poor environenmental condition and the operator performance make the actual mission longer than the ideal mission duration. Thereby, these effects are reflected into the mission reliability. In other words, it will take longer time for a system to accomplish its mission under unfavorable operator and environmental conditions than under favorable conditions. Therefore, the mission reliability tends to be decreased when the operator performance and the environmental condition are poor.

The probability distributions for the following random variables are assumed to be known:

(a) time interval between mission starts, V;

(b) time between failures induced by the hardware, Z^h;

(c) time between failures induced by the operator, Z^o;

(d) downtime, W;

(e) mission type, K;

(f) mission duration for a given type of mission k under ideal conditions, U_k; and

(g) environmental condition, Q, and the performance under this condition.

In addition, the functional form of the operator performance is assumed to be known.

3. Development of Model

3.1. Analytical Approach

A basic requirement for any mission to be successful is that the system must be operative upon demand and the system must accomplish its mission within the maximum allowable duration of time without any failure during this period. In probabilistic terms, the effectiveness of a mission, ME, under ideal conditions can generally be stated as

$$ME(t,u\,|\,\cdot)=P_r\{z(t)=1, u\leq u^{max}, T_f(t)>u^{max}\,|\,I(\cdot)\} \tag{1}$$

where $z(t)$ =the state of the system at time t,

0: down state

1: up state

$T_f(t)$ =time to failure measured from t,

u=mission duration under ideal conditions,

u^{max}=maximum allowable time specified for the mission in order for the mission to be successful, and

I(·)=totality of the required information such as the distributions for the failure and repair times, the initial state of the system, mission types, etc. In practice, however, the system performance is expcted to be different from one environmental condition to another. Also, the operator performance is expected to be different depending upon the qualifications of the operator as well as the retraining of the operator. Due to the effects of the operator and the environment, the actual mission duration, u', is expected to be longer than the mission duration under ideal condition, u, Replacing u by u' in eq. (1), mission effectiveness can generally be stated as

$$\begin{aligned} ME(t,u'\,|\,\cdot)&=P_r\{z(t)=1, u'\leq u^{max}, T_f(t)>u^{max}\,|\,I(\cdot)\} \\ &=P_r\{z(t)=1\,|\,I(\cdot)\}\,P_r\{u'\leq u^{max}, T_f(t)>u^{max}\,|\,z(t)=1;I(\cdot)\} \end{aligned} \tag{2}$$

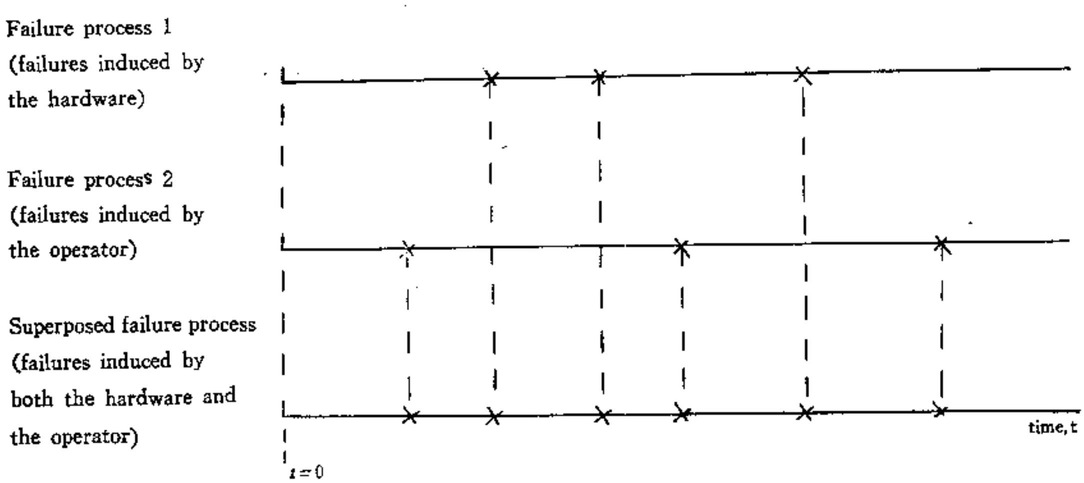


Fig. 1 Superposition of two failure processes

$P_r\{z(t)=1\,|\,I(\cdot)\}$ represents the pointwise availability whose expression varies depending upon $I(\cdot)$. In general, when the distributions for failures induced by the hardware and the operator are nonexponontial, it is difficult to obtain the analytic expression of the pointwise availability. However, if the system has been used for a long period of time, the pointwise availability may be approximated by the steady-state availability [4] as given by

$$A=\frac{MTBF}{MTBF+MDT} \tag{3}$$

where MTBF=mean time between failure of the system and

MDT=mean downtime of the system.

Since the system failures are caused by the hardware itself and the operator. MTBF of the system can be obtained by the superposition of these two separate failure processes. In Fig. 1, failure process 1 (failures induced by the hardware), failure process 2 (failures induced by the operator), and the superposed failure process (failures induced by both) are shown. If the failure processes 1 and 2 follow poisson processes with mean rates $1/MTBF_h$ and $1/MTBF_o$ respectively, then the superposition of these two processes also follows a Poisson process with mean rate $1/MTBF_h+1/MTBF_o$ [1]. Hence, the mean time between-failure of the system can be estimated by

$$MTBF=\frac{1}{1/MTBF_h+1/MTBF_o} \tag{4}$$

where $MTBF_h$=mean time between failure induced by the hardware and

$MTBF_o$=mean time between failure induced by the operator.

Although the availability expression given by eq. (3) is basically based on the exponential property, it can relatively be satisfactorily used in an approximate sense even for the nonexponential failure

Table. 1 Type of mission, K

Mission type, K	Maximum allowable time for a mission of type K to be successful, u^{max}	frequency, f
Type 1	u^{max}_1	f_1
Type 2	u^{max}_2	f_2
⋮	⋮	⋮
Type L	u^{max}_L	f_L

Table. 2 Mission duration for a given type of mission k under ideal conditions, U_k (k=1, 2, ……L)

u_k	frequency, f_k	$P_r(u_k)$
$(u_k)_1$	f_{k1}	$f_{k1}\Big/\sum_{i=1}^{j}f_{ki}$
$(u_k)_2$	f_{k2}	$f_{k2}\Big/\sum_{i=1}^{j}f_{ki}$
⋮	⋮	⋮
$(u_k)_j$	f_{kj}	$f_{kj}\Big/\sum_{i=1}^{j}f_{ki}$

and repair distributions if the mean values of thees distributions can be estimated.

$P_r\{u'\leq u^{max}, T_r(t)>u^{max}|z(t)=1;I(\cdot)\}$ represent the mission reliability. This is the conditional probability that the system will accomplish its mission within the maximum allowable duration of time, u^{max}, without any failure during this period, given the system was operative at the start of a mission, t. Since the system is required to carry out various types of a mission, which are determined based on the distribution given in Table 1, the mission reliability, R, can be expressed by

$$R=P_r\{u'\leq u^{max}, T_f(t)>u^{max}|z(t)=1, I(\cdot)\}$$

$$=\sum_{k=1}^{L}P_r(u'_k\leq u^{max}_k|\text{type}=k)\ \ P_r(\text{type}=k)P_r\{T_f(t)>u^{max}_k|z(t)=1,$$

$$\text{type}=k, I(\cdot)\} \qquad (5)$$

Suppose the distribution of mission duration for a given type of mission k under ideal conditions is given by Table 2 with the following functional form

$$g(u_k) \qquad i=1,2,\cdots\cdots J$$

$$=P_r\{u_k)_i\}=f_{ki}\int\sum_{j=1}^{j}f_{kn}'\ =P_r\{U_k\ =(u_k)_i\}=f_{ki}/\sum_{j=1}^{J}f_{kj} \quad k=1,2,\cdots\cdots L \qquad (6)$$

where u_k=mission duration for a given type of mission k (k=1, 2, ……L) under ideal conditions,

g=probability density function, and

f_i=frequency corresponding to $(u_k)_i$, i=1, 2, ……J

In eq. (5), u'_k denotes the actual mission duration for a given mission type k, which is expected to be longer than u_k due to the effects of the environment and the operator. Here, the environmental conditions are assumed to affect the system perfermance. Also, the operator performance is assumed to be affected by the qualifications of the operator as well as the retraining of the operator.

Once the environmental conditions encountered by the system is classified and each corresponding frequency and the system performance in each condition are observed as given in Table 3, then the average system performance under various environmental conditions, E, is

Table. 3 Environmental conditions, Q, and the system performance under this condition, P

Environment condition, Q	system performance, p	frequency f
1	p_1	f_1
2	p_2	f_2
⋮	⋮	⋮
I	p_I	f_I

$$E=\sum_{l=1}^{I}f_lp_l\ \Big|\sum_{l=1}^{I}f_l \qquad (7)$$

A typical functional form of the operator performance assumed in this study is (see Fig.2) [7]

$$y=y_1+y_2e^{-\beta t}$$

where y=operator performance at time t,

y_1=steady-state performance of the operator,

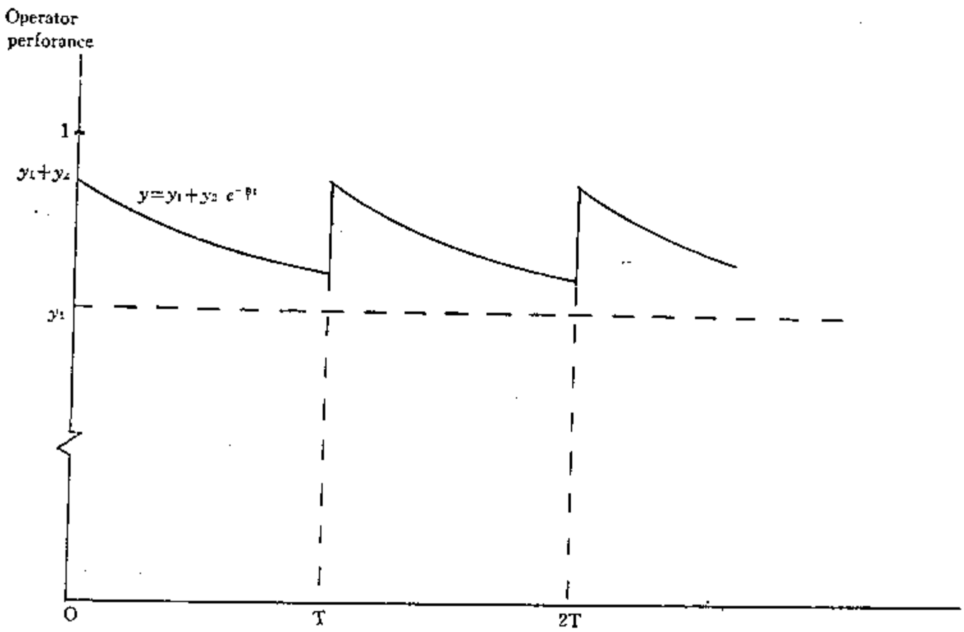
y_1+y_2=initial peak performance of the operator, and

β=decreasing rate of the operator performance.

Here, the retraining of the operator is performed every T time units and every retraining is assumed to bring the operator performance up to the initial peak level. Then, the average operator performance, p, as a function of the qualifications of the operator (y_1, y_2, and β) and the retraining period (T) becomes

$$p=\int_{0}^{T}ydt\ \Big/ T$$

Figure. 2 Operator performance as a function of the operator quality (y_1, y_2, β) and the retraining period (T).



$$=y_1+\frac{y_2}{\beta T}(1-e^{-\beta T})$$

Hence, the actual mission duration for a given mission type k, u'_k, is defined as

$$u'_k=u_k/E\cdot P,\ \ k=1,2,\cdots\cdots,L \tag{10}$$

By the change of variable technique [3], the distribution of u'_k is obtained as

$$\begin{aligned} g(u'_k) &= P_r\{U'_k=(u'_k)_i\} \\ &= P_r\{U_k=E\cdot P\cdot(u'_k)_i\} \\ &= f_{ki}\Big/\sum_{n=1}^{J} f_{kn},\ \ i=1,2,\cdots\cdots J \\ &\qquad\qquad k=1,2,\cdots\cdots L \end{aligned} \tag{11}$$

The distribution of u'_k is shown in Table 4. If U_k is a random variable of the continuous type having pdf $g(u_k)$, the pdf of $g(u'_k)$ can easily be obtained as

Table. 4. Actual mission duration for a given type of mission k, U'_k (k=1, 2, ………L)

$u'_k=u_J/E\cdot P$	frequency, f_k	$P_r(u'_k)$
$(u'_k)_1$	f_{k1}	$f_{k1}\Big/\sum_{i=1}^{J}f_{ki}$
$(u'_k)_2$	f_{k2}	$f_{k2}\Big/\sum_{i=1}^{J}f_{ki}$
⋮	⋮	
$(u'_k)_J$	f_{kJ}	$f_{kJ}\Big/\sum_{i=1}^{J}f_{ki}$

$$g(u'_k)=g\ \{u_k(u'_k)\}\left|\frac{du_k(u'_k)}{du'_k}\right|,\ \ \ k=1,2,\cdots\cdots L \tag{12}$$

Now, the first term in eq. (5) becomes

$$P_r(u'_k \leq u^{max}_k | \text{type}=k) = \sum P_r\{U'_k = (u'_k)_i\}$$

$$= \sum_i \left[\frac{f_{ki}}{\sum_{n=1}^{J} f_{kn}} \right], \; k=1,2,\cdots\cdots L \tag{13}$$

where the sum is taken over all indices i satisfying $(u'_k)_i \leq u^{max}_k$ For the continuous case,

$$P_r(u'_k \leq u^{max}_k | \text{type}=k) = \int_0^{u^{max}_k} g(u'_k) du'_k, \; \text{k}=1,2,\cdots\cdots L \tag{14}$$

The second term in eq. (5) becomes

$$P_r(\text{type}=k) = f_k \Big/ \sum_{m=1}^{L} f_m, \; k=1,2,\cdots\cdots, L \tag{15}$$

The third term in eq. (5) represents the conditional probability that the system will be operative for u^{max}_k, given the system was operative at the start of a mission, t. In general, it is difficult to find the analytic expression for this conditional probability if the failure distribution is nonexponential. Assuming the failure distribution is exponential with the parameter MTBF and the mission starting time t as a renewal point, this conditional probability is approximated by

$$P_r\{T_f(t) > u^{max}_k | z(t) = z(0) = 1, \; \text{type}=k, MTBF\} = exp(-u^{max}_k / MTBF),$$
$$k=1,2,\cdots\cdots L \tag{16}$$

Hence, the mission reliability, R, can be rewritten as

$$R = \sum_{k=1}^{L} \left\{ \left[\sum_i (f_{ki} \Big/ \sum_{n=1}^{J} f_{kn}) \right] \left[f_k \Big/ \sum_{m=1}^{L} f_m \right] \left[exp(-u^{max}_k / MTBF) \right] \right\} \tag{17}$$

Mission effectiveness, ME, therefore is expressed by.

$$\text{ME} = \text{A} \cdot \text{R} \tag{18}$$

where A and R are given by eqs. (3) and (17) respectively.

3.2. Simulation Approach

In section 3.1, analytical approach has a difficulty in handling non-exponential failure distrbution. Simulation approach discussed in this section, however, can handle any probability distribution even with emprical distributions with relative ease. Thus, the simulation approach can yield the pointwise availability and the conditional probability $P_r(T_f(t) > u'_k | z(t) = 1, \text{type}=k, I(\cdot)\}$ including the transient effects based on any observed probability distribution of failure times and downtimes. In this approach, the random variables are generated based on the observed distributions (a) through (g) listed in section 2. Then, the clock times related the missions, failures, and downtimes are computed. The model further computes the mission effectiveness of the system for each mission j, $(ME)_j$, defined by

$$(ME)_j = A_j R_j \tag{19}$$

where A_j is the availability of the system at the start of mission j and R_j is the mission reliability of the system for mission j.

A_j assumes the value 1 or 0 depending upon whether the system is in the upstate or downstate at the start of mission j, t_j, i.e.,

$$A_j = P_r\{z(t_j)\} = \begin{cases} 1 \text{ if } z(t_j) = 1 \\ 0 \text{ if } z(t_j) = 0 \end{cases} \tag{20}$$

To determine R_j, the type of mission j is first generated based on the distribution as shown in

Table 1. The mission duration for mission j for given type of mission k under ideal conditions, $(u_k)_j$, then is generated based on the distribution given in Table 2. Actual mission duration of mission j for given type of mission K, $(u'_k)_j$, is computed by considering the effects of the environment and the operator;

$$(u'_k)_j=(u_k)_j/E_jP_j \tag{21}$$

Here, E_j is the system performance under the environmental condition encountered during mission j, which is generated from the distribution given in Table 3. p_j is the average operator performance during $(u_k)_j$ and is expressed by

$$P_j=\frac{1}{(u_k)_j}\int_{t_j}^{t_j+(u_k)_j} y\,dt \tag{22}$$

Since the retraining at every T time units brings the operator performance up to the initial level, the clock time after every retraining needs to be set back to zero in evaluating P_j. Now,

$R_j=1$ if $(u'_k)_j\leq(u^{max}_k)_j$ and there is no failure during $(u'_k)_j$

0 otherwise

Using the above procedures the mission effectiveness for each mission can be estimated. If we run this simulation for M missions, the overall mission effectiveness of the system, ME, can be estimated by

$$ME=\sum_{j=1}^{M}(ME)_j/M$$

4. Conclusions and Discussion

The mission effectiveness model developed herein is applicable to a system which is required to carry out various types of a mission. The model has been illustrated by a numerical example. In this example, the empirical distributions are obtained for (a), (b), (c), (d), and (f) based on the data taken from the test report on XM198[6]. The subjective assessment is used to obtain the distributions (e) and (g) and the operator performance indices. The results obtained from the analytic and simulation approaches are compared.

One major disadvantage of an analytic approach is that this approach has a difficulty in handling nonexponential failure distribution. Thereby, in general, it is difficult to incorporate a transient effect into a model in an analytic approach. Therefore, the failure distribution is usually approximated by an exponential in an analytic approach. Simulation approach, however, can handle any probability distribution even with empirical distributions with relative ease. A transient effect, hence, is incorporated into the simulation model.

Furthermore, several options have been considered to observe what effects does the modification of input distributions have on the value of the mission effectiveness. By running the computer simulation model in each option, the sensitivity of each data element to the mission effectiveness can be observed. In this way, the critical data element can be identified early, and more efforts to this element can be allocated. If the cost information on each test procedure is available, the optimal test design can be determined by minimizing overtesting on less critical data element. This type of analysis is expected to provide the military agency in charge of operational test and evaluation with an optimal test design and system evaluation.

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