

Design Philosophy in Ship Structure

by

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1. Introduction

This report presents the modern technology on the stress analysis of ship in irregular ocean waves, and the design philosophy using reliability analysis of ship structure under fluctuating stresses.

In old time, the strength calculation of ship was carried out assuming a ship as a beam which is floating statically in the wave of which length is equal to ship length (L) and height is equal to $L/20$ or $1.1 \sqrt{L}$ (ft).

Nowadays, in the calculation of the longitudinal strength of ship, the irregular waves in ocean are used instead of regular waves, of which spectrum is known by the observation of waves in North Atlantic Ocean.

When an actual ship encounters the irregular waves in all directions, the ship rolls, heaves, yaws and surges. As a result of it, not only the vertical bending moment, but also the horizontal bending moment and the torsional moment are induced in the ship hull by the action of the hydrodynamic forces due to waves and the acceleration forces due to oscillation of the ship. The first half of this report indicates the method to predict the fluctuating stresses in irregular waves, the probability of occurrence of the stress and the maximum expected stress during the whole life time of the ship. This procedure of calculation is called the total system analysis of stresses.

Furthermore, in the traditional design procedure, the structure is designed comparing the obtained static stress with the traditional allowable stress which has been determined by many experiences of

ship damages.

Recently this process was improved and the true allowable stress is obtained by assigning the acceptable level of the probability of the fracture. For this purpose the fracture mechanics, the reliability analysis and the concept of the probability of fracture are introduced.

The design using above mentioned modern stress calculation and the concept of probability of fracture is called "direct design procedure". The latter half of this report mainly treats the outline of the direct design procedure and the associated problems.

2. Total System of Analysis on the Longitudinal Strength

2.1. Forward

Evaluation of reliability of a strength of ship structure and establishment of design criteria must be based on a rational and synthetic structural analysis of the ship as well as on the results of experience of ship's operation. For that purpose, first of all it is necessary to estimate accurately the stresses occurring in the structural members of a ship due to fluctuating wave loads during her navigation in ocean, and it is needed to develop such computer program as well directly as well as easily realize such purposes.

As a recent trend, on the other hand, the computer programs of this kind have been developed by the ship classification society of each country and presently the strength of ship structure is being examined synthetically by using such programs ⁽¹⁻⁶⁾. Likewise in Nippon Kaiji Kyokai, a computer program has been

* Nippon Kaiji Kyokai (Ship's Classification Society of Japan). Lecture delivered at the Special Lecture Conference for Ship Structure on April 21, 23, 1977

completed as a part of the research project for the development of a total system of computer programs for ship structural analysis ⁽⁷⁻⁸⁾. Refining on the method of the analysis currently employed on the longitudinal strength of ships, completion of this computer program has made it possible to realize the rational and consistent evaluation on the stresses caused in ship's hull girder.

In this system, firstly the theoretical analysis on the oscillating motions of the object ship under regular waves is performed and the fluctuating wave loads acting on the hull girder are obtained. Next, the fluctuating stresses caused under fluctuating loads in the longitudinal members of the ship are calculated based on the general bending and torsion theory, and then statistical analysis is performed with use of given wave spectrum and observation data on ocean irregular waves.

In the present paper, outline of the system and problems on the analysis are introduced and a discussion is made on the results of a series of calculation which are performed on the longitudinal strength of a bulk carrier by using this program.

2.2. Outline of the System

General process flow chart of the total system of analysis is shown in Fig. 1. The computer program of the system is written for use of the computer FACOM 230-55 of which the core memory is 512KB.

2.2.1. Input Data Processing

The input data processing program as indicated in step I of Fig.1 basically consist of two parts, i.e. data generator for the analysis of ship motion and that for the structural analysis of hull girder.

(1) Data Generator for Ship Motion

Analysis on the motions of ships is based on the strip method. The input data and their processing procedure are as follows.

(a) Necessary input data on the division of the strip is the ship's length only. The hull is equally subdivided into twenty strips between perpendiculars, and if additional data are provided, two strips after-ward A.P. and one strip forward F.P. may be added.

(b) Weight distributions and off-set table of the hull from are given as input data. Then the equilibrium condition of the ship under still water is deter-

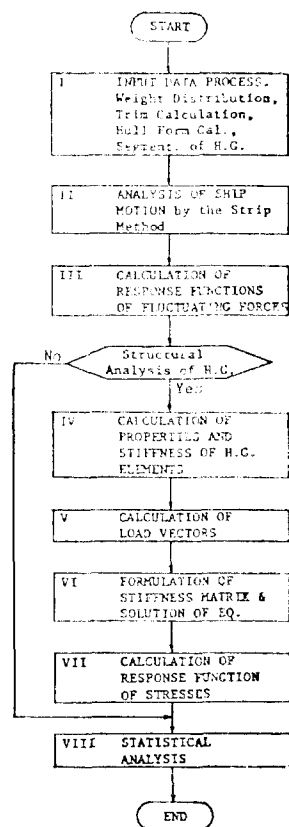


Fig. 1 Process Flow Chart of the System

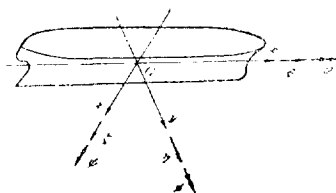


Fig. 2 Notation of Ship Motion and Coordinates Employed

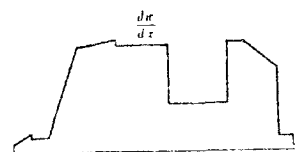


Fig. 3 Longitudinal Distribution of Weight

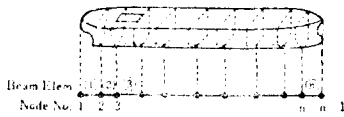


Fig. 4 Beam Element Division of Hull Girder

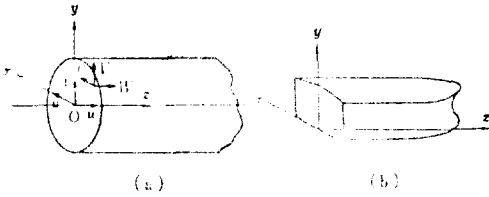


Fig. 5 Notation of Displacement and Coordinates Employed

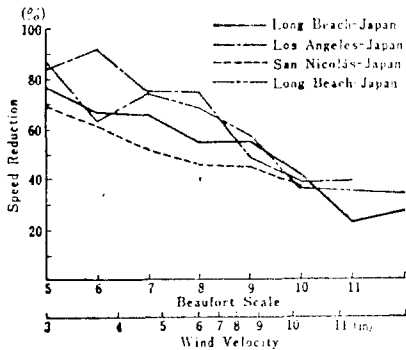


Fig. 6 An Example of Speed Reduction vs. Beaufort Scale

mined by fundamental calculations on the trim.

(c) Each cross section of the strips is generally transformed into Lewis form. As for a specially shaped cross section which can hardly be transformed into Lewis form such as a part of bulbous bow, it is modified to that having equal cross sectional area.

(d) With use of the result of trim calculation and the off-set table, the necessary data are generated of the cross sectional area, draft and the half breadth of water plane of each strip.

(e) Vertical height of the center of gravity KG is given as input data. Moment of inertia for pitching and yawing is generated in the program, while that

for rolling is to be given by manual input data for each strip.

(f) When consider non-linearity of rolling of ship, input data are to be given on the bilge keel.

(2) Data Generator for Structural Analysis

As for the input data for structural analysis, consideration has been so given as it would be sufficient enough only for basic scantling of the hull girder to be transferred from construction drawings. The input data and their processing procedures are as follows.

(a) The hull girder is simulated as a continuous beam with variable cross sections, and the full length of which is divided into certain numbers (maximum 30) of beam element with uniform cross section. The hull girder will be automatically divided by giving, as input data, the number of division and the cross section patterns.

(b) Position or co-ordinate along the longitudinal direction of the ship to indicate the specific cross section, location of the structural members to be analysed are given by frame number.

(c) Cross sectional properties of the beam element are calculated by finite element method, and mesh division in each cross section of the beam element is automatically generated for a given pattern of cross section such as cargo tank part of oil tankers, where only the basic dimensions and scantlings transferred from construction drawing are needed as input data.

(d) As for input data for the mesh division of the cross section in fore and after ends of the ship, the sectional properties of the beam elements are determined by simply multiplying the values for midship part by appropriate ratio depending on the location of the cross section and on existence of the deck opening.

2.2.2 Analysis of Ship Motion

The second step in the system is the analysis on the ship motion based on the strip method as indicated in step II of Fig.1. In this analysis the motion of ship due to each component wave is calculated for given values of the parameters such as ship's speed, angle of encounter with the component wave, length of the wave respectively, and accordingly the

response functions of ship's motion for regular wave of unit wave height are obtained. When considering the non-linear characteristics of rolling angle vs. wave height, the response functions are calculated for each regular wave of certain prescribed values of wave height.

2.2.3. Response Function of Forces

Next step of the analysis is the calculation of response functions of forces caused by the ship motion among regular waves. From the results of the analysis of ship motion, distribution of fluctuating forces acting on each strip are obtained as summation of fluid force and inertia force, and then resultant forces in each cross section are determined by integrating the distributed loads over the whole surface of the strip.

2.2.4. Calculation of Stiffness Coefficient for Structural Elements of the Hull Girder

For the purpose of torsional bending analysis of hull girder, the stiffness matrixes of the beam elements are formulated by calculating their cross sectional properties such as locations of geometrical center of gravity and of shear center, etc.

2.2.5. Calculation of Load Vectors

Equivalent nodal forces are converted from the fluctuating forces obtained in step III, and they are applied as concentrated loads at each joint of the beam element.

2.2.6. Structural Analysis of the Hull Girder

The stiffness matrix of the hull girder is formulated by superposing the matrixes of the beam elements. Then, a matrix equation is introduced with a set of the prescribed load vectors obtained in step V and is solved for unknown vectors of the nodal displacement at the joints. It should be mentioned here that the matrix equation must be solved for a large number of load vectors amounting to the total numbers of the calculations as mentioned in 2.2.2., and therefore, the decomposed stiffness matrix is stored in core memory while the load vectors are read repeatedly 10 by 10 from auxiliary memory of the system.

2.2.7. Calculation of Response Function of Stresses.

Fluctuating stresses at certain prescribed location in the hull girder are calculated from the nodal displacement obtained in the above step, and their re-

sponse function is formulated for any required points.

2.2.8. Statistical Analysis

Statistical Analysis Statistical analysis is then performed by using the results obtained in former steps on the response functions for ship motion, resultant forces in cross section, amplitude of the fluctuating stresses and so on. Short term distributions and probabilities of exceeding a certain prescribed level of each response are obtained with use of given wave spectrum and observation data on ocean waves.

2.3. Outline of the Method of Analysis

2.3.1. Ship Motion and Fluctuating Load

In the analysis of ship motion and fluctuating load, the strip method is applied in the system which has recently come into widely practical use. Ship motions of 6-degrees of freedom in 3-dimensional space are divided into the following three independent groups of oscillating motions:

- (1) heaving (ζ) and pitching (ψ)
- (2) swaying (η), yaying (ϕ) and rolling (θ)
- (3) surging (ξ). (see Fig. 2)

For each group, analysis is made by considering coupling effect of each motion of ship. Since the analysis on the vertical motion of ship has been studied for long time, usefulness of the strip method has been well confirmed by comparing with actual phenomena of ships. In this system, therefore, Fukuda's linear strip method⁽⁹⁻¹⁰⁾ is used for the analysis of heaving the pitching of the ships.

The analysis on the horizontal and rolling motion of ships is more difficult than for vertical one because of the damping resistance due to viscous effect in addition to the wave making resistance as in the case of heaving motion. In this system, the extinction in rolling oscillation is defined by the following relation; $A\dot{\theta} = a\theta_m + b\theta_m^2$ where θ_m is mean amplitude of rolling and the coefficients a and b are derived from Watanabe and Inoue's method⁽¹¹⁾. On the basis of above procedure, rolling resistance is represented as $A\dot{\theta}$ when rolling amplitude is θ . Above representation is linear form, but equation of motion becomes non-linear differential equation since the factor A is function of θ . In this system, Tsai's strip method⁽¹²⁾ is applied with above mentioned non-linear modification. Careful attention must be paid to that, on the influence of ship

speed and damping coefficient further research has to be made so that correspondence to actual phenomena should appear as good as in the case of vertical motion system. As for the third uncoupled surging motion, Motora's method⁽¹³⁾ based on only Froude-Kriloff hypothesis is applied in which added mass has been neglected. On this hypothesis, neither damping force nor restoring force is neglected, and therefore application of this method in the case of long period of wave encounter gives an unsuitable result. In this execution of the analysis, careful attention is paid so as to avoid such condition in each case.

In the next place, outline of the procedure on analysis of fluctuating load in a regular wave is given below.

Fluctuating load per unit length acting on a strip, dF/dx , is represented as the summation of fluid force dF_f/dx and inertia force dF_i/dx . The fluid force dF_f/dx is determined by the form of cross section under water plane, period of wave encounter, ship motion and others, and they are calculated to fairly good approximation except by those errors due to the conversion into Lewis form. As to the inertia force dF_i/dx , however, it is very difficult to obtain exact distribution of weight and moment of inertia about rolling motion in lengthwise direction of ship. Ordinarily, in the case of longitudinal strength calculation, each distributed weight is approximated as a trapezoid. This method is applied in this analysis, piling up each trapezoid to get longitudinal weight distribution. (see Fig. 3)

Vertical and horizontal distribution of weight is needed to get longitudinal distribution of moment of inertia about rolling motion. If such data are not available following estimation will be used as moment of inertia for rolling motion per unit length about given longitudinal co-ordinate x .

$$\frac{di}{dx} = \frac{\frac{dw}{dx} \cdot B(x) \cdot D(x)}{\int \frac{dw}{dx} \cdot B(x) \cdot D(x) \cdot dx} \cdot I_r \quad (1)$$

where $B(x)$ and $D(x)$ are breadth and depth of the section at x respectively, and I_r is moment of inertia for rolling motion about whole ship.

When wave forces in arbitrary cross section, con-

sisting of six components (Shearing force, bending moment etc.) are needed, they will be obtained by integrating above mentioned dF/dx from aft or fore end to given x .

2.3.2. Structural Analysis of Hull Girder

The purpose of the structural analysis of this system is to obtain structural response of the ship's hull subjected to fluctuating loads which are obtained from the ship's motion caused in many regular waves with parameters of ship's speed, angle of encounter and wave length being varied.

It is therefore necessary to solve the stiffness equation of the hull girder for a large number of the load vectors and accordingly in-core data handlings essentially needed for efficient processing of stiffness matrix by electronic computer.

Taking advantage of the reduced degree of freedom, a modified method of the finite element analysis based on the general bending theory for thin walled beams proposed by Kawai⁽¹⁴⁾ has been used in this system. In this method, the ship structure is simulated as a continuous beam with variable cross sections and full length of the beam is divided into certain numbers of beam element with uniform cross section as indicated in Fig. 4 and then displacement method of analysis on the bending and torsion of continuous beam is applied considering deformation due to axial thrust, shearing force and bending as well as warping torsional displacement.

In this method, displacement functions, U , V and W in the transverse section of the beam element are assumed as follows:

$$\begin{aligned} U(x, y, z) &= u(z) - y \cdot \theta(z) \\ V(x, y, z) &= w(z) + x \cdot \theta(z) \\ W(x, y, z) &= w(z) - x \cdot u'(z) - y v'(z) + \theta'(z) w_a \\ &\quad (x, y) \end{aligned} \quad (2)$$

where $u(z)$, $v(z)$ and $w(z)$ are displacement at the centroid of the section in x , y and z direction respectively (see Fig. 5 (a)); while $\theta(z)$ is the angle of rotation. $w_a(x, y)$ is the normalized warping function of St. Venant's torsion problem and is obtained by finite element technique⁽¹⁵⁾ with use of plate element. Then, various sectional properties of each beam element such as St. Venant's torsional stiffness

and warping torsional stiffness are evaluated in this program.

The displacement functions of the beam element along axial direction are assumed as polynomial of the 3rd order as follows:

$$\begin{aligned} u(z) &= a_0 + a_1 z + a_2 z^2 + a_3 z^3 \\ v(z) &= b_0 + b_1 z + b_2 z^2 + b_3 z^3 \\ \theta(z) &= c_0 + c_1 z + c_2 z^2 + c_3 z^3 \\ w(z) &= d_0 + d_1 z \end{aligned} \quad (3)$$

Stiffness matrix of the beam element (14 by 14) can be obtained by using the above mentioned displacement functions in which the degrees of freedom are taken such as lateral and vertical displacements and rotations, angles of torsion and warping and axial displacements at both ends of a beam element. Details of this method are fully explained in the reference⁽¹⁰⁾.

The number of division into the beam element must be appropriate enough so that hull girder which as a continuous beam with variable cross section can suitably be idealized as well as the solution can be obtained with a sufficient accuracy. In addition to these, the dimension of the formulated stiffness matrix must be within such limits that in-core data handling is possible as mentioned in the foregoing. The maximum number of beam element to be divided is 30 in this program. In this case, the size of the stiffness matrix is 217 with band width of 14, and computation with double precision can be performed within 131 KB core memories of electronic computer. The maximum number of load case which can be solved simultaneously in the matrix equation is 10 (the number of load vectors composed of amplitude and phase is 20). The above mentioned number of division of beam element is sufficient to obtain accurate result from practical view point of longitudinal strength analysis of hull girder.

As this analysis is based on a torsional bending theory of a beam, distribution of the shearing stress in the cross section due to external shear force can not be obtained. In this program, although each structural component in the hull girder is assumed to behave as a part of beam element, cross decks and deck girders in ships with large deck openings or

multi-row hatches like container ships deform complexly, not simply as a single unit beam of the main hull girder. Further improvement should be made in the program to evaluate suitable value of the stiffness of hull girder for such kind of ships.

2.3.3 Response Function of Fluctuating Stress

Fluctuating loads obtained by the analysis on ship motions in 2.3.1. are given in terms of amplitude and phase, both of which being varied in each component of the load and at joints of hull girder. It is therefore necessary to analyze the structural response by taking into consideration the phase lag effect. When fluctuating load F (amplitude of and phase of a component) is imposed on joints of hull girder, joint displacement d (amplitude d and phase of a component) is expressed by the following relation, where Λ (a being a component) is inverse of stiffness matrix of hull girder.

$$d = \Lambda F \quad (4)$$

It is otherwise read as

$$\begin{Bmatrix} d_1 \cos(w_e t - \varepsilon_{d1}) \\ d_2 \cos(w_e t - \varepsilon_{d2}) \\ d_3 \cos(w_e t - \varepsilon_{d3}) \\ \vdots \\ d_n \cos(w_e t - \varepsilon_{dn}) \end{Bmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \begin{Bmatrix} f_1 \cos(w_e t - \varepsilon_{f1}) \\ f_2 \cos(w_e t - \varepsilon_{f2}) \\ \vdots \\ f_n \cos(w_e t - \varepsilon_{fn}) \end{Bmatrix} \quad (4')$$

where w_e is circular frequency of ship's encounter with wave, and n is total degrees of freedom. With regard to a component of displacement, it is expressed as follows:

$$d_k \cos(w_e t - \varepsilon_{dk}) = \sum_{j=1}^n a_{ki} f_i \cos(w_e t - \varepsilon_{fi}) \quad (5)$$

Composing up each term in the righthand side of Eq. (5), the amplitude and the phase of a component of displacement at joints of hull girder are expressed as follows.

$$d_k = \sqrt{\left(\sum_{j=1}^n a_{ki} f_i \cos \varepsilon_{fj} \right)^2 + \left(\sum_{j=1}^n a_{ki} f_i \sin \varepsilon_{fj} \right)^2} \quad (6)$$

$$\epsilon_{dk} = \tan^{-1} \left(\frac{\sum_{j=1}^n a_{kj} f_j \sin \epsilon_{fj}}{\sum_{j=1}^n a_{kj} f_j \cos \epsilon_{fj}} \right) :$$

$$\sum_{j=1}^n a_{kj} f_j \cos \epsilon_{fj} \geq 0$$

$$= \tan^{-1} \left(\frac{\sum_{j=1}^n a_{kj} f_j \sin \epsilon_{fj}}{\sum_{j=1}^n a_{kj} f_j \cos \epsilon_{fj}} \right) + \pi :$$

$$\sum_{j=1}^n a_{kj} f_j \cos \epsilon_{fj} < 0$$

Further, from the displacement calculated as above can be obtained stresses in a certain location of a certain cross section of hull girder (axial stress, longitudinal bending stress, horizontal bending stress and warping stress). And furthermore as to the total longitudinal stress obtained by composing these stresses, it can be obtained as well by taking same procedure as above, considering phase difference among components.

2.3.4. Statistical Analysis

From the amplitudes of responses of ship in regular waves (fluctuating stress in longitudinal members, ship motions, resultant forces in cross section etc.) which were obtained by the calculation is calculated by using energy spectrum method. Then, with statistical data of waves in use, the longterm probability of extreme values of these responses is calculated as for wave spectrum, the following ISSC-1970 spectrum⁽¹⁷⁾ is used, and the statistical data of waves on significant wave height and mean wave period given by Walden as shown in Table 1 are used.

$$[f(w, x)]^2 = 0.11 H^2 x_1^{-1} (w/w_1)^{-5} \times \exp[-0.44(w/w_1)^{-4}] 8/(3\pi) \cos^4 x \quad (7)$$

Furthermore, in case of rough sea, artificial speed reduction by maneuvering as well as natural speed down due to increase of resistance are likely to occur, and therefore these speed reductions have also been taken into account on making statistical analysis.

In this case, correlation between Beaufort scale and speed reduction as indicated in Fig. 6 have been used, which have been prepared by the Society investigating

logbook of large exclusive vessels related with their navigating conditions at rough sea.

On the other hand, the correlation between Beaufort scale and significant wave height is given by ISSC-1970 report, and therefore, the responses of ship which have been calculated for several case of ship speed corresponding to a significant wave height, are statistically analyzed.

2.4. Analysis of Longitudinal Strength of Ore Carrier

2.4.1. Series of Calculations on Ore Carrier

A Series of calculation are performed on an ore carrier of 50,000 tons deadweight and principal dimensions are shown in table 2. In these analyses, the maxima exceeded during 10^8 cycles of wave encounter in the North Atlantic in winter, with wave date of Walden⁽¹⁸⁾ in use, are calculated for ship motions, moments, forces and stresses in the fully loaded condition at service speed of ships.

The moment of inertia for the rolling of ships is obtained from the formula given by Kato⁽¹⁹⁾. The N-coefficient for ship motion is modified by the method of Fukuda and others⁽¹⁰⁾, taking account of influence of the speed of ships. In the strength analysis of the hull girders, the hull girders are divided into thirty beam elements in the direction of ship length.

2.4.2. Results and Review

Fig. 7 shows statical shearing force and bending moment distribution in still water in the ship. From Fig. 8 and afterwards show response function of the ship against wave. In these figures, only the variation of the ship motion as well as stress in regular waves are shown, therefore, the total stresses are obtainable by adding the dynamic stresses to the still water stresses. In these figures, χ is the encounter angle of the ship to regular wave, so $\chi=0^\circ$ means the following sea and $\chi=180^\circ$ means the head sea.

The surging motion of the ship ξ , the sway motion η , the heaving motion ζ , the rolling motion θ , the pitching motion ϕ and the yawing motion ψ are shown in Figs. 8, 9, 10, 11, 12 and 13 respectively. In these figures, the amplitudes of motion are expressed in non-dimensional form using wave slope kh against root of ship-wave length ratio $\sqrt{L/\lambda}$ under several parameters of encountering angle χ , where k is $2\pi/\lambda$, λ

Table 1. Wave Frequency in the North Atlantic (According to Walden's Data)

		Wave Period (sec)								Sum over All Periods
		5	7	9	11	13	15	17		
Wave Hieght(m)	0.75	29.91	11.79	4.57	2.24	0.47	0.06	0.00	0.60	40.64
	1.75	72.78	131.08	63.08	17.26	2.39	0.33	0.11	0.77	287.80
	2.75	21.24	126.41	118.31	30.24	3.68	0.47	0.09	0.56	301.00
	3.75	3.28	49.60	92.69	32.99	5.46	0.68	0.12	0.27	185.09
	4.75	0.53	16.19	44.36	22.28	4.79	1.14	0.08	0.29	89.66
	5.75	0.12	4.34	17.30	12.89	3.13	0.56	0.13	0.04	38.51
	6.75	0.07	2.90	9.90	8.86	3.03	0.59	0.08	0.03	25.46
	7.75	0.03	1.39	4.47	5.22	1.93	0.38	0.04	0.04	13.50
	8.75	0.00	1.09	2.55	3.92	1.98	0.50	0.03	0.02	10.09
	9.75	0.00	0.54	1.36	2.26	1.54	0.68	0.20	0.04	6.62
	10.75	0.01	0.01	1.10	0.11	0.10	0.05	0.02	0.00	0.40
	11.75	0.00	0.00	0.03	0.08	0.17	0.06		0.00	0.34
	12.75		0.05	0.00	0.14	0.22	0.06	0.01		0.48
	13.75		0.02		0.07	0.09	0.03		0.01	0.22
	14.75				0.02	0.06	0.02	0.00	0.01	0.11
	15.75	0.00	0.02	0.00	0.01	0.01	0.02	0.01	0.01	0.08
Sum over All Heights		118.97	345.43	358.72	138.59	29.05	5.63	0.92	2.69	1000.00
Whole Year (for All Nine Weather Ships)										(277,497 Obs.)

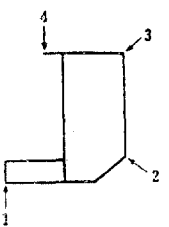
Table 2. Principal Dimensions etc. of the Ore Carrier Analyzed

L_{pp}	210.000	Trim	0.741
B	30.400	d_f	11.826
D	17.400	d_a	12.567
d	12.200	d_s	12.196
Δ	65663.80		
C_b	0.82256	$\boxtimes F$	-1.138
C_p	0.82974		
C_z	0.99168	$\boxtimes B$	2.473
C_w	0.89243	KB	6.418
C_{vp}	0.92201		
KM_L	277.568	$\boxtimes G$	2.439
KM_T	12.551	KG	9.110
GM_T	3.441	K_L/L	0.224
		K_T/B	0.366

is wave length and L is ship length respectively.

The forces and moments acting in the midship-section of the ship are shown in from Fig. 14 to Fig. 19. The horizontal shear F_H , the vertical shear

Table 3. Maximum Expected Stresses in Midship Section for 10^8 cycles

kg/mm ²	point	σ_a	σ_{bx}	σ_{by}	σ_w	σ_n
	1	1.161	10.003	0.000	0.000	11.609
	2	1.161	8.826	5.682	5.940	15.794
	3	1.161	10.166	5.682	1.362	10.604
	4	1.161	10.788	2.029	10.701	13.825

F_v , the axial force F_A , the vartical bending moment M_v , the horizontal moment M_H , the torsinoal moment around shear center are shown in Figs. 14, 15, 16, 17, 18 and 19 respectively. In these figures, the forces and the moments are expressed in non-dimensional form dividing $\rho ghLB$ and ρghL^2B , were ρ is density

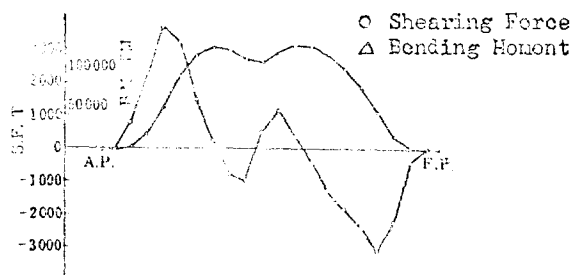


Fig. 7. Shearing Force and Bending Moment in Still Water

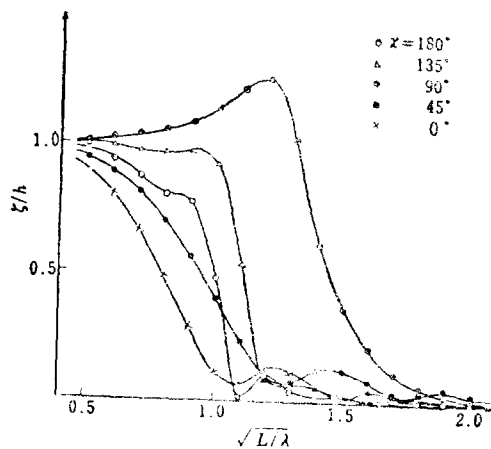


Fig. 10. Heaving Motion

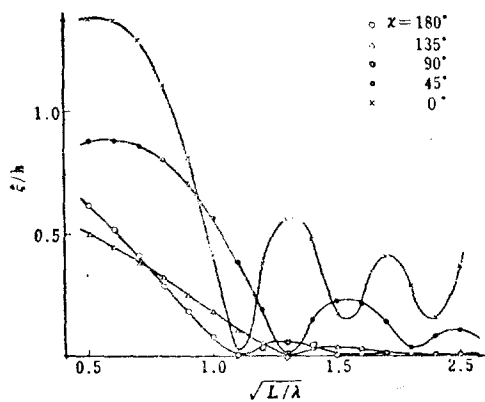


Fig. 8. Surging Motion

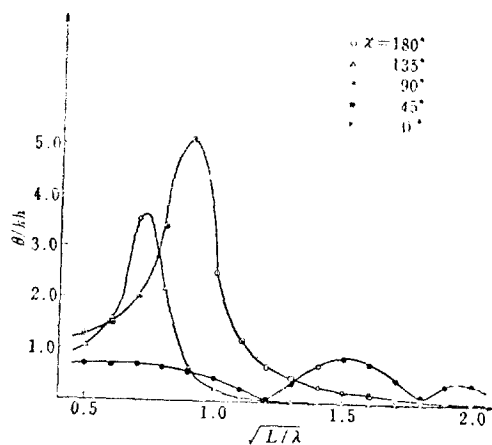


Fig. 11. Rolling Motion

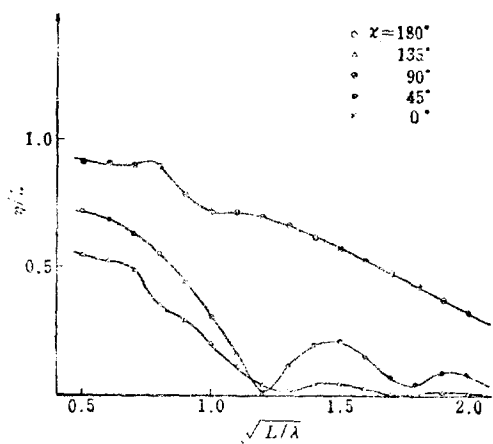


Fig. 9. Sway Motion

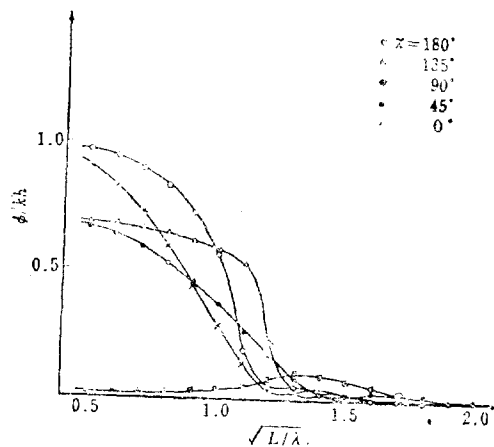


Fig. 12. Pitching Motion

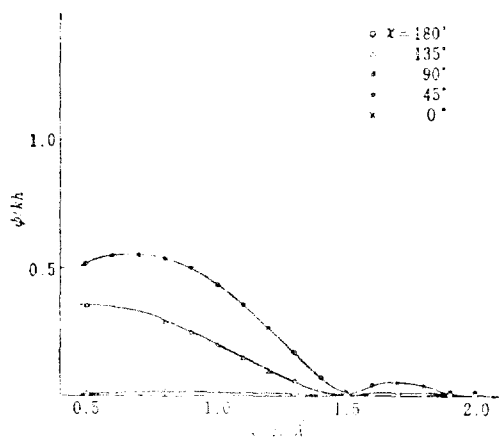


Fig. 13. Yawing Motion

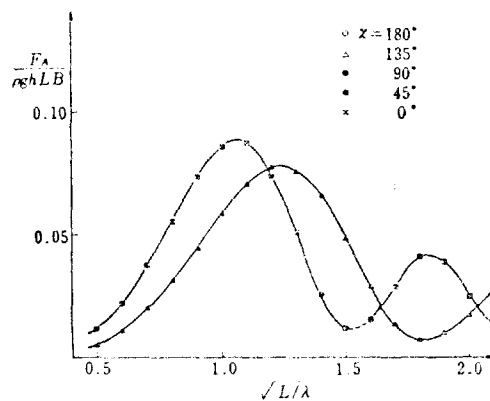


Fig. 16. Response Function of Axial Force Amidship

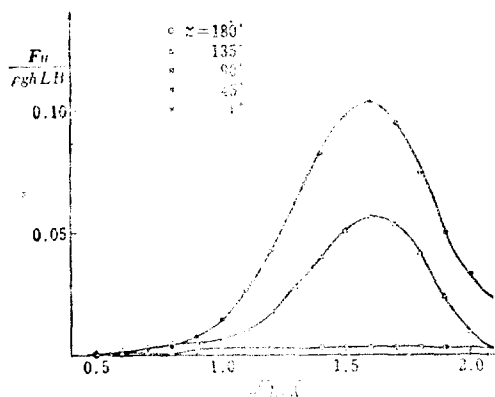


Fig. 14. Response Function of Horizontal Shearing Force Amidship

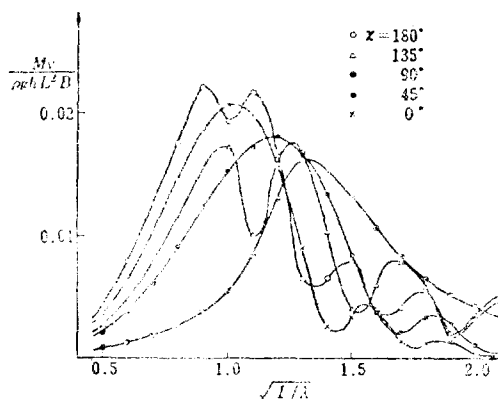


Fig. 17. Response Function of Vertical Bending Moment Amidship

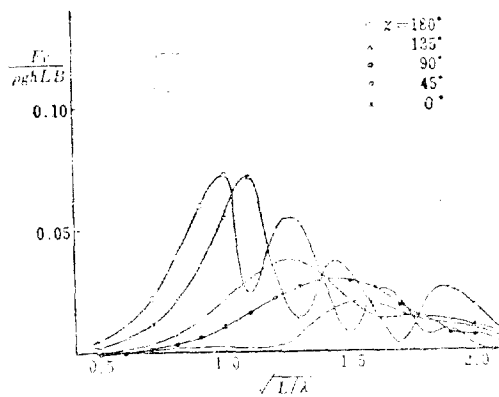


Fig. 15. Response Function of Vertical Shearing Force Amidship

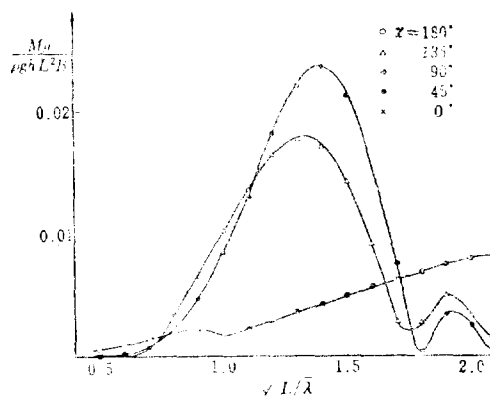


Fig. 18. Response Function of Horizontal Bending Moment Amidship

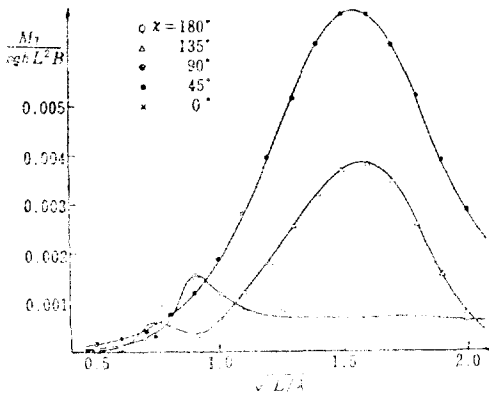


Fig. 19. Response Function of Torsional Moment Amidship Around Shear Center

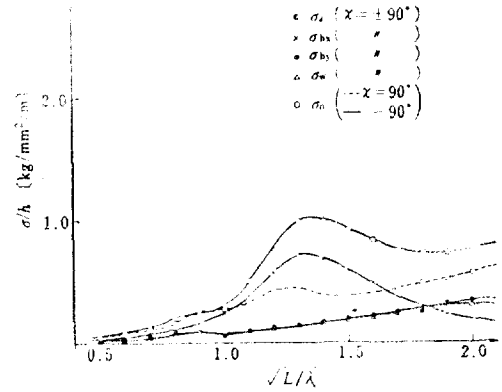


Fig. 22. Response Function of Several Stresses at Bilge Part Amidship for Directional Angle \$90^\circ\$

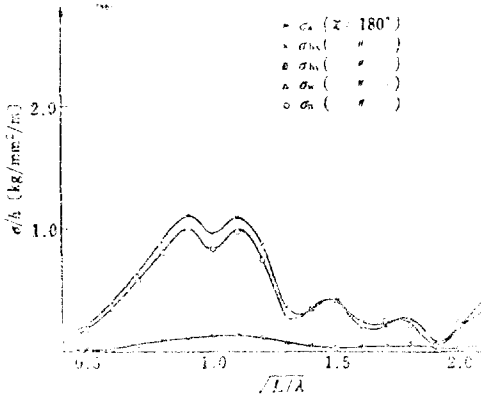


Fig. 20. Response Function of Several Stresses at Bilge Part Amidship for Directional Angle \$180^\circ\$

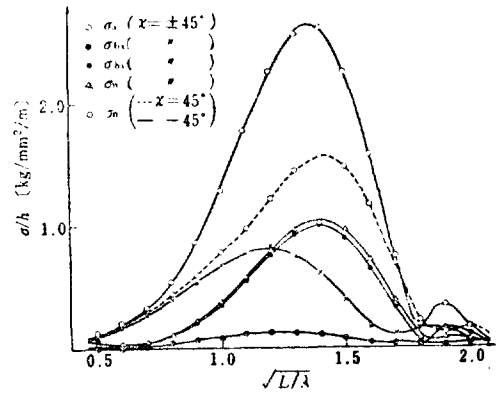


Fig. 23. Response Function of Several Stresses at Bilge Part Amidship for Directional Angle \$45^\circ\$

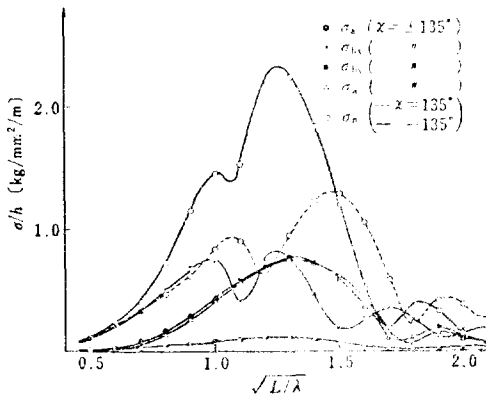


Fig. 21. Response Function of Several Stresses at Bilge Part Amidship for Directional Angle \$135^\circ\$

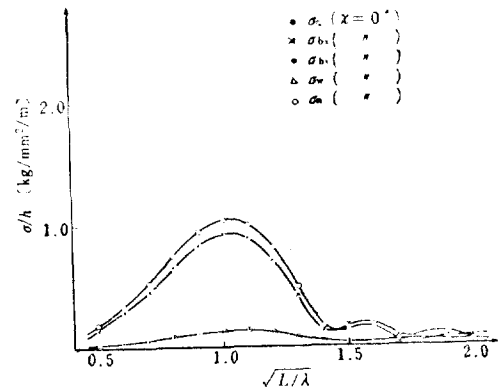


Fig. 24. Response Function of Several Stresses at Bilge Part Amidship for Directional Angle \$0^\circ\$

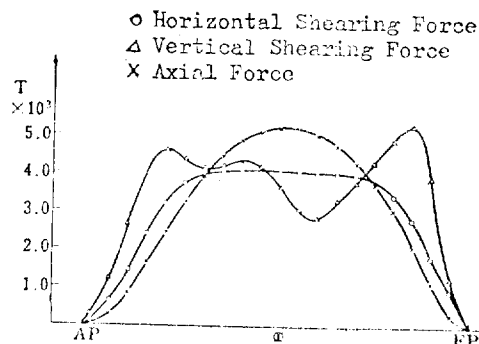


Fig. 25. Maximum Expected Shearing Force and Axial Force for 10^8 cycles

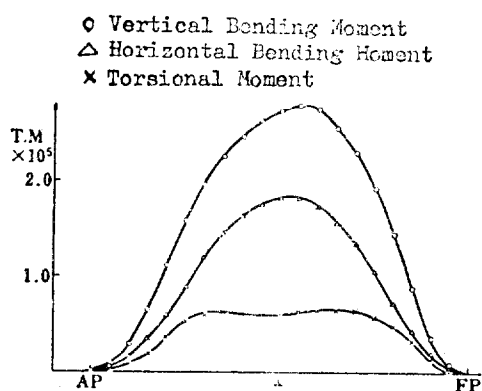


Fig. 26. Maximum Expected Bending and Torsional Moment for 10^8 cycles.

of water, g is acceleration due to gravity and B is breadth of the ship respectively.

Fig. 20, 21, 22, 23, and 24 show the response function of several kind of stress acting at the bilge part of the ship at midship, where σ_a is axial stress, σ_{bx} is vertical bending stress, σ_{by} is horizontal bending stress, σ_w is warping stress and σ_n is total combined axial stress respectively against five values of λ . In these figures some stresses, for instance σ_a , σ_{bx} , σ_{by} , σ_w Fig. 21 show the symmetry against $\pm\lambda$, and the others show unsymmetry.

By performing statistical analysis in irregular waves having ISSC spectrum and frequency distribution in North Atlantic Ocean given by Walden ⁽¹⁸⁾, the maximum expected value of stress is obtainable. Fig. 25 shows the distribution of the maximum expected values of shearing forces and axial forces during 10^8 cycle of wave forces which indicates approximately

○ Maximum Expected Total Stress (Excluding Still Water Stress)
△ Still Water Stress

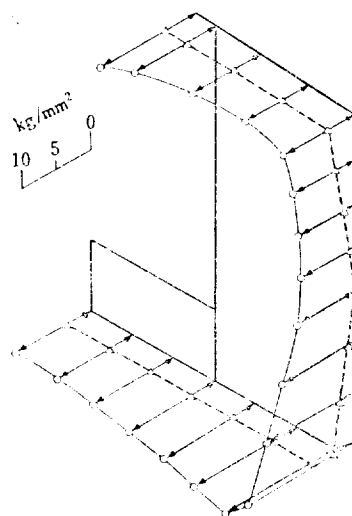


Fig. 27. Maximum Expected Total Stress Amidship in Longitudinal Direction for 10^8 cycles

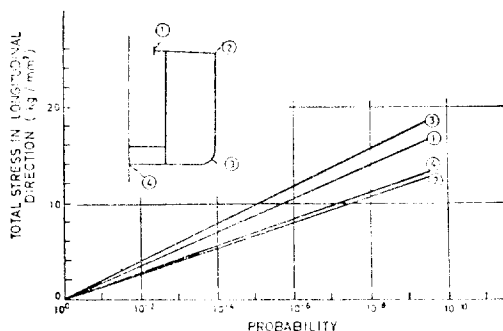


Fig. 28. Probability of Total Stress Amidship in Longitudinal Direction for 10^9 cycles

the total cycle during twenty years of ships whole life. As shown in Fig. 25, the maximum stress is seen in the bilge part. Table 3 shows the maximum expected total stress at several part of the ship during 10^8 cycles.

Fig. 28 shows another example of the maximum expected total stress in a tanker as function of the probability of occurrence.

The total consuming time of the above calculation with FACOM 230-55 was about 2 minutes in CPU time and 10 minutes in elapse time.

3. General Design Procedure

At the 1969 International Ship Structural Congress in Tokyo, the method of the "direct design" was proposed as first time⁽²⁰⁾. Many researches⁽²¹⁻²³⁾ have been carried out since then in this direction

The direct design method is much useful particularly in case of designing U.L.C.C., LNG carriers, container ships etc., for which design people have generally very few experience of fabrication and navigation.

3.1. Direct Design

In the direct design method, the pressure distribution acting on ships hull in regular waves is obtained by using the strip method, then the stress response in ship structural members is obtained through the comprehensive structural analysis and finally the statistical frequency of the stress variations during ship's life is calculated using proper sea spectrum and its distribution. This step is called "system analysis" or total system of stress analysis as indicated by the dotted line in the left upper square block of Fig. 29.

The second step of the direct design method is called "design criteria," which consists of the application of fracture mechanics and reliability analysis. In this step, the probability of failure is obtained using the frequency distribution of initial imperfections or initial deformations in the fabrication process of ships, according to their mode of fracture, i.e., fatigue, brittle fracture, buckling, yielding and ultimate strength of ship structures. When the probability of failure shows satisfactory level, the assumed local scantlings become acceptable.

3.2. Semi-Direct Design and Conventional Design

As the direct design method necessitates generally considerable amount of time and man-power, more simplified method called "semi-direct design method" is needed from view point of its practical use. Several types of procedures may be possible in this case, and after selecting a suitable one, the semi-direct design method will become effective tools for the modernization of the ship design in shipyards.

Besides, these direct and semi-direct design methods, the traditional and conventional design method in

design office should be improved to a more rational one based on the concept of the direct design method, and will also be utilized for practical use.

The flow chart of the direct design method and two examples of the semi-direct design methods (1) and (2) are given in Fig. 29. As a typical shipyard practice, principal dimensions and arrangement of ship are determined in the beginning of design stage by investigating the minimum cost for both building and navigation of the ship under given conditions of type of ship, dead weight capacity, speed, draft and route of ship.

This step is followed by the structural design of the ship. The local scantling is assumed first, and hull weight is estimated secondly, then IMCO-conditions etc. are checked. If the answer is OK, the analysis of structural systems and design criteria are set up according to the direct design method as stated previously.

When the obtained probability of failure remains within an acceptable level, the other combination of the local scantling is compared with the obtained scantling in order to optimize the weight or cost of ship.

3.3. Examples of Semi-Direct Design

With regards to the semi-direct design method (1), the stress obtained by the system analysis previously mentioned is compared with the newly defined "allowable stress". This stress, however, is not always the same as the traditionally used allowable stress which has generally no definite meaning in statistical sense.

The value of the allowable stress defined here is generally derived from the calculation of probability of failure in typical structure, as shown in the dotted thick arrow in Fig. 1. In the case of fatigue, for instance, when the allowable level of probability of failure is given, the maximum expected stress and its frequency distribution (demand) can be determined by calculating cumulative damage based on S-N curve of fatigue tests with scatter band concept on typical welded structure of ships (capability), e.g. butt welded face plate of the transverses of tankers. As the results, the maximum expected value of stress is defined as the allowable stress in fatigue. The similar procedure can be applied for buckling, collapsing and so on.

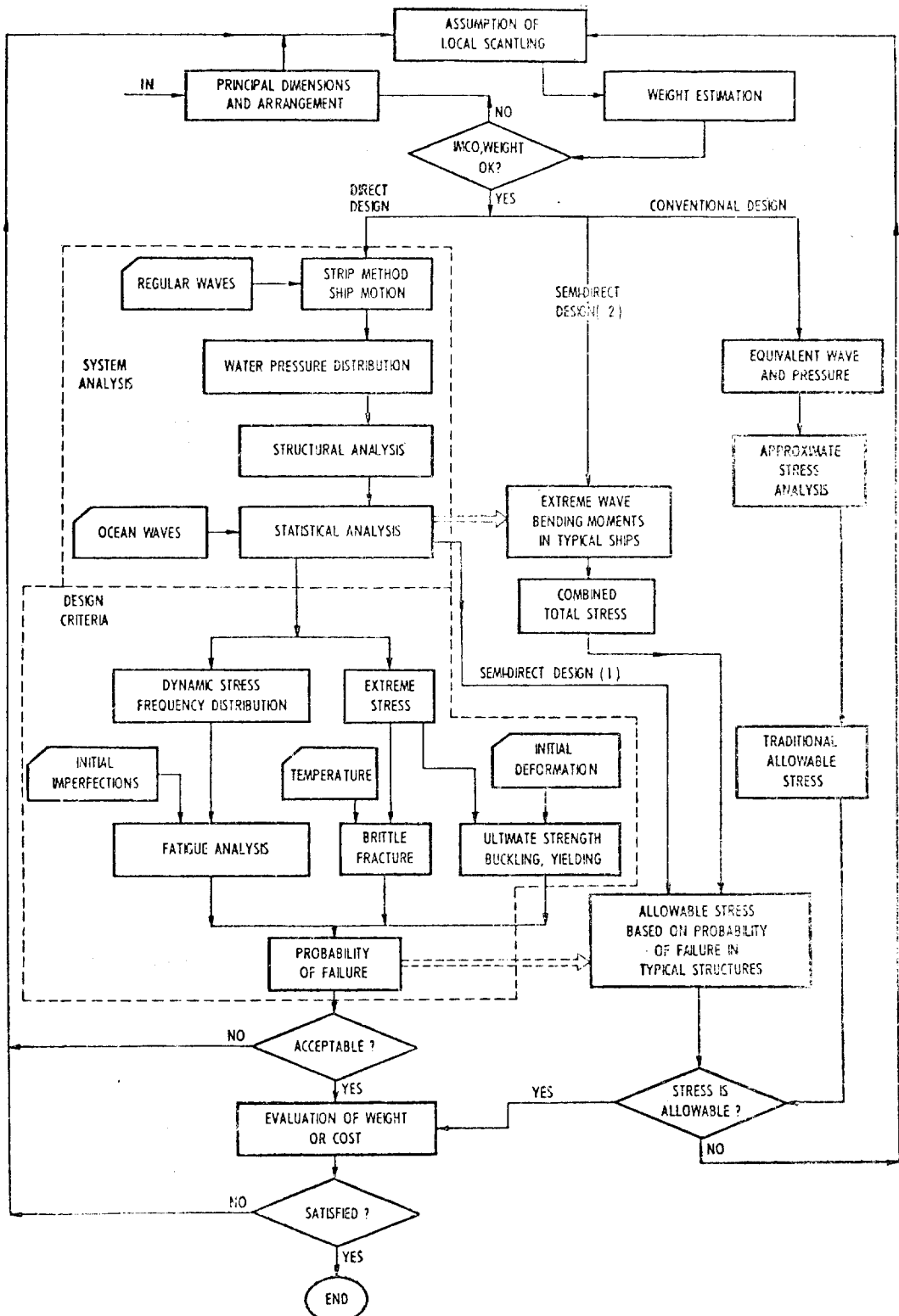


Fig. 29. Flow Chart of General Design Procedure

In Fig. 29, another example of the semi-direct design method (2) is also shown. In this method, the statistical extreme value of wave moment and shear force are given without performing detailed calculation of the system analysis.

In the case of longitudinal strength, for example, the empirical formulae of non-dimensional statistical extreme values of vertical bending moment, horizontal bending moment as well as torsional moment are prepared previously from many calculated examples of the system analysis for typical ships. These non-dimensional values are expressed in terms of type of ship, ship length, block coefficient, etc. The approximate statistical moments of a given ship are obtainable using these empirical formulae. These process is shown in Fig. 29 by dotted thick arrows.

Since the extreme values of the above-mentioned components of moments in this case are given independently, their phase lag is unknown. Therefore, the combined stress from these components of moments is obtained approximately as a square root of a sum of squares of the individual stress components by assuming their phase lag is random. According to the reports⁽²⁴⁾, the above mentioned combined stress showed satisfactory approximation to the exact value which is calculated by taking into considerations of the phase lag.

The combined stress values thus obtained may be compared with the previously mentioned allowable stress as in the case of the semi-direct design method (1).

It should be mentioned here that further investigations are needed on the detail of the semi-direct design method, and also it is necessary to establish the rational level of the probability of failure by considering total economy of the ship construction and its maintenance as well.

4. Failure Probability of Fatigue

4.1. General

In this article, the outline of the fatigue design is discussed on the basis of statistical concept.

For the design criteria based on the fatigue strength of ships, first of all, initiation of fatigue crack is to

be checked in the primary structural members, such as deck, side shell plating, water-tight bulk-heads, webs, girders and transverses, whereas in the case of the secondary members, such as stiffeners, brackets and so on, the crack propagation becomes of practical importance from view point of their structural functions.

The fatigue strength of the structures should be determined, ideally speaking, from the results of a simulated fatigue test on the structural components under programmed load conditions, but now it may be obtained conventionally by using SN-curves of welded butt joint specimens under zero-tension load, taking into account the effect of initial imperfections and of corrosive atmosphere, etc.

4.2. Capability

The report of the Committee 11 of ISSC Hamburg⁽²⁵⁾ showed a proposed fatigue design curve for both sound and defective butt joints, which indicates the effect of weld defect severity as well as the effect of porosity of upper range and lower range of quality.

Fig. 30(a) is a typical fatigue curve of welded butt joints under zero-tension load⁽²⁶⁾, showing the scatter mainly due to the difference of microstructure in weldment. In the case of structural members of ships, wider scatter band will appear in the diagram due to the imperfection and misalignment of the weldment.

The fatigue strength of butt weld joint in ship structure is given by a logarithmic linear relationship. This can be expressed by the following equation, where N denotes the failure life of crack initiation, σ_a denotes the amplitude of applied stress (one half of stress range) and A & B are coefficients in the zero-tension test, respectively.

$$\log_{10} N = B - A \log_{10} \sigma_a$$

In the case of alternating load with mean stress σ_m (i.e. still water stress in ships), the fatigue life is determined approximately by applying Goodman's method as shown in Fig. 30(b), where the correction due to the mean stress is made at any specified numbers of cycle N_1 and N_2 .

Due to the scatter of the fatigue test results, the coefficients A and B are regarded as random variables, of which the distribution is assumed normal (the

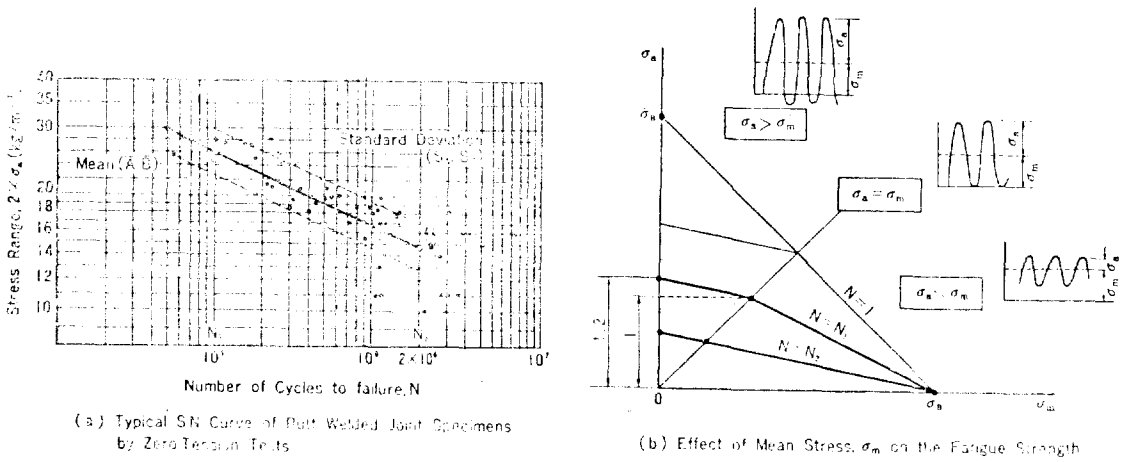


Fig. 30. Fatigue Strength of Butt Welded Joints

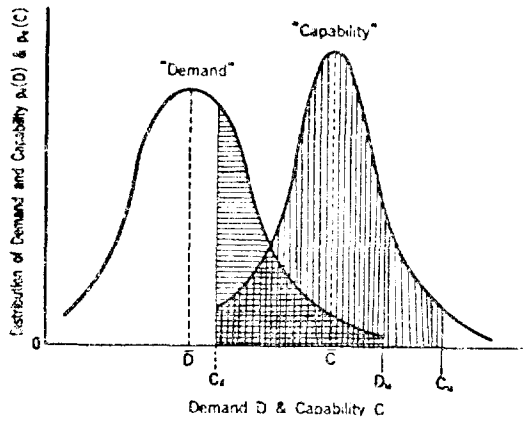


Fig. 31. Probability of Fatigue Crack Initiation

mean values are A and B , the standard deviations S_A and S_B , respectively). These values are determined as shown in Table 1, considering the proposed fatigue curve⁽²⁵⁾ and the test data⁽²⁶⁾, with further modifications for the effect of misalignment at welded joints. It is also assumed that the fatigue life of crack initiation N is approximated by one half of the fatigue life of fracture given by the test results of Fig. 30 (a).

4.3. Demand

In the "demand" of the failure probability analysis, two independent variables are considered; namely the structural response in still water and that caused among waves. The former one is directly related to loading condition of ships, of which the statistical

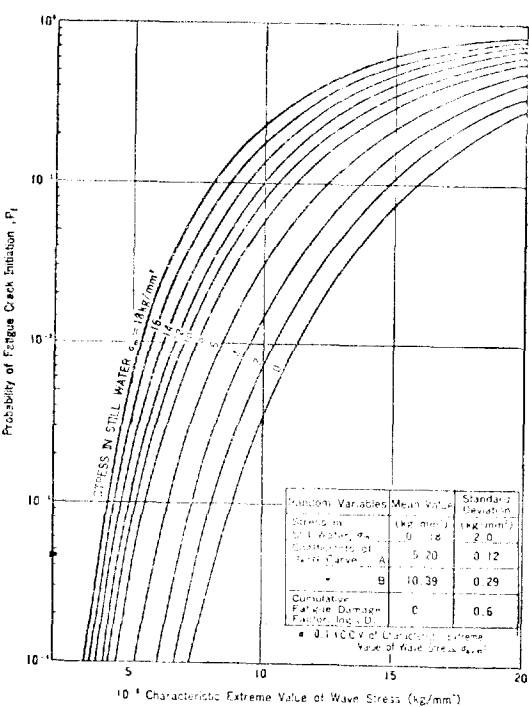


Fig. 32. Probability of Fatigue Crack Initiation at Butt Welded Joint during for 10^8 cycles

distribution is assumed to be normal (the mean value $\bar{\sigma}_m$ and the standard deviation S_{σ_m}). The influence of the still water stress σ_m on the fatigue strength of the structural members of ships has been considered as mentioned above.

On the otherhand, the long term distribution of

the amplitude of fluctuating stress σ_a is calculated by means of a system analysis computer programme, of which the frequency of occurrence $p(\sigma_a)$ is approximated by the following exponential distribution.

$$p(\sigma_a) = \frac{1}{\lambda} \exp(-\sigma_a/\lambda)$$

where, λ is the mean value in the long term distribution of σ_a . When considering the fatigue failure in ship's life, being assumed say 20 years of 108 encounters of the fluctuating stress, the following characteristic extreme stress $\sigma_{a,ext}$ is introduced.

$$\sigma_{a,ext} = \lambda \cdot \ln 10^3$$

This characteristic extreme stress $\sigma_{a,ext}$ is regarded as a random variable due to variation of sea zone and weather conditions in each ship, which belongs to the same population. It is then assumed that the distribution of $\sigma_{a,ext}$ is logarithmic normal distribution of the mean value $\sigma_{a,ext}$ and the coefficient of variance α . When the "demand function" D is defined such that

$$D(X_i) = \log_{10} \sigma_{a,ext}$$

the distribution of the demand $p_d(D)$ can be determined by the mean value \bar{D} and the standard deviation S_D .

4.4. Cumulative Damage

The cumulative fatigue D , as defined by Miner's law, of the welded butt joint of ship structure subjected to 10^8 pulsating wave loads is given by the following equation.

$$D = \int \frac{10^8}{N} \frac{p(\sigma_a)}{\sigma_a} d\sigma_a$$

When assuming that fatigue limit does not exist in the welded joint due to effect of corrosive atmosphere, then it follows that⁽²⁷⁾

$$D = 10^{(8-B^*)} \cdot \left(\frac{\sigma_{a,ext}}{\ln 10^8} \right)^{A^*} \cdot \Gamma(A^*+1)$$

where Γ is gamma function, A^* and B^* are the coefficients of σ_a-N relationship for alternating load with mean stress σ_m .

Fatigue failure criterion is expressed in term of cumulative fatigue damage factor.

$$D = D_f$$

The cumulative fatigue damage factor D_f is regarded as a random variable, on the basis of subjective uncertainty concept, of which the distribution is assumed to be of logarithmic normal (mean value $\log_{10} D_f$ and standard deviation $S_{\log_{10} D_f}$). It should be mentioned here that this distribution is to be determined statistically by relevant data of model tests on welded joints under simulated load condition. However, few informations are available on this subject, and therefore it is assumed that D_f varies from 0.25 to 4.0 (mean value being 1.0 and $S_{\log_{10} D_f} = 0.6$). To examine the results of this analysis, further computation is also made for $D_f = 0.4$, which is obtained from the test data on steel round bar specimens⁽²⁹⁾.

4.5. Capability Analysis

The "capability function" C on the fatigue strength of butt welded joints of ship structure is defined as follows⁽²⁸⁾.

$$C(Y_i) = \frac{1}{A^*} \{ \log_{10} D_f + B^* - 8 - \log_{10} T \\ (A^*+1) \} + \log_{10} (L_n 10^8)$$

where, the random variables are taken such that $Y_1 = \sigma_m$, $Y_2 = A$, $Y_3 = B$ and $Y_4 = \log_{10} D_f$.

In order to formulate the statistical distribution of the capability, it is assumed that C is approximated by a linear function $C^*(Y_i)$ of random variable Y_i 's.

It is then ready to determine the distribution of the capability $P_c(C)$, of which the mean value is \bar{C} and the standard deviation is S_c .

4.6. Probability of Fatigue Failure

With use of the statistical distribution of both "demand" and "capability" in ship structural members, the fatigue failure probability of butt welded joint P_f can be obtained by the following formula.

$$P_f = \int_{C_l}^{C_u} \int_{D_l}^{D_u} p_d(D) \cdot p_c(C) \cdot dD \cdot dC$$

where D_u is upper bound of the demand, and C_l as well as C_u are lower and upper bound of the capability, respectively, as shown in Fig. 31.

Then, the numerical analysis is performed by using a computer programme, of which the results are

shown in Fig. 32. This diagram illustrates the probability of initiation of fatigue crack at welded butt joint P_f for a given characteristic extreme wave stress $\bar{\sigma}_{a,ext}$ during 10^8 wave encounters and mean still water stress $\bar{\sigma}_m$, respectively.

Furthermore, a sensitivity analysis is made to examine the influence of variations in the mean value as well as the standard deviation of the random variables $\sigma_{a,ext}$, σ_m , A , B and D_f on the results of the failure probability. Table 4 summarizes the above mentioned results as compared with those for the standard case given in the extreme right column. It can be seen from this comparison that the theoretical estimate by this analysis gives a significant figure of P_f , except for the cases of low stresses both in still water and among ocean waves.

Table 4. Effect of Variations in Random Variables on the Estimate of Fatigue Failure Probability, P_f

Variations in Random Variables		$S\sigma_{a,ext}$	$S\sigma_m$	S_A	S_B	B	$\log D_f$	Standard Case
$\sigma_{a,ext}$ (kg/mm ²)	σ_m (kg/mm ²)	($\alpha=0.3$)	(10kg/mm ²)	(0.48)	(0.58)	(10.69)	(-0.4)	$\alpha=0.1$ $A=5.20$ $B=10.39$ $10\log D_f=0$ $S\sigma_m=2\text{kg/mm}^2$ $S_A=0.12$ $S_B=0.29$
5	2	2.39×10^{-4}	3.57×10^{-4}	2.81×10^{-5}	1.31×10^{-4}	3.15×10^{-7}	3.77×10^{-5}	3.01×10^{-6}
	8	3.03×10^{-3}	3.59×10^{-3}	1.06×10^{-3}	2.53×10^{-3}	9.93×10^{-5}	2.77×10^{-3}	3.80×10^{-4}
15	2	1.56×10^{-1}	1.90×10^{-1}	1.38×10^{-1}	1.61×10^{-1}	4.96×10^{-2}	2.55×10^{-1}	1.12×10^{-1}
	8	3.18×10^{-1}	3.38×10^{-1}	3.15×10^{-1}	3.32×10^{-1}	1.80×10^{-1}	5.21×10^{-1}	3.04×10^{-1}

Table 5 calculated Objective and Total Uncertainties

Mode		$\bar{\sigma}_u$ (tsi)	$S=0.1\%$ lower probability σ_u (tsi)	Objective $V_{si}\%$	Total $V_s\%$
FRIGATE hogging:	Upper deck yield	22.2	17.5	6.5	7.1
	Bottom beam-column	18.6	13.0	7.1	10.1
FRIGATE sagging:	Bottom yield	22.2	17.5	6.4	7.1
	Upper deck strut-penel	11.9	8.6	5.7	9.2
	Superstructure deck grillage instability	18.9	13.5	4.2	9.5
TANKER hogging:	Deck yield	16.5	12.0	8.6	9.1
	Bottom beam-colum	15.0	9.9	8.8	11.3
TANKER sagging:	Bottom yield	16.5	12.0	8.6	9.1
	Deck strut-penel	15.4	10.0	8.8	11.3
	Deck grillage instability	16.5	11.0	8.6	11.2

* Assuming 50 ft draught (15.2m) and middle tank empty.

note, 1 tsi=15.44N/mm²

Recently the effect of the distribution form of the long-term stress distribution was shown by Bennet⁽³⁰⁾. If σ_a is not exponential but Weibull, which means that

$$p(\sigma_a) = \frac{k}{\lambda} (\sigma_a/\lambda)^{k-1} \cdot \exp\{-(\sigma_a/\lambda)^k\}$$

then the cumulative damage is given as

$$D = 10^{8-B^*} \{ \sigma_{a,ext} / (\ln 10^8)^{1/k} \}^{A^*} \cdot \Gamma(A^*/k+1)$$

The results shows D will be more than three times higher for $k=1.25$ than for $k=1$.

It is therefore concluded that the fatigue probability on the structural members of ships can be well estimated for practical design purpose by using the results obtained from the present analysis.

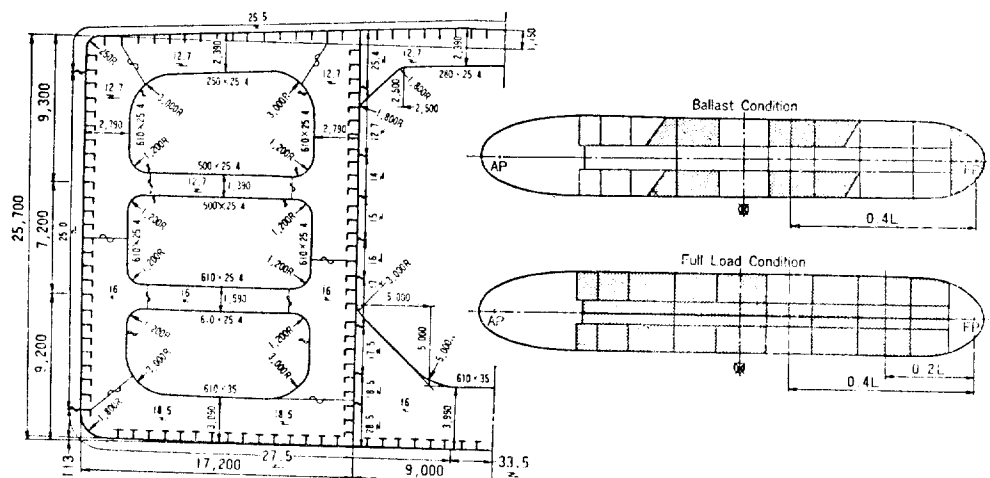


Fig. 33. 240,000 DWT Oil Tanker

5. Application in Longitudinal Strength of Ships

5.1. General

Although a complete method of the direct design procedure in ship structures has not been so far established yet, several comprehensive studies have been carried out on the longitudinal strength of ships from view point of the direct design procedure based on the probability concept of the reliability analysis.

At the stage of the structural design of ships, the longitudinal strength is usually checked by considering still water loading and quasi-static wave load as well as slamming and springing phenomena and thermal effect due to temperature difference of the hull girder in air and sea water. Among these, as for the slamming load, a basic study was made by Lewis⁽²³⁾, and its application of the statistic considerations on the design procedure is discussed in ISSC 1973 Repors of Committee 10⁽³¹⁾. In this field, however, still remains many problems which have to be investigated by progressive research works and with further data accumulation.

This article provides typical examples of the direct design procedure based on the reliability analysis,

where the quasi-static wave load and the still water bending of hull girder are taken into considerations; and then a brief discussion is made on the semi-direct design method, which is approximate but more simple, realistic and practical for design use in shipyards.

5.2. Analysis Example 1

A series of investigations have been made on the reliability of the longitudinal strength of a naval frigate, a large tanker^(32,35), and of marinertype ship⁽³³⁾ by Mansour and Faulkner, finding out the maximum expected moment of those vessels from the results of system analysis and evaluating the failure probability in a 20 year ship life.

The principal dimensions of the frigate are 360.0ft × 41.0ft × 28.9ft × 12.0ft and the displacement is 2800 tons, while those of the oil tanker are 775ft × 105.5ft × 62.5ft × 47.0ft and the displacement is 91,200 tons. The ship's route covers the North Atlantic, the Mediterranean, Indian Ocean and the West Pacific Ocean.

As of the "demand" in the reliability analysis, the most probably extreme value Z_n of the total bending moment distribution is given by the following formula⁽³⁴⁾.

$$Z = m_o + \lambda \cdot \log_{10} n$$

where, m_0 is still water bending moment, λ is the total expected value of the bending moment among waves during life time and n is the number of periods of time or number of encounters.

Capability analysis is then made for each ductile mode of failure, including tension yield in upper deck (hogging) or in bottom (sagging), as well as strut-panel, beam-column and grillage instability in deck or bottom structures.

5.3. Factor of Uncertainty

Several kinds of uncertainties usually exist in design process, and they are divided, into "objective" according to Ang's suggestion. The objective uncertainty is measured and include as-built dimension, difference in material properties and manufacturing imperfection, etc. The subjective uncertainty, on the other hand, cannot be measured and are associated with lack of perfect knowledge and with assumptions regarding load and response of ship structure. Then, the total uncertainty is obtained as the root mean square of these two uncertainties, and the calculated values on the frigate and the tanker examples are as illustrated in Table 5.

5.4. Failure Probability

When assuming the strength or the capability of the ship as a normally distributed one, the risk of failure P_f are given by the following formula.

$$P_f = 1 - \frac{1}{\delta \sqrt{2\pi}} \int_{-\infty}^{\infty} [1 - \exp\{-(\zeta + m)^2\}]^n \cdot \exp\left\{-\frac{1}{2}\left(\frac{\zeta}{\delta}\right)^2\right\} \cdot d\zeta$$

The integration extends over the possible range of total bending moment. The nondimensional parameters are defined for all possible mode of failure by

$$m = (\bar{S} - m_0)/k, \quad \delta = |S_s/k|$$

where k , 1 are parameters in the Weibul distribution, and \bar{S} is the mean of the strength, S_s is the standard deviation of the strength, respectively.

The result of the safety analysis on the frigate are summarized as follows:

- (a) failure by tensile yield in upper deck (hogging)
..... $P_f^a = 4 \times 10^{-8}$
- (b) failure by tensile yield in bottom (sagging)

- $P_f^b = 1 \times 10^{-7}$
- (c) strut-panel failure in upper deck (sagging)
..... $P_f^c = 5 \times 10^{-4}$
- (d) beam-column failure in bottom (hogging)
..... $P_f^d = 2 \times 10^{-5}$

It can, therefore, be shown that the bounds on the probability of failure are:

$$\begin{aligned} \max(P_f^a, P_f^b, P_f^c, P_f^d) &\leq P_f \leq \\ &(P_f^a + P_f^b + P_f^c + P_f^d) \\ 0.00050 &\leq P_f \leq 0.00052 \end{aligned}$$

When the effectiveness of deckhouse is considered, the probability of failure is considerably less:

$$P_f \simeq 0.00001$$

The long-term probability of failure of the tanker is computed using a similar procedure as for the frigate. In full load condition, the probability of failure during lifetime is found to be of the order 1.3×10^{-8} , whereas in ballast condition about 6×10^{-11} . For 50% full load 50% ballast operational condition, the final probability of failure is of the order about 1×10^{-8} .

Furthermore, sensitivity analysis of the strength parameters for strut-panel deck failure indicates that

1. decreasing the strength mean 10% increases the probability of failure to 1×10^{-3} for the frigate, and to 5×10^{-8} compared with 1×10^{-8} for the tanker.
2. increasing the c. o. v. by 25% increases the probability of failure to 5.4×10^{-4} for the frigate, and to 1×10^{-5} , three orders of magnitude higher for the tanker.

5.5. Analysis Example 2

This part presents the second example of the analysis on the failure probability of the longitudinal strength members of oil tankers of oil tankers, which has been discussed in the Japan Shipbuilding Research Association⁽²⁸⁾.

In this reliability analysis, the demand is basically defined as a parameter R , which represents either loads or the structural responses of the Ships. In general, of random variables X_i 's, such as stress components in structural members or the resultant forces and moments in hull girder sections, etc.

$$R = R(X_1, X_2, \dots)$$

Statistical distributions of each random variable X_i is readily obtained by the total system analysis computer program, which has been developed for the longitudinal hull girder analysis of ships⁽³⁷⁾.

The system analysis provides the long term distribution (or the frequency of occurrence) of the among-wave response X_i approximated by the following exponential function,

$$P(X_i) = 1/\lambda_{xi} \cdot \exp(-X_i/\lambda_{xi})$$

where, λ_{xi} is the mean value of X_i .

When considering a structural failure during ship's life, being assumed say 20 years of 10^8 encounters of waves, the following characteristic extreme value $X_{i,ext}$ is introduced in the analysis.

$$X_{i,ext} = \lambda_{xi} \cdot \ln 10^8$$

Since the structural failure by yielding, buckling and plastic collapse of the members is governed by the maximum value of the response, $X_{i,max}$ it is necessary to consider the statistical distribution of the extreme value. In this analysis, it is assumed that the distribution of the extreme value is approximated by a normal distribution, of which the mean value is $X_{i,max}$ and the standard deviation $S_{xi,max}$ given by the following relations⁽³⁸⁾.

$$\bar{X}_{i,max} = X_{i,ext} + \gamma \lambda_{xi}$$

$$S_{xi,max} = \pi / \sqrt{6} \cdot \lambda_{xi}$$

where, γ is Euler constant ($=0.5772$).

It is then assumed that the maximum value of the structural response R_{max} occur when $X_i = X_{i,max}$, simultaneously, and the demand D is defined as follows.

$$D = R_{max} = R(X_{i,max}) \quad (i=1, 2, \dots)$$

A normal distribution of the demand $P_d(D)$ is assumed with the mean value \bar{D} and the standard deviation S_D , respectively.

$$\bar{D} = R(\bar{X}_{i,max})$$

$$S_D^2 = \sum (\partial R_{max} / \partial X_{i,max})^2 \cdot S_{xi,max}^2$$

In the case of the fatigue failure analysis, however, the cumulative fatigue damage concept is applied instead of the maximum value of the response, and the characteristic extreme value of the wave stress is selected as the demand in the analysis.

The capability function C , on the other hand, is formulated in terms of random variables Y_i 's, such as geometrical scantlings of structural members i.e. thickness of plate, initial deformations of the components and the material properties including the yield stress, characteristic values of S-N diagram, cumulative fatigue damage factor, etc., of which the distribution $P_c(C)$ is assumed to be of normal one⁽³⁹⁾.

$$C = C(Y_i)$$

Then, with use of the probability distribution of both the demand and the capability of ship structures, the probability of failure P_f can be calculated by the following formula.

$$P_f = \int_{C_l}^{C_u} \int_{D_l}^{D_u} P_d(D) \cdot P_c(C) \cdot dD \cdot dC$$

where, D_u is the upper limit of the demand, while C_u and C_l is the upper and the lower limits of the capability, respectively.

Numerical calculations are made on the following failure modes of the longitudinal strength members of a 240,000 DWT oil tanker, of which the principal dimensions are 304M00 \times 52M4 \times 19M8, respectively. The midship section of the ship and the loading condition are shown in Fig. 33.

5.6. Plastic Collapse of Hull Girder

The total collapse of hull girder due to longitudinal bending of the ship or jackknifing is considered at first. When the hull girder consisting of the longitudinal strength members of the ship, which is subjected to bending moment M and axial thrust T , fails into total plastic collapse in its certain cross section, it is assumed that the following failure criterion is generally satisfied.

$$R = \{(M/Z_p)^2 + \alpha^2 (T/A_e)^2\}^{1/2} \geq \sigma_y$$

where, Z_p and A_e is the fully plastic modulus and the effective cross-section area of the hull girder, respectively and α is an interaction coefficient depending upon the shape of the cross section.

Then, the demand and the capability is defined such that

$$D = R_{max}(M, T)$$

$$C = \sigma_y$$

In this analysis, an assumption is made that the total collapse of the hull girder does not accompany local buckling failure of deck or bottom plating of the ship within elastic range of the material. The results of the analysis are shown in Table 6, where the failure probability is given in the case of the fully loaded condition. In this analysis, the correlation between random variables M and T is taken into consideration, but there found very little influence of the axial thrust on the failure probability of the hull girder.

5.7. Yield and Buckling Failure of Deck and Bottom Plating

As the second example of the analysis, local yield and buckling failure of deck and bottom plating of the oil tanker is considered by analysing a stiffened plate panel surrounded by transverses, longitudinal bulkhead and side shell plating, which is subjected to axial compression σ due to longitudinal bending of the ship and water pressure q .

The deck and the bottom plating is assumed to be an orthogonally anisotropic rectangular plate, which is simply supported at four edges with initial deflections of a single curvature in the direction of the water pressure.

The maximum compression stress R caused on the surface of the plate at center, namely a combined stress of the compression and bending, is obtained by the ordinary plate bending analysis of large deflection theory⁽⁴⁰⁾.

When taking the extreme value of this maximum stress during the ship's lifetime as the demand of the reliability analysis, it varies generally with those random variables such as the axial compression stress σ , the water pressure q as well as the initial deflection of the panel w_0 , and thickness and the dimensions of the structural components.

$$D=R_{\max} (\sigma_{\max}, q_{\max}, w_0, \text{ scantlings of components})$$

The axial compression stress σ and the water pressure q are the sum of those in still water and among-waves, respectively, which are assumed to be independent random variables and their extreme values σ_{\max} and q_{\max} during the ship's life occur

Table 6. Failure Probability of Hull Girder for 10^8 cycles

Location of Cross Section		Midship	L/6 Aft from Midship
Z_p	(mm ³)	$8,490 \times 10^{10}$	
A_e	(mm ²)	$7,910 \times 10^6$	
a		1.0	
Still Water B.M./ Z_p (kg/mm ²)		1.42	3.70
10^{-8} Charact. Ext. Value of Wave Stress, σ_{ext} (kg/mm ²)		10.78	8.13
Demand (kg/mm ²)	Mean Value, D	12.53	12.08
	Stand. Deviation, S_D	0.750	0.565
Capability (kg/mm ²)	Mean Value, C	28.86	28.86
	Stand. Deviation, S_C	2.17	2.17
Failure Probability P		5.8×10^{13}	3.7×10^{14}

Table 7. Yielding and Buckling Failure Probability of Deck and Bottom Plating for 10^8 cycles (0.4L from F.P.)

Location	Deck		Bottom		Notes
Load Condition	①	②	③	④	
Full Load	1×10^{-1}	2×10^{-1}	8×10^{-1}	8×10^{-1}	
Ballast	1×10^{-1}	1×10^{-1}	1×10^{-1}	7×10^{-1}	

Table 8. Probability of Instability Failure of Bottom Longitudinals for 10^8 cycles

Load Condition	Bm. Longl. Location	High Tensile Steel $840 \times 200 \times 17.5/30$	Mild Steel $840 \times 200 \times 20/39$
Full Load	Wing Tank	7.0×10^{-3}	5.5×10^{-2}
Ballast	Center Tank	3.1×10^{-6}	2.7×10^{-5}

Table 9. Fatigue Failure Probability at Butt Welded Joint of Longitudinal Strength Members for 10^8 cycles

Panel No.	Deck		Bottom		Notes
Load Condition	①	②	③	④	
Full Load	3.2×10^{-8}	4.5×10^{-7}	6.1×10^{-8}	2.0×10^{-11}	
Ballast	7.4×10^{-8}	6.3×10^{-8}	9.4×10^{-11}	1.5×10^{-11}	

simultaneously.

The mean value \bar{D} and the standard deviation S_D of the demand are given as described previously, and numerical calculations are performed by using a computer program to evaluate \bar{D} and $\partial D/\partial X_i$ values in S_D .

As for the capability of the structure, defined is the yield stress of the material σ_Y . The failure criterion for yielding of the panel is then given by the following relation.

$$R_{\max} \geq \sigma_Y$$

Table 7 summarises the results of the analysis on the failure probability of deck and bottom plating in both center and wing tank of the oil tanker. It can be seen from this table that the failure probability is comparatively high on the bottom plate panel in void wing tank under full load condition, and on the deck plate panel in the case of the ballast condition of the ship.

5.8. Lateral Instability Failure of Longitudinals

When a longitudinal of bottom plating is subjected to axial compression stress σ due to longitudinal bending of the ship together with water pressure q (in meter aqua), the ultimate load carrying capacity of the longitudinal Mult is given approximately by the following formula.

$$Mult \cong (\sigma_Y - \sigma) \cdot K \cdot Z_p$$

where, K is a reduction factor of the plastic modulus of the longitudinal Z_p , when the axial thrust vanishes, and is obtained numerically by a computer program⁽⁴¹⁾.

External moment due to water pressure q , on the other hand, is given by the following formula, when the longitudinal is fixed at both ends of the span.

$$M = \frac{1}{12} \cdot s \cdot L^2 \cdot q \cdot 10^{-3} \quad (\text{in kg-mm})$$

where, s and L are the space and the span of the longitudinal, respectively. Then, the failure occurs when the following relationship holds.

$$M \geq Mult$$

Using the extreme values of the water pressure q_{\max} , and of the axial compression stress σ_{\max} caused

during the ship's life, the demand and the capability are defined as follows.

$$D = \alpha \cdot q_{\max} + K \cdot \sigma_{\max}$$

$$C = K \cdot \sigma_Y$$

where,

$$\alpha = \frac{s \cdot L^2 \cdot 10^{-3}}{12 \cdot Z_p}$$

The reduction factor K varies with random variable such as the initial lateral deflections of the longitudinal, thickness of the web and of the flange.

Table 8 shows an example of the results of the analysis on the bottom longitudinals of the tanker. It is seen from this table that the failure probability of the bottom longitudinals is fairly high and is of almost same order as that of yielding of the bottom plate panel.

5.9. Fatigue Failure

General discussions have been made in FAILURE PROBABILITY OF FATIGUE of this report based on the criterion of crack initiation at welded butt joint of the primary members of ships. A series of numerical calculations have been carried out on the reliability analysis, of which the results have provided the failure probability of the structural members of ships in the lifetime, being expressed in terms of the mean value of still water stress $\bar{\sigma}_m$ and the 10^{-8} characteristic extreme value of wave stress $\bar{\sigma}_{a,ext}$ of the members. (see Fig. 32)

As a typical example of the evaluation on the fatigue failure probability, computations are made on the longitudinal strength members on deck and bottom of the oil tanker.

The still water stress in these members are determined from each loading condition, and the characteristic extreme values of the wave stress are calculated by the total system analysis program as mentioned previously.

The results of the analysis on the fatigue failure probability of these members in ship's lifetime (loading condition is assumed constant) are summarized in Table 9. It is clearly seen from this table that the fatigue failure probability of deck plating and of bottom plating at bilge is of order of 10^{-1} under both full load and ballast conditions.

5.10. Semi-Direct Design Method

As has been described in GENERAL DESIGN PROCEDURE of this report, several attempts have been made to formulate approximately the maximum expected value of the wave bending moments (vertical & horizontal), torsional moment and shearing force, which are obtained by system analysis and statistic approach, into a functional relationship in terms of such parameters as ship's length, breadth, depth, draft, block coefficient, etc. However, such approximate formula have not been so far published yet.

5.11. Approximate Probabilistic Method

As one of the semi-direct design method, an approximate probabilistic method is presented in 1973 by Mansour⁽⁴²⁾, where the structural design is performed by considering the random variables in the procedure as of distribution-free for both the random variables and the probability of failure.

If Z is a random variable representing the amplitude of the total bending moment and S is a random variable representing the strength of a ship, then the safety margin is defined as follows:

$$M = S - Z$$

Failure occurs when, the bending moment exceeds the ship strength, i.e., when the margin M is negative. Therefore, the probability of failure P_f is,

$$\begin{aligned} P_f &= P[M < 0] \\ &= P\left\{ \frac{M - m_M}{\sigma_M} \leq -\frac{m_M}{\sigma_M} \right\} \\ &= P[G \leq -\gamma] = F_G(-\gamma) \end{aligned}$$

where, m_M = mean of the safety margin $= m_s - m_z$

m_s = mean of the strength

m_z = mean of the total bending moment

σ_M^2 = variance of the safety margin $\approx \sigma_s^2 + \sigma_z^2$

G = standardized safety margin

$$= (M - m_M) / \sigma_M$$

γ = safety index $= m_M / \sigma_M$

F_G = the distribution function of G

From the above, each value of the safety index γ is associated with some probability of failure. Even in the absence of information on the type of distribution function F_G , the safety index could be used as a measure of the probability of failure.

The safety index γ can be expressed as the next equation

$$\gamma = \frac{m_M}{\sigma_M} = \frac{m_s - m_z}{\sqrt{\sigma_s^2 + \sigma_z^2}} = \frac{\theta - 1}{\sqrt{\theta^2 V_s^2 + V_z^2}}$$

where σ_s^2 and σ_z^2 are the variances of the strength and the total bending, and V_s and V_z are the coefficients of variation of the strength and the total bending moment, respectively, and θ is the central safety factor: $\theta = m_s / m_z$. The first step of the design procedure is to specify the values of λ , V_s , V_z and to obtain θ from above equation. Then, the mean of the strength is obtained from θ and m_z .

$$m_s = \theta \cdot m_z$$

Finally the section modulus of ship can be determined from m_s :

Required Section Modulus

$$= \frac{m_s}{\text{The average failure stress}}$$

Analysis of eighteen ships of different types are made in order to serve as a preliminary investigation of the appropriate level of safety as measured by a proposed safety index ranging from about 4 to 6.5.

Another study on an approximate probabilistic method in the design of the longitudinal strength of ship is made by Stiensen and Mansour, considering the effect of slamming and springing as well as the low frequency wave induced moments on the longitudinal bending of hull girders⁽⁴³⁾.

6. Application in Transverse Strength of Ships

6.1. Foreword

The application of direct or semi-direct calculation methods to transverse strength of ships is still nowadays a relatively unexplored field and the publications on this subject are very limited.

This part presents an example of the analysis on the failure probability of the transverse strength members of oil tankers by applying the "direct design procedure" on ship structures, which has been investigated in the Committee No. 134 of the Japan Shipbuilding Research Association⁽⁴⁴⁾.

The working stress in the transverse members of

ships is calculated by using the total system analysis program, of which the outline is described in the Report of the above Committee⁽⁴⁵⁾.

In this total system analysis program of transverse strength, considered are the non-linear damping effect in rolling of ships as well as the non-linear and non-sinusoidal stress variation due to local effect of water pressure at ship side, being not proportional to wave height. The structural model in this program is of two tank length, of which boundary condition is given by the total system analysis program of longitudinal strength⁽²²⁾, and still water stress as well as the fluctuating wavestress are obtained by finite element technique, where the influence coefficient method is used for both transverse symmetric load and anti-symmetric load, respectively.

6.2. Yield Failure of Transverse Members

Numerical analysis is made on the probability of failure due to yielding in face plate of the transverses and struts of the 240,000 DWT tanker as described in the previous chapter.

As the demand in the reliability analysis, defined is the maximum value of the normal stress σ_{\max} , i.e. the sum of still water stress and wave stress, of which the 10^{-8} characteristic extreme value is obtained directly by utilizing the above mentioned total system analysis computer program.

$$D = \sigma_{\max}$$

The normal stress in face plate is calculated for a designed scantling of the transverses (breadth and thickness of the face plate B_f and t_f , respectively). Since the cross-sectional properties of the face plate of the transverses are regarded as random variables, the yielding failure criterion for actual members is given by

$$\sigma_{\max} B_f \cdot t_f / B_f t_f \geq \sigma_Y$$

Then, the the capability of the transverse members is defined by the following formula,

$$C = \sigma_Y \cdot (B_f / B_f) \cdot (t_f / t_f)$$

The probability of failure P_f is calculated in the similar procedure given in the preceding article, where the mean value and the c.o.v. of the random variables

σ_Y , B_f / B_f , t_f / t_f are chosen as 28.86, 1.0, 1.0 and 0.0752, 0.0028, 0.0145 respectively.

It should be also noted here that the effect of the stress concentration at mid-breadth of the face plate in round corners of the transverse members is considered approximately by introducing the effective breadth coefficient.

A typical example of the results of the analysis is illustrated in Fig. 34, where the yield failure probability of face plate, P_f is shown in logarithmic scale, the base line being taken along the face plate of the members. It can be seen from this figure that the failure probability is relatively high at gunwale corner of the deck transverse in wing tank, at ends of struts on side transverses and at the longitudinal bulkhead corner of the bottom transverse under fully loaded condition.

6.3. Plastic Collapse of Transverse Members

The total plastic collapse is considered to evaluate the failure probability of the transverse strength members, such as struts, side and bottom transverses of the oil tanker.

In this analysis, an assumption is made that the plastic collapse of the transverse members occurs after shear buckling of web plate has taken place in the vicinity of the corners at the ends of the span, and thereby forming real hinges of simply supported ends.

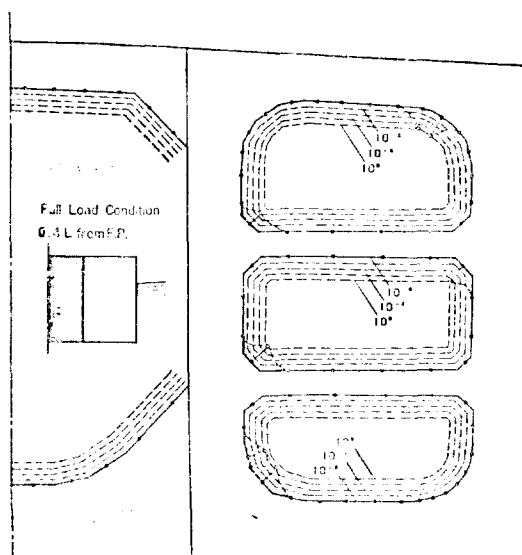








Fig. 34. Yield Failure Probability of Face Plate of Transverse Members for 10^8 cycles

Table 10. Collapse Mode and Failure Probability of Transverse Members for 10⁸ cycles

Collapse Mode		Failure Probability	
		Full Load	Ballast
A		0	0
B		0	0
C		0	0
D		7.6×10^{-2}	3.4×10^{-3}
E		7.2×10^{-16}	3.8×10^{-6}
F		9.5×10^{-1}	1.0×10^0

When considering the plastic collapse of the transverse members subjected to the water pressure q which is the sum of the still water pressure and the fluctuating wave pressure, the maximum value q_{max} during the ship's lifetime is defined as the demand of this reliability analysis, and its 10⁻⁸ characteristic extreme value is obtained by the total system analysis computer program.

$$D = q_{max}$$

Ordinary method of the plastic analysis is then applied for evaluating the collapse load due to water pressure, q_c for each possible mechanism of the transverse ring, which is given in terms of yield stress of material σ_Y , plastic modulus Z_p and length of span L of each member.

The capability of the transverse members is then defined such that,

$$C = q_c(\sigma_Y, Z_p)$$

where, σ_Y and Z_p are random variables in this analysis.

The plastic modulus of the members is approximately

given by the following formula, in which the random variables, t_w and t_f denote the thickness of web and of flange of the members, respectively, and a_1, a_2 and a_3 are the coefficients.

$$Z_p \approx a_1 t_w + a_2 t_f + a_3$$

Then, the plastic collapse of the transverse occurs, when

$$q_{max} \geq q_c$$

and the probability of failure, P_f is obtained by the general method described in the preceding article.

Numerical calculations on the failure probability are made on the transverse ring of the oil tanker for each mode of the mechanisms, and some of the results are shown in Table 10. It is clear that the failure probability of plastic collapse of the transverse members is generally very small, except for such mode of a frame-type mechanism of the wing tank construction (Mode D) where the supporting effect of longitudinal bulkhead and side shell plating is neglected. It is also seen from this table that the failure probability of Mode F is approximately equal to 1.0, when the upper and lower struts in wing tank are assumed not effective due to their premature buckling prior to the mechanism failure.

It should be born in mind, however, that the results obtained results obtained from this analysis would be, in general, an unsafeside estimate because of neglecting the effect of local or lateral instability of the members on the ultimate strength of the transverse ring.

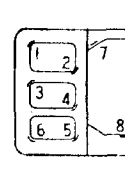
6.4. Fatigue Failure

As a typical example of the evaluation on the fatigue failure probability, computations are made on the welded butt joints in face plate of the transverse strength members of the oil tanker.

The results of the analysis on the fatigue failure probability of these members are summarized in Table 11. It can be seen from this table, that the fatigue failure probability of the welded butt joint in face plate of the transverse strength members varies widely in the range from 10⁻¹ to less than 10⁻⁸, depending on their location and the loading condition of the ship.

Table 11. Fatigue Failure Probability P_f for 10^8 cycles (Face Plate of Transverse Strength Members of the Oil Tanker)

Load condition	Distance from F.P.	Location of the Face Plate							
		1	2	3	4	5	6	7	8
Full Load	0.2L	4×10^{-1} (8×10^{-1})	3×10^{-4} (3×10^{-1})	*	*	*	1×10^{-2} (1×10^{-1})	5×10^{-3}	3×10^{-3}
	0.4L	8×10^{-3} (2×10^{-1})	1×10^{-7} (1×10^{-2})	*	*	*	3×10^{-6}	4×10^{-6}	2×10^{-8}
Ballast	0.4L	2×10^{-2} (5×10^{-1})	4×10^{-7} (8×10^{-2})	2×10^{-7} (4×10^{-2})	5×10^{-6} (2×10^{-1})	1×10^{-4} (3×10^{-3})	1×10^{-5}	*	5×10^{-8}



Notes: * mark means that P_f is below 10^{-8}

Values in () indicate the fatigue failure probability when considering the effect of stress concentration at mid-breadth of the face plate in round corners.

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