論 文
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電子計算器에 依한 原子爐最適制御

Optimal Control of Nuclear Reactors by Digital Computer

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Abstract

In this paper a method is presented for the optimal control of a nuclear reactor at equilibrium state by use of a digital computer. Using the optimal control theory, we formulate the control problem of the reactor as a discrete-time linear regulator problem. A quadratic performance index is defined. The effects of choosing different performance index weighting matrices to the feedback gain matrix and reactor transient responses are studied for the deterministic optimal control with all state variables accessible to measurement.

1. Introduction

The design of the optimal computer control system of nuclear reactors or nuclear power plants are of great importance, because it has been completely accepted that safety, reliability and economy of their operation depended to a great extent on their control system.

There have been many studies undertaken in the past years on the computer control of nuclear reactors or nuclear power plants by means of advanced control concepts.[1)-6)]

In this paper a method is presented for the optimal control of a nuclear reactor to regulate it at the constant power level. Using the deterministic reactor model and the quadratic performance index, we obtained the optimal feedback gain matrix for each control stage by means of Lagrange multipliers and Riccati transformation[7)] and assuming all state variables to be accessible to measurement. Following this, we examined the dependence of the feedback gain matrix and reactor transient responses upon the weighting factors in the performance index.

2. Formulation of Closed-loop Control System

2.1 Description of System

We consider a closed-loop control system controlled by a digital control system shown in Fig. 1. The reactor system is controlled by the optimal control u_0, which is stored in the digital computer memory. When the trajectory deviates from the precalculated optimal trajectory x_0 on account of some external disturbances, the control system detects the deviation and modifies u_0 to $u_0+\delta u$ in such manner as to minimize a given performance index.

Comparing the output of the detector system with the optimal trajectory x_0, we obtain the erro signal

$$\delta x(k)=x(k)-x_0(k), \tag{1}$$

with the optimal feedback matrix precalculated and memorized in the computer memory,

$$\delta u(k)=H(k)\delta x(k) \tag{2}$$

where $H(k)$ is the optimal feedback matrix. Then $u(k)$ is obtained as the sum of $u_0(k)$ and $\delta u(k)$ and serves as the control input to the reactor system.

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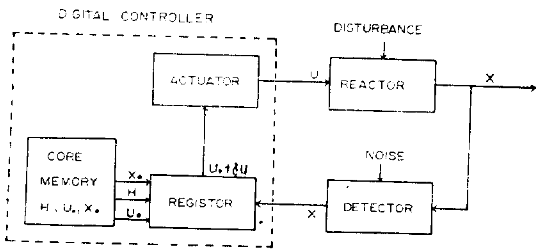

Fig. 1. Digital control of a nuclear reactor system

2.2 Reactor Dynamics

The point-model kinetics equations for a nuclear reactor with one-group delayed neutrons are:[8]

$$\frac{dn(t)}{dt}=\frac{\delta k(t)-\beta}{l}n(t)+\lambda c(t), \tag{3}$$

$$\frac{dc(t)}{dt}=\frac{\beta}{l}n(t)-\lambda c(t), \tag{4}$$

where

$n(t)$=neutron density

$\delta k(t)$=reactivity

β=total delayed neutron fraction

l=neutron lifetime

λ=decay constant of neutron precursor

$c(t)$=concentration of delayed neutron precursor

If the following variables are defined

$$\alpha \triangleq \beta/l \tag{5}$$

$$\rho(t)\triangleq \delta k(t)/\beta \tag{6}$$

$$z(t)\triangleq \frac{\lambda}{\alpha}c(t) \tag{7}$$

and substituted into Eqs. (3) and (4), a set of normalized equations is obtained. Thus,

$$\frac{dn(t)}{dt}=\alpha\rho(t)n(t)-\alpha n(t)+\alpha z(t) \tag{8}$$

$$\frac{dz(t)}{dt}=\lambda\{n(t)-z(t)\} \tag{9}$$

$\rho(t)$ is defined in terms of the control variable u, for which we adopt the rate of change in reactivity to prevent the occurrence of abrupt changes of reactivity in the control sequence:

$$\frac{d\rho(t)}{dt}=\xi u(t) \tag{10}$$

where ξ is an arbitrary given constant.

The Eqs. (8)-(10), expressed in vector notations, become

$$\left.\begin{array}{l}\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x},u)\\ \boldsymbol{f}=[f_1\ f_2\ f_3]^T\\ \boldsymbol{x}=[n\ z\ \rho]^T\end{array}\right\} \tag{11}$$

where

$$f_1=\alpha\rho(t)n(t)-\alpha n(t)+\alpha z(t)$$
$$f_2=\lambda\{n(t)-z(t)\}$$
$$f_3=\xi u(t)$$

In Eq. (11), the vector $\boldsymbol{x}^T$ and $\dot{\boldsymbol{x}}$ respectively denote the transpose and time derivative of the state variable $\boldsymbol{x}$.

The control system considered here is one that controls the deviations of actual trajectory $\boldsymbol{x}$ from the optimal trajectory $\boldsymbol{x}_0$ by the action of unforeseen external disturbances imparted to the reactor system. Consequently, we will derive a system equation that describes the deviations of $\boldsymbol{x}$ from $\boldsymbol{x}_0$.

Linearizing Eq. (11) around $\boldsymbol{x}_0(t)$ and $u_0(t)$, we get the linear vector equation:

$$\delta\dot{\boldsymbol{x}}=\boldsymbol{A}\delta\boldsymbol{x}+\boldsymbol{B}\delta u \tag{12}$$

where $\delta\boldsymbol{x}\triangleq\boldsymbol{x}-\boldsymbol{x}_0$, $\delta u\triangleq u-u_0$,

$$\boldsymbol{A}\triangleq\left[\frac{\partial f}{\partial x}\right]_{u_0}^{x_0},\quad \boldsymbol{B}\triangleq\left[\frac{\partial f}{\partial u}\right]_{u_0}^{x_0}$$

The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are generally dependent on $\boldsymbol{x}_0$ and u_0; however, we treat them as constants evaluated at a particular set of $\boldsymbol{x}_0$ and u_0. In this case we choose the condition that the reactor system is at equilibrium. Therefore, the optimal trajectory is found by solving

$$\boldsymbol{f}(\boldsymbol{x}_0,\ u_0)=\boldsymbol{0}. \tag{13}$$

Thus we get

$$\boldsymbol{x}_0=[1.0\ 1.0\ 0]^T$$
$$u_0=0$$

If the reactor control should be implemented on a digital computer, it is necessary to make a discretization of Eq. (12). If $\delta u(t)$ is a discrete-time input with a sampling period T,

$$\delta u(t)=\delta u(kT)\ \text{for}\ kT\leq t<(k+1)T \tag{14}$$

Eq. (12) can be converted to the discrete form[9]

$$\delta\boldsymbol{x}(k+1)=\boldsymbol{\Phi}\delta\boldsymbol{x}(k)+\boldsymbol{G}\delta u(k) \tag{15}$$

where

$$\left.\begin{aligned} &\boldsymbol{\Phi}=e^{AT}=\boldsymbol{I}+AT+\frac{1}{2!}(AT)^2+\cdots,\\ &\boldsymbol{G}=\boldsymbol{A}^{-1}(\boldsymbol{\Phi}-\boldsymbol{I})\boldsymbol{B}\\ &\quad=T\Big[\boldsymbol{I}+\frac{1}{2!}AT+\frac{1}{3!}(AT)^2+\cdots\Big]\boldsymbol{B},\\ &\boldsymbol{I}=\text{unit matrix} \end{aligned}\right\} \qquad (16)$$

Typically, $\beta=0.0065$, $l=10^{-3}$sec, and $\lambda=0.0775$ 초 for a commercial nuclear reactor.[10] Assuming $\xi=1$ and $T=1$ sec, we calculate transition matrix $\boldsymbol{\Phi}$ and input driving matrix $\boldsymbol{G}$ by a digital computer.

$$\boldsymbol{\Phi}=\begin{pmatrix} 1.3158\text{E-}02 & 9.8684\text{E-}01 & 1.0518\text{E+}00\\ 1.1766\text{E-}02 & 9.8823\text{E-}01 & 0.4959\text{E-}02\\ 0 & 0 & 1.0000\text{E+}00 \end{pmatrix}$$

$$\boldsymbol{G}=\begin{pmatrix} 1.6660\text{E-}01\\ 2.8417\text{E-}02\\ 1.0000\text{E+}00 \end{pmatrix}$$

The discrete time linear model, Eq. (15) will be the basis for the feedback control that is to be described in the next sections.

2.3 Optimal Feedback Control System

In formulating the deterministic feedback control system, we begin by assuming all state variables to be accessible, and by neglecting the system noise and the time lag in the control system. We define a performance index of the form

$$J_N=\sum_{k=0}^{N-1}\Big[\frac{1}{2}\delta\boldsymbol{x}^T(k)\boldsymbol{Q}\delta\boldsymbol{x}(k)+\frac{1}{2}R\delta u^2(k)\Big] \qquad (17)$$

where the weighting matrix $\boldsymbol{Q}$ and constant R can be chosen freely, required that $\boldsymbol{Q}\geq\boldsymbol{0}$ and $R>0$. The problem is to find the optimal control that will minimize J_N. This problem is solved by use of Lagrange multipliers and Riccati transformation.[7] The solution is

$$\delta u(k)=\boldsymbol{H}(k)\delta\boldsymbol{x}(k). \qquad (18)$$

The feedback gain matrix $\boldsymbol{H}(k)$ is obtained from

$$\boldsymbol{H}(k)=-\frac{1}{R}\boldsymbol{G}^T[\boldsymbol{\Phi}^T]^{-1}[\boldsymbol{P}(k)-\boldsymbol{Q}] \qquad (19)$$

where $\boldsymbol{P}(k)$ is computed backwards, starting with $\boldsymbol{P}(N)=\boldsymbol{0}$ as follows;

$$\boldsymbol{P}(k)=\boldsymbol{Q}+\boldsymbol{\Phi}^T\boldsymbol{P}(k+1)\Big[\boldsymbol{I}+\frac{1}{R}\boldsymbol{G}\boldsymbol{G}^T\boldsymbol{P}(k+1)\Big]^{-1}\boldsymbol{\Phi}. \qquad (20)$$

The weighting matrix $\boldsymbol{Q}$ and constant R have to be assigned by trial and error and by considering the eigenvalues of the closed loop system, $\boldsymbol{\Phi}+\boldsymbol{GH}$.

2.4 Nuclear Performance Indices

The general performance index given by Eq. (17) can be written in expanded from as a function of the neutron density deviation, reactivity, and reactivity rate:

$$J_N=\frac{1}{2}\sum_{k=0}^{N-1}\Big[\delta n^2(k)+a\rho^2(k)+bu^2(k)\Big] \qquad (21)$$

where a and b are weighting coefficients. By comparing Eq.(17) with Eq. (21) weighting matrix $\boldsymbol{Q}$ and constant R are obtained as follows:

$$\boldsymbol{Q}=\begin{pmatrix}1&0&0\\0&0&0\\0&0&a\end{pmatrix},\quad R=b$$

The optimal control obtained by using the performance index, Eq. (21), will minimize the sum of the squares of neutron density deviation and the reactivity at the sampling instants.

3. Results and Discussions

3.1 Feedback Gain Matrik H

The feedback gain matrix $\boldsymbol{H}(k)$ is given as a

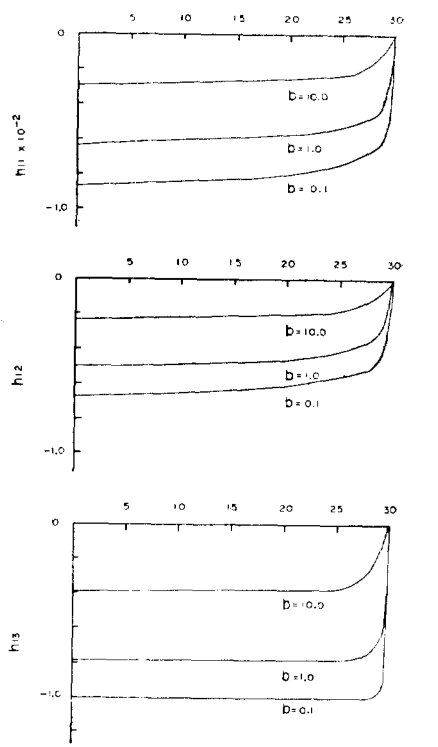

Fig. 2 (a) Elements of feedback gain matrix $\boldsymbol{H}$ (N=31, a=1) solution to Eq. (19) for the sampling period $T=1$ sec. With Φ and G determined, the numerical solution of $H(k)$ to the recurrence equations (19) and (20) can be calculated, letting $P(N)=0$ for given values of Q and R in the interval $k=N\sim 0$. In order to examine the effects brought upon feedback matrix $H(k)$ by changes in the weighting matrix Q, $H(k)$ was calculated for various values of Q, with the result shown in Fig. 2.

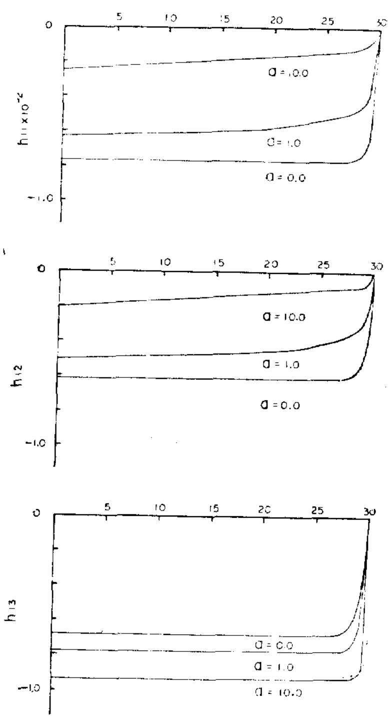

Fig. 2 (**b**) Elements of feedback gain matrix $\boldsymbol{H}$ (N=31, $b=1$)

Fig. 2 (a) reveals that the absolute value of h_{ij} decreases with increasing parameter b. And since, from Eq. (18), change in $H(k)$ brings about a proportional change in the control δu, the δu consequently depends upon the value of b. This means that we can restrict the value of δu through the appropriate choice of b. Thus with a suitable value assigned to b as a function of the control stage k, the control $\delta u(k)$ will be constrained within a desired limit over the whole control interval.

The result shown in Fig. 2 (b) indicates that the absolute value of feedback gains h_{11} and h_{12} decreases with increasing parameter a, while the absolute value of h_{13} increases.

The closed-loop control system incorporating $H(k)$ determined in this manner should be asymptotically stable, since this $H(k)$ would become a constant matrix for an infinite control interval.[11]

3.2 Simulations of Closed-loop Control System

To examine the performance of an optimal feedback control system embodying the optimal con troller described in secs. 2.3 and 3.1, the transient responses of the closed-loop control system to a step reactivity disturbance are simulated for the case of state variables accessible to measurement, for various magnitudes of disturbance, and for various values of the feedback gain matrix $H(k)$.

The method of simulation is illustrated by the block diagram shown in Fig. 3. In the digital

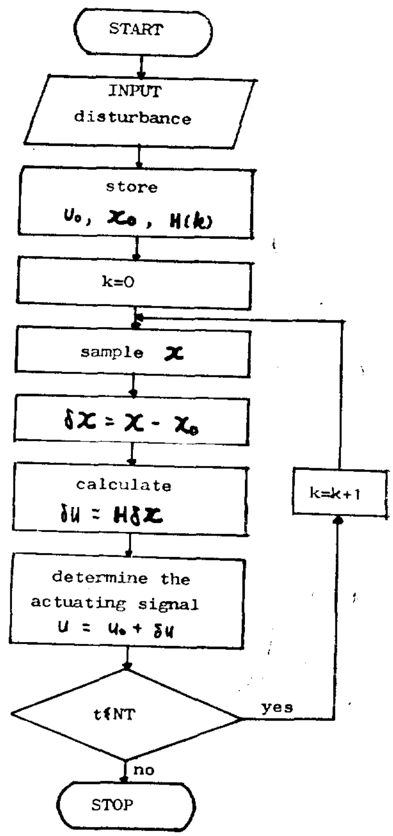

Fig.3. Flow chart of digital simulator

simulations, we neglect the computing delay and control delay to simplify the simulation study.

The feedback control system simulated in this section is the optimal control system theoretically since all states are assumed to be accessible and disturbances are deterministic. Therefore, we intend to use the results for examining the performance of the proposed closed-loop control system, which aims at the establishment of the feedback control system to remove effects of various disturbances as possible.

Figs. 4 show the reactor transient responses with an initial disturbance of $\rho(0+)=0.1$, $\delta z(0+)=0$, and $\delta n(0+)=0.11$ and various weighting coefficients. At $t=(0-)$, the reactor is at equilibrium, which corresponds to $\rho(0-)=0$, $\delta z(0-)=0$, and $\delta n(0-)=0$. At $t=(0+)$, a step change of reactivity occurs which gives rise to the prompt jump in neutron density.[12]

With the same magnitude of reactivity disturbance, the effect of different values in weighting coefficients is illustrated in Figs. 4 (a)-(c), which reveal that to reduce the control value which is applied to correct a disturbance, the coefficient

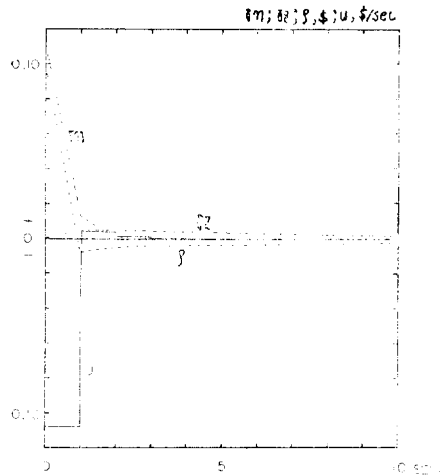

Fig.4 (a) Transient response for $a=0$, $b=0.1$, $\delta n(0+)=0.11$, $\delta z()+)=0$, $\rho(0+)=0.1$

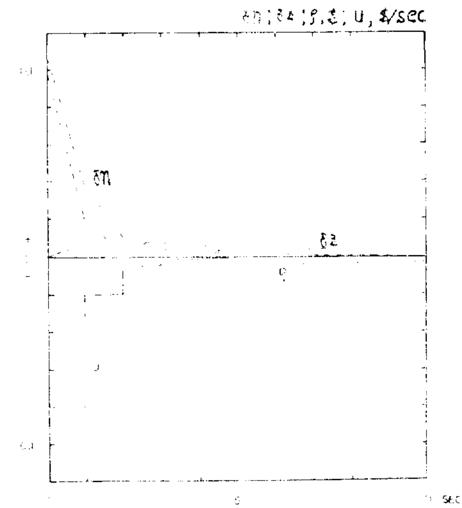

Fig. 4 (b) Transient response for $a=0$, $b=1.0$, $\delta n(0+)=0,11$, $\delta z(0+)=0$, $\rho(0+)=0.1$

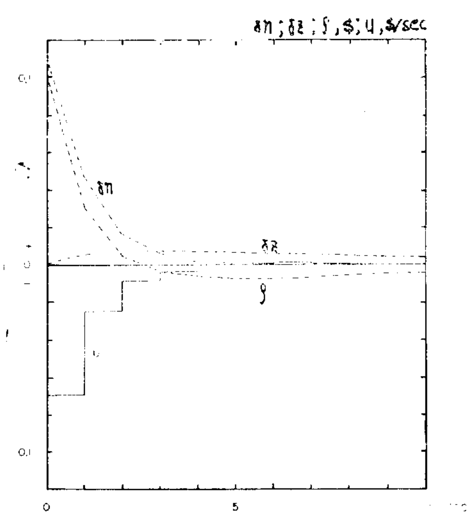

Fig. 4 (c) Transient response for $a=1$, $b=1$, $\delta n(0+)=0.11$, $\delta z(0+)=0$, $\rho(0+)=0.1$

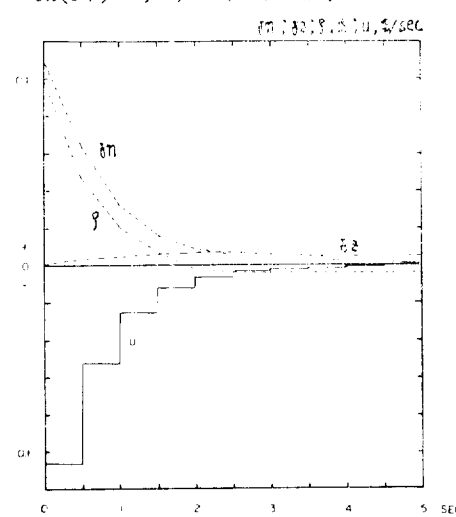

Fig. 4 (d) Transient response for $a=1$, $b=1$, $\delta n(0+)=0.11$, $\delta z(0+)=0$, $\rho(0+)=0.1$

b is increased, and similarly, reactivity is reduced to zero more quickly after a disturbance by increasing the coefficient a.

1 sec is selected for control sampling period T in Figs. 4 (a)-(c), while Fig. 4 (d) presents the reactor transient response with sampling period $T=0.5$ sec. If the sampling period is decreased, the initial control value is increased.

4. Conclusions

The aim of this control system is to minimize the effect of disturbances imparted to a reactor at equilibrium state. We have presented a method for elaborating a digital closed-loop control system for a nuclear reactor with a one-group delayed neutron model.

Numerical solutions were obtained for the optimal feedback gains by means of Lagrange multipliers and Riccati transformation method. These solutions indicated that the optimal control value would be constrained within a desired limit over the whole control interval.

Many problems remain to be investigated. Some of them are the following:

(1) Clear guidelines for the optimal choice of weighting matrices are not developed yet.

(2) Since the computer program used to compute the optimal control input was not compiled for minimum time execution, no conclusions can be made concerning real-time control capability.

(3) The actual characteristics of the detectors and actuators should also be included to obtain more realistic results.

(4) The optimal control input for inaccessible case and the question of system noise remain to be solved, which we believe, can be done by applying the Kalman filter; but these call for further study.

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