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⟨Original⟩

A Quasi-stationary Random Simulation of Three Standard SAE Load Spectra; Part. I.

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3 SAE 표준하중 스펙트럼에 대한 의사 정상과 씨뮬레이숀 : 제1부

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요 약

확율적 진동 이론을 도입하여 의사 정상과 모델을 세우고 이를 자동차부품들로부터 측정된 미국자동차공학자 협회의 표준 이력 하중의 씨뮬레이숀에 적용하였고 모델의 타당성을 수치적으로검토하였다. 이 이론 결과는 실험실의 피로 수명의 예측에 응용될 수 있다.

NOMENCIATURE

A=maximum stress peak/RMS of RMS

B=overall all amplitude RMS/RMS of instantaneous RMS

b=Weibull exponent

N=number of cumulative occurrences of exceedance

N_μ=number of occurrences at a given reference mean

P_{max}=probability of occurrences of the maximum peak

R=irregularity ratio

S=stress peak level

S_{max}=maximum stress peak level

T=RMS of all amplitude/RMS of positive peaks

T_{min}=lower bound of T

σ=RMS

σ_{0_v}=overall all amplitude RMS

σ_σ=RMS of instantaneous RMS

μ=reference mean stress level

1. Introduction

The most difficult problem in laboratory fatigue testing has been to simulate as accurately and realistically as possible the irregularly varying load which structure encounter during service. For programmed block type variable amplitude tests (1—5) factors like sequence, number of load levels, and block size (6) were found to influence the fatigue life. In the past two decades, the approach to this problem has

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been continually refined using the theory of random processes. In the so-called ICR test (7), the continuous load spectra were replaced by discrete load spectra where the discrete load levels of varying amplitude were called up in a computer-randomized sequence. In a recent refinement an analogous random process (ARP) (8, 9, 10) has been proposed. Although ARP tests seem to provide a better simulation of service conditions, they are still incapable of simulating many actual service loads, for example, broad-band signals (8) with varying mean and root mean square (RMS).

The objective of this investigation was to describe the actual service loading process in a better way so that any service load history of high cycle can be simulated in a laboratory fatigue test. The present Socity of Automotive Engineers (SAE) Cumulative Fatigue Damage Committee Study (11) presents an excellent opportunity to test the simulation techniques developed in this research. The SAE study involves three completely unrelated load histories which are extremely different in character.

Ⅱ. Theory of Quasi-stationary Random Process

The concept of randomness and some important statistcal properties of random processes have been well documented (12). The quasi-stationary random (QSR) process is defined as a superposition of stationary random processes of different RMS and mean values which combine to give a fixed overall RMS and mean value (13, 14, 15). Figures 1 and 2 show a photograph and a simplified diagram of a QSR process. $\sigma_1, \sigma_2, \cdots\cdots, \mu_1, \mu_2, \cdots\cdots$ etc. are the RMS and mean values of individual stationary process.

Since the peak stress distribution[15,16,17,18] is ultimately responsible for fatigue failure the cumulative peak stress distribution of exceedance in QSR processes can be expressed mathematically as follows:

$$P(s>S) = \int_{\mu\,\min}^{\mu\max} \int_{\sigma\min}^{\sigma\max} \int_{s}^{\infty} h(\mu) g(\sigma) f(\mu, s, \sigma)\, ds\, d\sigma\, d\mu \quad (1)$$

where

$P(s>S)$ = cumulative probability of exceedance (CPE) at stress peak level S

$g(\sigma)$ = probability density distribution (PDD) function of RMS levels

$f(\mu, s, \sigma)$ = PDD function of peaks for a stationary Gaussian random (SGR) process with fixed μ and σ

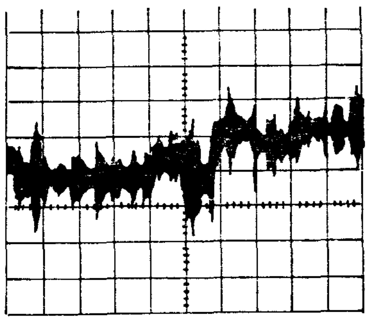

Fig. 1. quasi-statationary random signal

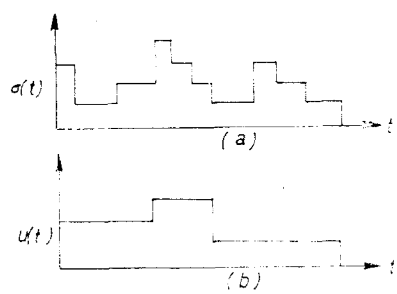

Fig. 2. RMS (a) and mean (b) variations of a quasi-stationary random process

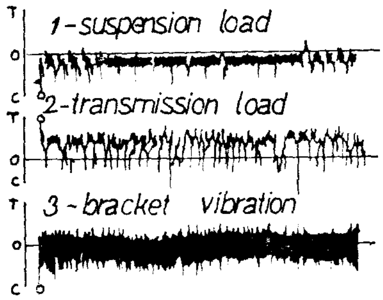

Fig. 3. Three SAE load spectra

$h(\mu)$ = PDD function of mean levels

s = peak stress

σ = RMS of stress

μ = mean stress

For the case of symmetric load distribution (i.e., distributions of peaks and valleys symmetric with respect to fixed reference mean), Eq. 1 reduces to the following equation:

$$P(s>S)=\int_{\sigma min}^{\sigma max}\int_S^{\infty} g(\sigma)\ f(\mu,s,\sigma)\ ds d\sigma \quad (2)$$

which for the case of SGR process reduces further to the familiar expression[19] of

$$P(s>S)=\int_S^{\infty}\Big\{\frac{K_1}{\sqrt{2\pi}\,\sigma}\exp\Big[-\frac{(s-\mu)^2}{2K_1^2\sigma^2}\Big]$$

$$+R\cdot\frac{(s-\mu)}{\sigma^2}\cdot\exp\Big[-\frac{(s-\mu)^2}{2\sigma^2}\Big]\Phi\Big(\frac{s-\mu}{K_2\sigma}\Big)\Big\}ds$$

$$=1-\Phi\Big(\frac{S-\mu}{K_1\sigma}\Big)+R\exp\Big[-\frac{(S-\mu)^2}{2K_1^2\sigma^2}\Big]$$

$$\Phi\Big(\frac{S-\mu}{K_2\sigma}\Big)=F(S,\mu,\sigma) \quad (3)$$

where

$K_1=[1-R^2]^{\frac{1}{2}}$

$K_2=K_1/R$

$\Phi(Z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{Z}\exp\ (-y^2/2)dy$

R = irregularity ratio.

When $R=1$ (narrow-band process), the PDD function $f(\mu,s,\sigma)$ becomes a Rayleigh distribution. When $R=0$ (limit of wide-band process), $f(\mu,s,\sigma)$ becomes a Gaussian distribution [19,20].

In order to evaluate $g(\sigma)$ and $h(\mu)$ for a particular service load, one may assume appropriate distribution functions which are dependent on certain free parameters. Under certain circumstances, $g(\sigma)$ and $h(\mu)$ can be measured directly.

Ⅲ. Proposed Models for PDD Functions of σ and μ

The assumed PDD functions of RMS load $g(\sigma)$ and reference mean load $h(\mu)$ can be used in Eqs. 1, 2, and 3 to obtain an integrated CPE curve which can be compared to the actual load history CPE curve. The free parameters in the PDD function can be varied until a good comparison between CPE curves is obtained. Experience has shown that $g(\sigma)$ decreases rapidly with increasing σ. This observation indicates that an exponential decay type of PDD function should be used for $g(\sigma)$.

At the present time, little is known about the PDD function $h(\mu)$. Examination of CPE curves for field data tends to indicate that either the mean value is relatively constant with little variation or is relatively constant at two or three distinctly different levels. Based on these observations, the following two statistical models are proposed for the QSR simulation process.

Model A

Assumptions:

i) Fixed reference mean

ii) Varying RMS has a Weibull distribution[21]

iii) Fixed irregularity ratio

In the same manner as before, the CPE

was found to be given by

$$P(s>S)=\int_{\sigma_{min}}^{\sigma_{max}} F(S,\mu,\sigma)\cdot\frac{b}{\sigma_\sigma}g_2(b)\cdot \left[\left(\frac{\sigma-\mu_\sigma}{\sigma_\sigma}\right)g_2(b)+g_1(b)^{b-1}\cdot \exp\left[-\left(\left(\frac{\sigma-\mu_\sigma}{\sigma_\sigma}\right)g_2(b)+g_1(b)\right)^b\right]d\sigma \tag{4}$$

where $g_1(b)=\Gamma\left(1+\frac{1}{b}\right)$

$$g_2(b)=\left[\Gamma\left(1+\frac{2}{b}\right)-\Gamma^2\left(1+\frac{1}{b}\right)\right]^{\frac{1}{2}}$$

Γ=gamma function

$\sigma_{min}=\mu_\sigma-(g_1/g_2)\sigma_\sigma$.

Let σ_{0V} denote the overall (all amplitudes) RMS measured from the entire load history. Then, with some mathematical manipula tion[15], it can be shown that σ_{0V} has the following relationship with σ_σ and μ_σ

$$\sigma_\sigma^2=\sigma_{0V}^2-\mu_\sigma^2. \tag{5}$$

The corresponding dimensionless parameters for this case become

$\overline{T}=\sigma/\sigma_{0V}$

$\overline{S}=S/\sigma_{0V}$

$\overline{\overline{\mu}}=\mu/\sigma_{0V}$

b=Weibull exponent

$B=\sigma_{0V}/\sigma_0$

$T_{max}\simeq 4.0$

The only free parameters are B and b.

Model B

Assumptions:

i) Varying reference mean has a skew discrete (two or three levels) distribution

ii) Varying RMS has either a positive Gaussian distribution or a Weibull distribution

iii) Fixed irregularity ratio

The expression for $P(s>S)$ is exactly the same as in Eq. 1 where $h(\mu)$ and $g(\sigma)$ take the assumed distributions given above. An iterative scheme was employed to determine the optimum shape of $h(\mu)$.

IV. Numerical Procedures for Simulation

Nine numerical steps were employed to obtain a simulated load history from the actual load history. Any one of the two models (A, or B) could be used in these numerical procedures. In order to make the procedures more easily understood, load spectrum 11 is used as an example as illustrated in Fig. 4.

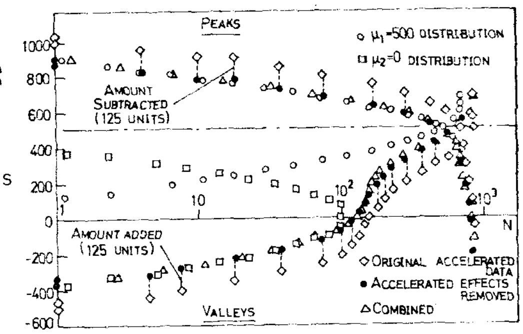

Fig. 4. Load spectrum II showing simulation procedure

1. Subtract the minimum stress range from all stress ranges in range-mean table of references[15,24]. This step is equivalent to subtracting a half of the minimum stress range from the positive peaks and adding same amount to the valleys (see Fig. 4).

2. Measure the overall reference mean (μ_R) experimentally. When this can not be done (for example, in an accelerated test data with small amplitude stresses eliminated), an approximation for the reference mean can be made by taking an average value of all instantaneous mean values from the

range-mean table.

3. Check the symmetry of the statistical distribution of peaks and valleys with respect to the reference mean. If symmetric, assume a fixed reference mean. If not symmetric, assume either a continuous or a discrete nonsymmetric distribution of reference means. An assumption of simple discrete distributions was found to be satisfactory for this study. An educated guess can be made from the range-mean table. (For example: two reference means (500 and 0) were chosen from the range-mean table of load spectrum 11. The two subdivided distributions of peaks with two reference means are shown in Fig. 4. Also, the number of occurrences with these means is chosen from the same table. Then check the following relationship:

$$\mu_{R1}\cdot N_1+\mu_{R2}\cdot N_2+\cdots\cdots=\mu_R\cdot N? \qquad (6)$$

where μ_{R1}, μ_{R2} are individual reference means, μ_R is reference mean, and $N_1, N_2, \cdots\cdots N$ are numbers of occurrences. In the case of load spectrum 11, N_1 and N_2 were chosen to be 701 and 158 ($701\times500+0=350500$? $409.7\times859=349375$).

4. Trial and error procedures are repeated with newly assumed values of $N_1, N_2, \cdots\cdots$ until the summation of the subdiAided individual symmetric distributions yields close duplication of the actual load history as shown in Fig. 4. Here, the interaction between the distributions is negligible (consequently, subdivision is easy) because the two individual reference means are quite far apart from each other. Now, these respective symmetric distributions are analyzed as described in the following steps.

5. From the distribution of peaks, calculate σ_R, the RMS value of peaks.

6. The irreglarity ratio R can be measured from the individual pieces (short time samples of constant RMS) of the entire service load history of the structure. If this can not be done (for example, when the low range stresses are eliminated from the entire load history), the following method is recommented. A first approximation to the irregularity ratio R is made from the ratio of the number of positive stress peaks above reference mean $N_{s\geq\mu}$ to the total number of positive stress peaks (N total) i.e., $N_{s>\mu}/N$ total$=(1+R)/2$; see reference 15.

7. From the R vs. σ_p/σ data (Table 1), determine σ, the RMS value of all amplitudes. (NOTE: σ_p/σ remains the same for a quasi-stationary process with a fixed reference mean.) Since σ_{0V} is equal to σ which can be measured experimentally from the field data, then σ_p/σ will serve as information for cross-checking the validity of the chosen value of R. Plot the cumulative occurrences of exceedance (S vs. N) for each mean value.

8. The free parameters for Models A and B are varied in order to optimize the theoretical quasi-stationary fit to the experimental statistical peak distributions for each mean value. This means that optimum valus are picked for weibull parmeters″ b' and B ($\sigma_{oV}/\sigma_\sigma$) for Medel A or B where σ_σ is the RMS value of instantaneous RMS values. In addition, the value of σ_{oV} can be changed slightly to obtain a better fit.

9. If the fit is good (e.g., as judged from the chi-square criterion), stop and sum the individual symmetric distributions of peaks to obtain the entire nonsymmetric distribution of peaks.

V. Service Load Histories vs. Numerically Simulated Load Histories

The numerical procedures described above were applied to the simulation of the three SAE load spectra· Model *B* (a Weibull distribution of RMS) was employed with an assumption of a varying reference mean (i. e., a few different values of reference mean). The resulting cumulative occurrences of exceedance plot for the simulated load histories of SAE loadsp ectra Ⅰ, Ⅱ, and Ⅲ are shown in Figs. 5, 6, and 7 along with the actual distributions. Even though rough estimates rather than best fits were used, fairly good results were obtained.

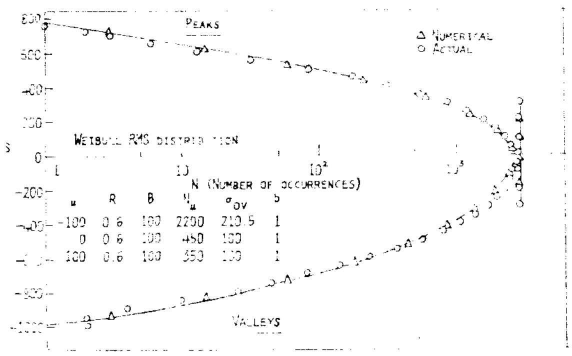

Fig. 5 Numerical simulation of SAE load spectrum Ⅱ (Weibull)

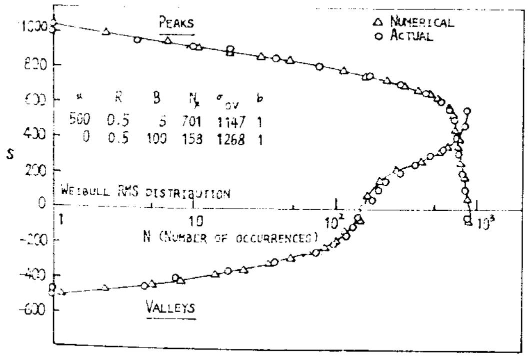

Fig. 6 Numerialof SAE load spectrum Ⅱ (Weibull)

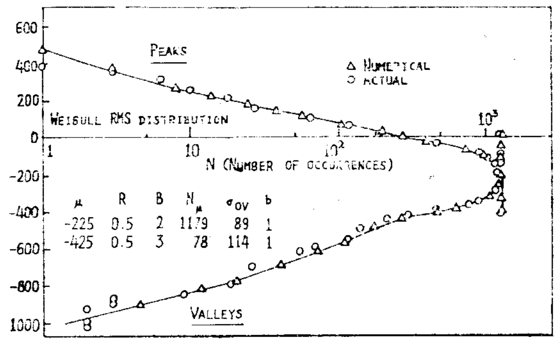

Fig. 7 Numerical simulation of SAE load stectrum Ⅲ (Weibull)

An alternative simulation approach was made for comparison purposes. If there is no reason to believe the nonsymmetric nature of a load spectrum (for example, load spectrum Ⅰ), a simpler simuation procedure can be applied to the assumed symmetric distribution with a single fixed reference mean load. The first scheme shown in Fig. 8 emphasizes the nature of an unbiased statistical best fit where the parmeters are adjusted to fit the experimental data as closely as possible. In the second scheme shown in Fig. 9, a more conservative approach is emphasized where the parameters are adjusted so that most, if not all, experimental points are included as shown near regiion A.

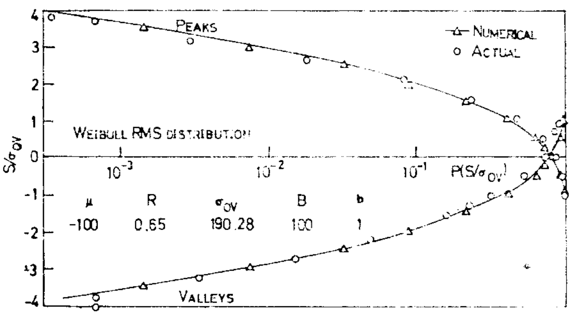

Fig. 8 Unbiased numerical simulation of SAE load spectum Ⅰ

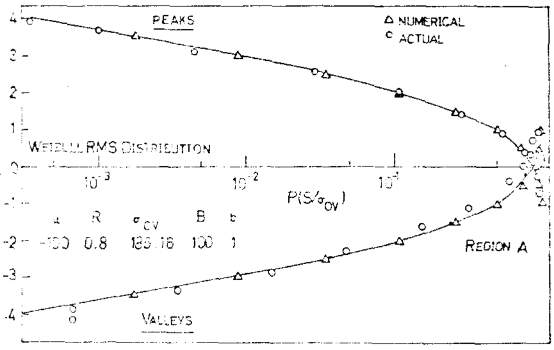

Fig. 9 Conserative numerical simulation of SAE load spectrum I

Ⅵ. Conclusions and recommendations

A theoretical quasi-stationary simulation technique based on recent random vibration and statistical theories was successfully applied to three SAE standard load profiles. The assumptions of a Weibull distribution of RMS coupled with a discrete distribution of reference mean values, were found to be satisfactory. For the proposed simulation technique, load analysis data are required such as statistical distributions of load obtained from the rainflow counting method as indiviual pieces (short time samples of constant RMS) of the entire service load history of the structure for determination of the irregularity ratio. An accelerated test can be achieved along with the simulated fatigue test through the superposition of a pseudo-random signal. There may be more efficient methods for evaluating the RMS and reference mean distributions. One method could be a direct measurement of the time histiores of RMS and reference mean reather than indirect enalation from tqe overall probability distribution of load amplitude. Finally, the adequacy of this simulation technique needs to be verified by comparison of the fatigue lives of standard SAE specimens subejcted to the original SAE load histories and the simulated load histories.

In part 11 of this paper, experimeneal techiques and methods of implementation are discussed which show experimentally that the theoretical manipulations are indeed correct for the three SAE load histories cited here.

In addition, a successful scheme is described which allows for elimination of small stress flouctuations which are less than 25 percert of the maximum stress. This is accomplished through use of a pseudo-random signal superposition onto the original radom signal.

Table 1 R vs. σ_p/σ

R	σ_p/σ
0	1.0
0.050	0.993
0.100	0.992
0.150	0.989
0.200	0.985
0.250	0.979
0.300	0.971
0.350	0.962
0.400	0.951
0.450	0.939
0.500	0.924
0.550	0.908
0.600	0.890
0.650	0.870
0.700	0.847
0.750	0.822
0.800	0.795
0.850	0.765
0.900	0.731
0.950	0.694
1.0	0.655

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