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THE MULTI-MODEL COMPARISON AND COMBINED MODEL ANALYSIS OF AN AGGREGATE SCHEDULING DECISION**

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Abstract

Given a fixed production process and facility capacity, the ability to respond to market fluctuations in terms of changes in production, work force, and inventory is the major task of production management. The costs involved are primarily payroll (regular and overtime), inventory carrying, and hiring and firing. The magnitude of these costs is usually a significant portion of the operating costs of the firm and consequently a small percentage saving due to astute aggregate scheduling can mean substantial absolute saving. At least three demonstrably optimal techniques have been developed for solving this aggregate scheduling problem. These three optimal are apparently LDR, PPP, and SDR. By combining these three different approaches, another optimal solution was obtained by me. I call this CDR (Combined Decision Rule). This approach appears to be useful. This approach may be generalizable to aggregate scheduling involving a short term resourecs.

INTRODUCTION

Aggregate scheduling is the planning of capacity adjustments to meet seasonal or repetitive changes in demand. Capital and obsolescence costs must be considered as well as the costs associated with storage, insurance, and handling. The aggregate scheduling problem is primarily concerned with the anticipation and adaptotion of resources to this aggregate demand, i.e. implicit in planning is an interaction between the time horizon of the anticipation and the ability to make the appropriate adjustments. Having established that our major concern in aggregate scheduling is the adaptation of relatively fixed resources to a seasonal or fluctuating demand pattern, we will now consider the variables which are available to us as follows: (1) Variation in work force size (hiring & firing,or addition or deletion of shift workers), (2) Variation in production rate (overtime or undertime), (3) Variation in production amounts, (4) Variation in inventory size to absorb fluctuations. Holt, et al. (16, 17, 18) have proposed that this problem be handled by a form of quadratic programming they have entitled the linear decision rule (LDR). Charnes, et al. (6) have suggested a linear programming approach. Briefly, the first technique assumes all costs are quadratic and then solves the set of linear equations formed when the partial derivatives of total cost with respect to each period inventory are set equal to zero. The second presents the cost and production relationships in linear form and from the set of values which satisfy these equations, selects the values which

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minimize the objective cost function. Besides the mathematically optimal decision rule approach, there are search decision rule (22, 23) and heuristic decision rule approach. In the heuristic decision rule approach, at least two optimal approaches have been developed: one is parametric production planning (PPP, 21) by Jones and another one is management coefficient model by Bowman (4). Briefly the first presents heuristic decision rules by obtaining coefficients by simulation of cost model with various coefficient combinations. The second technique gets heuristic decision rules by obtaining coefficients by regression analysis of historical data and postulated decision rule equations. One of the most recent heuristic approaches to the aggregate planning is Jones' PPP model. PPP model postulates the existence of two linear feedback rules. One rule provides the number of workers and the second the production amounts. Each rule contains two parameters. The rules are formulated to include the full range of possible decision. This approach searches the four dimensional universe of possible parameters which gives the lowest cost over time to the particular firm. Many sets of parameters are applied to a likely sequence of forecasts and sales to provide many series of work force and production level decisions. The evaluation of each set of parameters is performed by costing out the series of decisions by the cost structure of the particular firm. Another one of the most recent approaches to the aggregate planning problem has been through optimal seeking computer search methods. Taubert (22, 23) developed the basic SDR methodology in 1967 using the paint company data.

This work developed the basic search decision rule with the LDR optimum objective cost function and the Jones' PPP for obtaining optimum work force and production rate per month as test functions, using the same paint data which have been used by many researchers. The result of the development is a combined decision rule (CDR) which will then be compared with the other aggregate scheduling decision models.

DESCRIPTION OF EXPERIMENT

This section describes how I did operations research approaches to a specific aggregate scheduling situation. The cost structure is approximated by a quadratic function which is then minimized mathematically to give a set of optimum. PPP estimates for scheduling production and work force. I will experiment by combining the basic search decision rule methodology using the paint company data with a quadratic cost function and optimum scheduling production and work force estimates obtained by paramertic production planning(PPP).

The Economic Environment and Cost Structure:

It is rather common for a company to emphasize the use of only one or two methods to absorb pronounced fluctuations in demand during the year. Examples are: work force variations in a medium sized candy producer or inventory variations in shoe company, or inventory and production rate variations in container company. In many firms the production plan is based on the sales forecast plus or minus the planned change in inventory levels. Production management must decide how demand requirements are to be allocated to producing periods, i.e. given the monthly demands for the firm's products, what should be the monthly production and work force levels? The operations research approaches to this specific aggregate scheduling situation is based on an economic criterion such as the minimization of costs which are influenced by the size of work force, production, production rate, inventory in the short term. There are a number of optimization techniques which have been proposed such as linear programming, dynamic programming, and linear decision rule to determine the cost structure of the situation. I will use liear decision rule which is one of the typical cost function. The applicability of the revised decision rules is a function of the validity of the cost function and for optimization, is with respect to the cost function used in the formal analysis. In reality the cost function is the model and thus validity relates to the validity of the cost function. For the purpose of analysis the perceived cost function is approximated to make it tractable while still retaining the essential features of the environment. To approximate the structure of cost, we should know the following: (1) the working hours of regular time employment per certain interval, (2) the average payroll cost per regular time employment, (3) the overtime component of the payroll cost, (4) the cost of hiring-an addittional worker consists of the notification required and the inefficient production experienced after a change in level, (5) notification fee for hiring, (6) laying-off cost-includes lay off processing and terminal cost required by contract, and (7) the inventory connected costs-the stockout cost or the spoilage cost. The next task was to translate the above described economic environment into a cost function which was valid for the operating range and tractable for analysis. This was accomplished by plotting the specific points identified and the acceptable range of the variables. A quadratic function was fitted to relate the effect of variation in work force on cost and production. The form of the quadratic function penalizes larger changes more heavily which complies with management's feeling about lost production due to major changes. Holt, et al. (16, 17, 18) formulated the quadratic formulation of objective cost function for the paint factory was:

Regular payroll costs $340.0W(t)$,

Hiring & layoff costs $64.3(W(t)-W(t-1))^2$,

Overtime costs $0.2\ (P(t)-5.67W(t))^2$
$+51.2P(t)\ -281.0\ W(t)$,

Inventory costs $0.0825\ (I(t)-320.0)^2$

where W=work force in men,

P=production in gallons,

I=net inventory in gallons,

S=sales in gallons.

Therefore,

Min $f(C(t))$

subject to $I(t-1)+P(t)-S(t)=I(t)$

where $C(t)=\sum_{t=1}^{12}(C_{RP}+C_{H\ L}+C_{OT}+C_I)$

The Optimum Production Scheduling and Work Force:

PPP estimates developed by Jones (21) postulated the existence of two linear feedback rules One rule provides the number of workers and the second the production rate. Jones pointed out that the LDR would be formulated as two feedback rules where the work force (and production quantity) responds to some fraction of the difference between the long term work force requirements (or production requirements) indicated by a weighting of future sales forecasts and the existing work force(or production quantity). One of the first steps in setting up feedback rules is to determine the base from which the discrepancy from the desired level is to be measured. Previous research has used both the rate in the previous period and the planned rate for the decision period. In our work force we have chosen to use the work force on hand at the end of the preceding period rather than the work force planned as the base. In the production rule, the important deviation is between the actual production and the optimal production for the work force on hand. It is this difference that determines the amount of overtime or undertime and the production inefficiencies. I propose, therefore, to calculate the desired long term resource first. The work force will then be converted to a lowest unit cost production rate which will be the base for the second rule. The specific calculations necessary in this instance to derive the rules to be presented in the next were performed on a computer. Input consists essentially of the 13 cost coefficients which describe the quadratic cost function. Output consists essentially of three parts: a program trace, the work force and production rule coefficient, and the sales forecast weights for the two rules. The derived rules were as follows:

$$W(t)=W(t-1)+.2685(\text{expb}/5.67-4.59 -W(t-1)+.2364(320-I(t-1)/5.67)),$$

$$P(t)=5.67(W(t)+4.59)+.9475(\text{expd} -5.67(W(t)+4.59)+.5309(320-I(t-1)),$$

where expd, expb are the linear function of sales on each month.

Description of Search Heuristic Decision Rule:

This section proposes a search decision rule to the optimum work force and production rate of Jones (21) and the objective function of the classical "HMMS" (17, 18) paint factory scheduling problem. By means of the search decision rule approach it should be possible to eliminate the restrictions imposed by linear and quadratic cost models and thereby purse a more general and realistic approach to the aggregate scheduling problem. This problem is concerned with the determining what production and work force level should be set at time $t=1$ in order to minimize the total expected cost associated with producing a product over a planning horizon $t=1, 2, \cdots, n$. The production and work force decision which are decisioned first by PPP method are made periodically again by SDR and each time the planning horizon is adjusted to cover n units of time into the planning horizon is adjusted to cover n units of time into the future. Rigorous mathematical solutions of an optimal nature have been obtained only for simplified models based on linear and quadratic cost relationships. At first the monthly production and work force which were determined by parametric production planning of Jones are treated as "given" independent variables with trial values supplied by the SDR heuristic, following the objective function of "HMMS". Month end inventory levels are computed using the recursive relationship between sales, production, and inventory. The objective function is numerically evaluated using the trial values and recursive relationships for each month in the forecast horizon. The total cost objective function, $C(n)=\sum_{t=1}C(t)$, can be expressed in more general terms as a function of production and work force levels and inventory levels:

$$C(n)=F(P(1), W(1), I(1);\ P(n), W(n), I(n)).$$

The SDR test which uses pattern nonlinear search (19) for the "HMMS" paint factory objective function started in one month using beginning inventory, work force, and sales distribution data given in the original "HMMS" article (17). The initial starting vector was set at $W(1)=77.6$ men, $P(1)=471.9$ gallons/moth, and $I(1)=304.9$gallons/month. The PPP searched the optimum solution of $P(t)$, $W(t)$, then I used the pattern search routine for the resulting response surface and a final solution vector of $P(t)$, $W(t)$ for each month in the 12 months planning horizon. The upper and lower bounds of the feasible hyperspace were set at minimum data and maximum data. I think the upper production bound is restricted by the maximum of the factory productivity and the upper inventory bound is restricted by the capacity of storage and the shelf life of commodity. A seperate computer program was written to calculate the parametric production planning of $W(t)$, $P(t)$, and $I(t)$ for the identical sales pattern and initial condition assumed in the pattern search program. This routine uses the equations for $W(t), P(t)$, and $I(t)$ given on previous section. And the objective cost function of "HMMS" gives payroll expenses, hiring and firing costs, overtime costs, and inventory carrying charges for comparative purposes.

COMPARISON OF RESULTS AND COST EVALUATION

The well-known Holt, et al. (17) paint factory example was selected as a test vehicle which would permit a comparison among the known optimal decisions by the LDR method, near-optimal decisions made by the SDR, another known optimal decisions made by the PPP, and new decisions made by the combined decision rule(CDR).

Experimental Results:

The basic input to the evaluation procedure was the sales forecast for the test period. The perfect forecast situation is represented by the using the actual sales realized and assuming that this information was known is advance. All of these four models were used in conjunction with the sales forecast and actual sales to generate the system behavior for the test period. These simulations were achieved with the aid of the Taubert (22, 23) Fortran subroutine and the CDC computer facility at University of Washington which produced a standard output for all the models which was accurate, rapid and amenable for analysis and further economic evaluation. The work-force and production projections in the solution vector for months two through twelve can also be seen along with the resulting inventory levels. Because this new combined method used the SDR methodology, production is not only provided with a set of decisions for the current month, but is also provided with a set of projected decisions and their economic consequences for each additional month in the planning period. This forward-looking feature could prove to be a valuable aid in short range production and employment planning. On the basis of models, it was decided to plot the results for the following four models: LDR, PPP, SDR,and CDR. A comparison data is given in Table 1 in terms of production rate, work force, inventory level for each month of the 12 month period of simulated paint factory operation.

TABLE 1. A Comparison of LDR, PPP, SDR, and CDR.

		LDR	PPP	SDR	CDR
Month	Sales	Production (Gallons)			
1	430	467.72	461.26	472	461.33
2	447	441.32	440.50	443	440.50
3	440	414.88	417.11	418	414.88
4	316	379.83	380.39	385	379.83
5	397	375.28	379.80	376	375.80
6	375	367.09	317.34	366	366.00
7	292	358.51	360.91	360	358.57
8	458	380.57	390.73	382	380.57
9	400	376.80	385.71	379	376.80
10	350	366.70	372.14	366	366.00
11	284	366.59	367.31	359	359.00
12	400	404.00	408.72	401	401.00
Inventory (Galllons)					
1		292.72	286.26	305	286.38
2		289.03	281.76	301	282.00
3		263.92	258.88	279	259.02
4		328.75	324.25	348	324.25
5		309.03	309.06	327	309.12
6		301.12	305.39	318	301.15
7		369.64	376.30	386	369.64
8		295.21	312.03	309	295.27
9		272.01	297.74	288	272.10
10		283.71	314.88	304	283.74
11		365.30	397.09	379	365.30
12		366.24	400.91	380	366.24
Work Force (Men)					
1		78.63	78.56	78	77.63
2		75.32	75.37	74	74.11
3		72.24	72.44	71	70.61
4		69.55	69.82	68	67.32
5		67.61	67.98	66	64.53
6		66.29	66.74	64	62.07
7		65.66	66.13	63	60.23
8		65.87	66.53	63	58.72
9		66.49	67.25	64	57.07
10		67.68	68.39	64	62.07
11		69.67	70.17	67	70.06
12		72.62	72.93	70	70.00

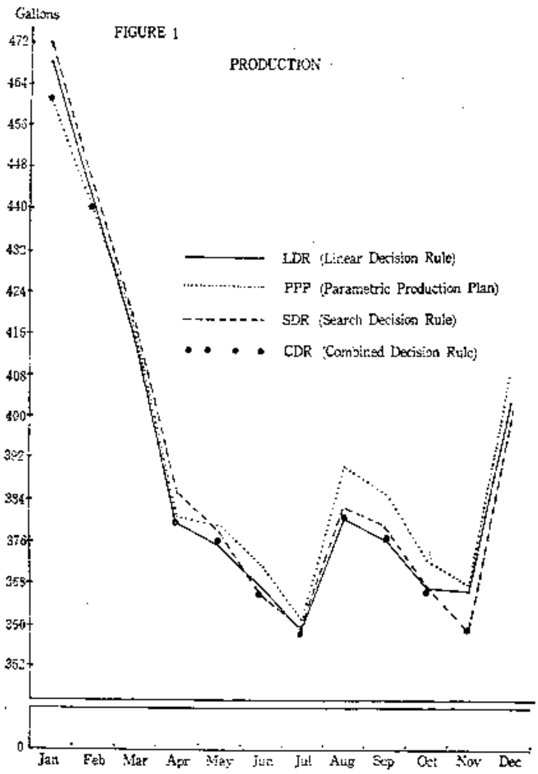
TABLE 2. Cost Analysis

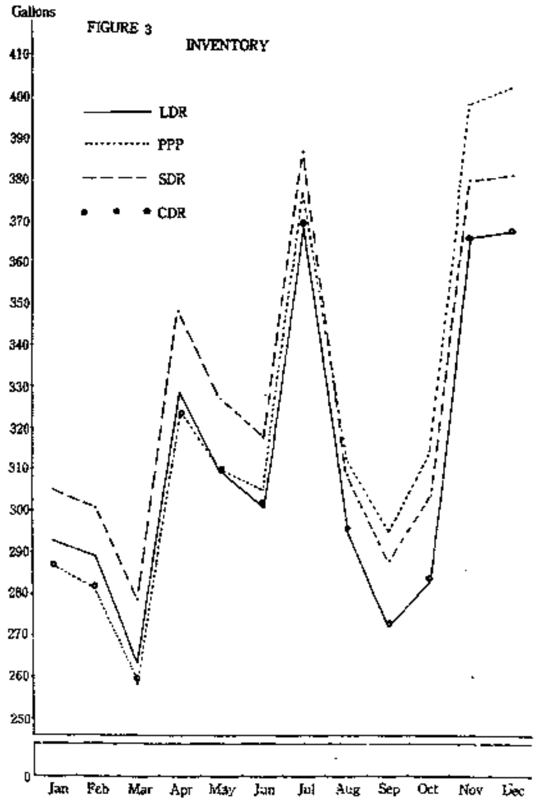
Month	LDR	SDR	CDR
1	$ 29,348	29,705	28,383
2	27,797	27,930	27,345
3	26,294	26,460	25,860
4	24,094	24,415	23,721
5	23,504	23,415	23,206
6	22,879	22,672	22,556
7	22,614	22,539	22,216
8	23,485	23,382	23,315
9	23,367	23,331	23,107
10	22,846	22,569	22,411
11	23,408	23,004	22,932
12	25,750	25,654	24,891
Total	$ 295,386	$ 295,371	$ 289,893

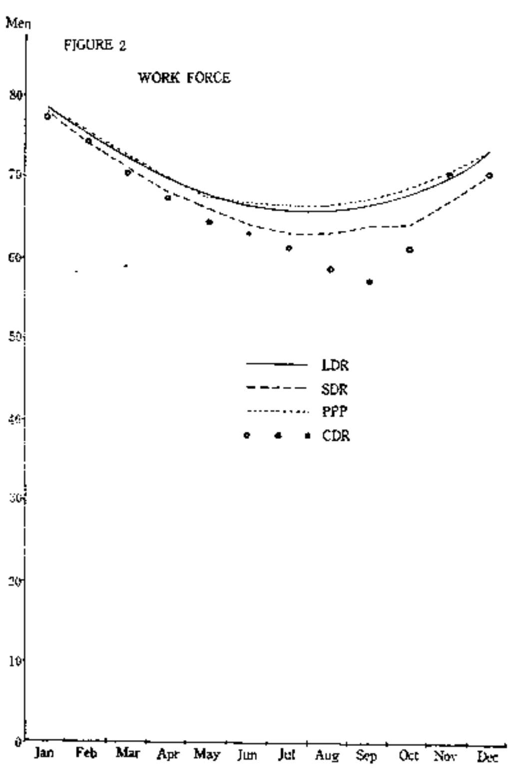
These results are plotted in Fig. 1, 2, 3 and 4 which show the behavior of production, work force, inventory, and overtime respectively for these four models for the 12 months test period. Fig. 2, for example, shows the trace of the production called for in each month of the test period by each of the four models.

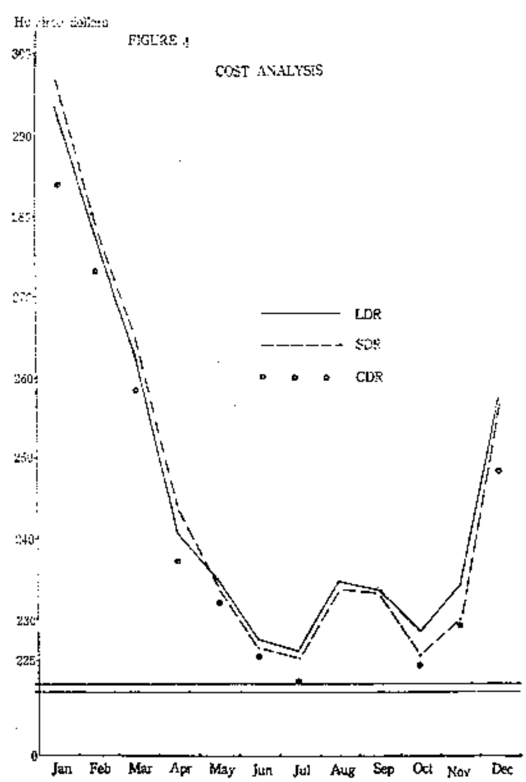
Comparison of the Cost Evaluation:

The final form of the quantitative evaluation of the performance of the various decision models was to assign a cost to each models for the full cycle's operation. The cost structure used to assign this cost was that nominally referred to as the perceived cost structure which preceded the quadratic approximation. This cost structure is essentially linear. Natually the use of the quadratic cost approximation would be guaranteed to give the minimum cost for the linear decision rules and we have explicitly admitted that this approximation is analytically attractive but not necessarily a more realistic representation of the


Gallons
FIGURE 1
PRODUCTION
LDR (Linear Decision Rule)
PPP (Parametric Production Plan)
SDR (Search Decision Rule)
CDR (Combined Decision Rule)
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec


Gallons
FIGURE 3
INVENTORY
LDR
PPP
SDR
CDR
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec


Men
FIGURE 2
WORK FORCE
LDR
SDR
PPP
CDR
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec


COST ANALYSIS
LDR
CDR
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

true cost structure. Ideally we would like to have the true cost structure to evaluate the performance of the models but we must settle for our perception of the cost structure which is in a form amenable to evaluation. The results of these evaluations are shown in Table 1 which gives a complete summarization of the evaluation of the performance of the four models.

Analysis of Performance:

As originally stated the purpose of this research was to develop a number of different decisions models of the same empirical situation. It was hypothesized that the result of such a research effort would be a more complete understanding of the decision process and the empirical situation. As an initial screening, the total cost comparison for the four models shows that the new combined decision model outperforms the other schedules by 1.6% at the same paint company. Gordon (13) indicated the normative model outperforms the actual schedule by 10% at Chain Brewing Co. The major difference in the cost components of the four performance models arises in the payroll and change of work force. Thus, while a cost improvement is provided by the new CDR, further improvements could be gained by the use of refined estimates from the LDR, SDR, and PPP models.

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