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AN APPLICATION OF EXTREMAL LENGTH METHOD TO THE BOUNDARY BEHAVIOR OF ANALYTIC FUNCTIONS

BY UN HAING CHOI

The purpose of this note is to present an example of a simple application of extremal length method to the boundary behavior of analytic functions.

THEOREM: *Let $f(z)$ be a bounded single-valued analytic function in the complement of E, where E is a totally disconnected compact set of positive capacity in the complex plane. Then it is not the case that for each z in E, except for those z in a set of capacity zero, there exist two arcs in the complement of E at z on which $f(z)$ has the limits a and b, $a \neq b$.*

Proof. Assume that the statement is not true. Choose a point in E where exist two arcs A and B in $\mathscr{C}E$ on which $f(z)$ has the limits a and b respectively.

Select a Jordan curve J in $\mathscr{C}E$ containing arcs A and B, and enclosing a subset E_J of E of positive capacity.

Let L be a subarc of $f(J)$ and consider the family $\mathfrak{F}$ of all curves with end points in L and $\partial(f(\mathscr{C}E)) - \{a, b\}$. Then the extremal length of $\mathfrak{F}$ is finite, and it follows from remark on p. 84 in [1] that the extremal length of $f^{-1}(\mathfrak{F})$ is also finite.

Let $G = \{z \in E_J$: a curve in $f^{-1}(\mathfrak{F})$ ends at $z\}$. Then G is of positive capacity by Lemma 1 in [2]. On the other hand G must be a countable set by remark 4 in [3]. Thus we have arrived at a contradiction. This completes the proof of the theorem.

References

[1] Ohtsuka, M. *Dirichlet Problem, Extremal Length and Prime Ends*, Van Nostrand, New York 1970.

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[3] McMillan, J. E. *Arbitrary Functions Defined on Plane Sets,* Michgan Math. J. **14** (1967). 445–447.

Inha University