目的函數가 고려된 適應샘플링技法에 의한 샘플링効率에 관한 硏究

논 문
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On the Sampling Efficiency by the Adaptive Sampling Technique based on Performance Index

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Abstract

In this paper we deal with that the performance indices by the three adaptive sampling control laws are computed and compared. It shows that the most effective control law is the integral input difference method. The techniques of simulation by Analog/Hybrid computer are presented and the results of the output illustrate that the maximum and minimum sampling interval can be applied to the time sharing of digital controller or computer.

I. Introduction

Various adaptive sampling criteria have been suggested in the past, largely on the basis of increasing sampling efficiency of error sampled data control systems.[1~4] But the sampling efficiency has not been defined and sampling has only been considered to be more efficient when similar output response characteristics are obtained with fewer samples for a given observation interval. In this manner, the sampling efficiency by various adaptive sampling methods cannot be compared with each other.

This paper gives an analytical apprach to increase the efficiency by introducing the mean sampling period which is the figure of merit under the constraint of performance index. Three kinds of sampling control laws[2~4] are described, and their characteristics and the method of increasing their efficiencies are presented. And also the simulation technique of adaptive control systems by Analog/Hybrid computer is shown. The results of the performance index by the computer are described and compared with each other.

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II. Adaptive Sampling Criteria

In this section, three kinds of adaptive sampling criteria[2~4] are presented and their characteristics and the method of improving their efficiencies are described.

1. Integral input difference method

Ciscato and Mariani[4] suggested this method to be highly immune to noise and quite easy to implement, and considered the effective behavior of the system during sampling interval.

The equation to determine the sampling instants is

$$I_d = \left| \int_{t_k}^{t_{k+1}} [m(t) - \overline{m}(t)] dt \right| \tag{1}$$

where, $m(t)$: input function

$\overline{m}(t)$: the output of hold device

The next sampling decision can be stated as follows;

$$I_d < A, \text{ no sampling occurs} \tag{2}$$

$I_d \geq A$, or $t-t_k$=Tmax, sampling occurs
Where A is a positive threshold constant and Tmax is the maximum allowable duration of the sampling interval based on the stability consideration.

Consider the Taylor series expansion of $m(t)$ as follows,

$$m(t)\Big|_{t>t_k} = m(t_k)+\dot{m}(t_k)(t-t_k)+\frac{\ddot{m}(t_k)}{2!}(t-t_k)^2+\cdots$$

In a system using zero order hold device,

$$m(t)=m(t_k)$$

and,

$$I_d=\left|\int_{t_k}^{t_k}[m(t)-\overline{m}(t)]dt\right| = \left|\frac{\dot{m}(t_k)}{2!}T_k^2+\frac{\ddot{m}(t_k)}{3!}T_k^3+\cdots\right| \quad (3)$$

where,

$$T_k=t_{k+1}-t_k$$

In the case that $\dot{m}(t_k)$ is large, the sampling period will be very short. For improving the sampling efficiency, it is necessary to use the hold device described in the following form, i.e.,

$$\overline{m}(t)=m(t_k)+\dot{m}(t_k)(t-t_k) \quad (4)$$

Then, the eq. (3) becomes

$$Id=\left|\int_{t_k}^{t_{k+1}}[m(t)-\overline{m}(t)]dt\right| = \left|\frac{\ddot{m}(t_k)}{3!}T_p^3+\frac{\dddot{m}(t_k)}{4!}T_k+\cdots\right| \quad (5)$$

and the effect of $\dot{m}(t_k)$ is cancelled.

In eq. (4), it is reasonable not to use the differentiation element $\dot{m}(t_k)$ because of the noise effect. In actual systems the output of hold device is often that of D/A convertor, and the appoximation of eq.(4) can be represented by the following first order hold equation,

$$\overline{m}(t)=m(t_k)+\frac{m(t_k)-m(t_k-1)}{T_{k-1}}(t-t_k) \quad (6)$$

for $t_k \leq t < t_{k+1}$

The result of the output using the first order is compared with that using zero order hold by an illustrative example in section Ⅳ. Of course, it is desirable to use higher order hold device to improve the sampling efficiency, but it is not available because of the problems in hardware and noise-sensitivity.

2. Amplitude sensitivity method

This method is presented by Tomovic and Bekey[2]

The sampling algorithm is derived from the sensitivity of the output of a sampled-data system to change in hold circuit output level. The characteristic is that it does not require differentiation of input variable in the control system. But it can be applied only to the system using zero order hold device. And it cause large error at the moment when the sensitivity is zero and the rate of the sensitivity is large.

The sensitivity function is defined as

$$U_k=\frac{\partial y}{\partial a_k}[u(t-t_k)-u(t-t_{k+1})]$$

$$U^{(r)}{}_k(t_k)=0,\ \text{r } 0,1,2,\ \cdots,\ n\text{-}1 \quad (7)$$

where a_k is the hold device output at $t_k \leq t < t_{k+1}$, i.e.,

$$A_k=\overline{m}(t_k)$$

By Taylor series expansion,

$$y(t)\Big|_{t>t_k}=y(t_k)+U_k(t)[\overline{m}(t_k)-\overline{m}(t_{k-1})]+\frac{\dot{U}_k(t_k)}{2!}[\overline{m}(t_k)-\overline{m}(t_{k-1})]^2+ \quad (8)$$

The change in system output since the last sample, $\triangle y_k(t)$ can be approximated by

$$y_k(t)\cong U_k(t)[\overline{m}(t_k)-\overline{m}(t_k)-\overline{m}(t_{k-1})] \quad (9)$$

and the control law can be stated as follows,

if $y(t)<C$, then no sampling occurs
if $y(t)\geq C$, or $(t-t_k)$=Tmax, then sampling occurs (10)

To reduce the error caused by the zero sensitivity, it is reasonable to consider the second term of equation(8) ; that is,

$$\triangle y_k(t)\cong U_k(t)[\overline{m}(t_k)-\overline{m}(t_{k-1})]+\frac{\dot{U}(t)}{2!}[\overline{m}(t_k)-\overline{m}(t_{k-1})]^2 \quad (11)$$

The disadvantages of this method are that it is necessary to find the sensitivity function to each system and is much more difficult than the integral input difference method.

3. Local sensitivity method

This method is also proposed by Bekey and

Tomovic.[3]

In such cases that the output is connected with the integrator (1/s), it is easy to realize the control law because the input of the integrator is the component of the local sensitivity function. Otherwise, it is difficult to set up the equation of control law and also to realize it. As in the amplitude sensitivity method, it is very errorneous when the sensitivity function is zero and the rate of it is pretty large. The result of above effect is shown in Fig. 12. (b), (c), ection V.

Local sensitivity function is defined as

$$U_k=\frac{\partial X(t_k)}{\partial t}=\lim_{\Delta t\to 0}\frac{X(t_k+\Delta t)-X(t_k)}{\Delta t} \quad (12)$$

This equals to the time derivation of the state vector evaluated at the sampling instants. The control law[3] which controls the sampling interval is

$$T_k=t_{k+1}-t_k=\frac{1}{A|U_k|+B} \quad (13)$$

where A can be considered as the threshold constant and the constant bis selected to make the system stable with the maximum sampling interval. It is also resonable to consider the rate of sensitivity U_k for the reduction of error caused by the zero sensitivity point.

Ⅲ. Analog/Hybrid Computer Simulation

Any adaptive sampling control system which does not contain differentiation can be simulated by Analog/Hybrid computer. In this section, Analog/Hybrid computer simulation of sampling control laws, hold device and plant is described by an illustrative system.

The plant transfer function of the system is given by:

$$G(s)=\frac{14(s+10)}{S^2} \quad (14)$$

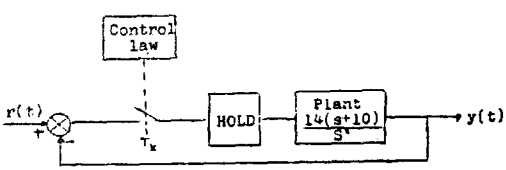
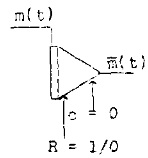


Fig. 1. Adaptive sampling control system.

which is the same considered by Tomovic and Bekey, Ciscato and Mariani. The block diagram of the system is illustrated in Fig. 1.

1. Sample/Hold simulation

In Fig. 2 the simulation of sample/zero order hold device using an integrator is shown



R	1	0
Mode	RS	HD

$$\overline{m}(t)=\begin{cases} m(t), & R=1 \\ m(t_k), & R=0 \end{cases}$$

Fig. 2. Sample/zero order hold simulator

when $R=1$, the integrator is reset and the output $\overline{m}(t)$ follows the value $m(t)$. As R changes the state abruptly from 1 to 0, the output $\overline{m}(t)$ holds the value at the transition time t_k.

It is considered to be advantageous to maintain the reset period as short as possible. But it is reqiued enough time to settle the output as the value of input. Hence the settling time of an integrator is the minimum reset time to be maintained. As a result it is the first thing to measure the setting time of the integrator, and time scaling is also needed for the reduction of relative error caused by the sampling period.

2. Sample/First order hold simulation

Another sample/hold device is sample/first order hold whose simulation is shown in Fig. 3. It is a direct decomposition of the following equation.

$$\overline{m}(t)=m(t_k)+\frac{m(t_k)-m(t_{k-1})}{T_{k-1}}(t-t_k).$$

where $T_{k-1}=t_k-t_{k-1}$

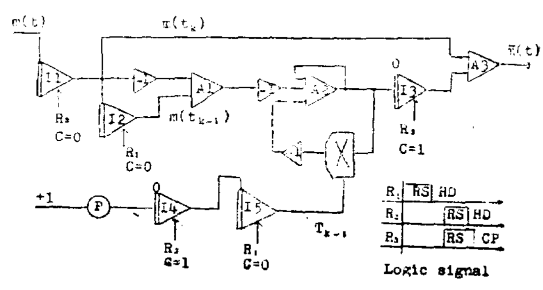


Fig. 3. Simulation of sample/first order hold

3. Plant simulation

The simulation method of the plant $G(s)$ is well know by the usual analog computer. As shown in Fig. 4. the output of the plant should be the output of an integrator to get the local sensitivity function U_k from the input of the last integrator.

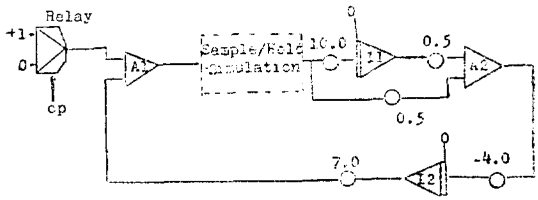


Fig. 4. Simulation of plant $G(s)=\frac{14(S+10)}{S^2}$

4. Simulation of integral input difference control law

The schematic diagram of simulation is shown in Fig. 5. It is the realization of eq. (1) and(2).

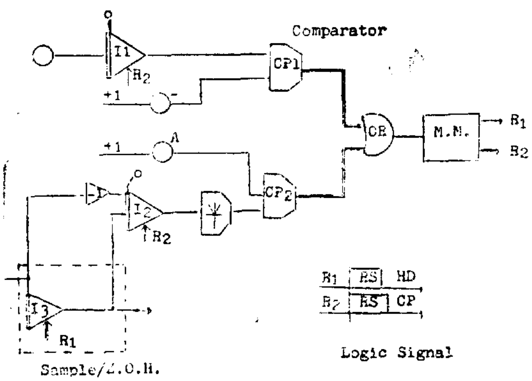
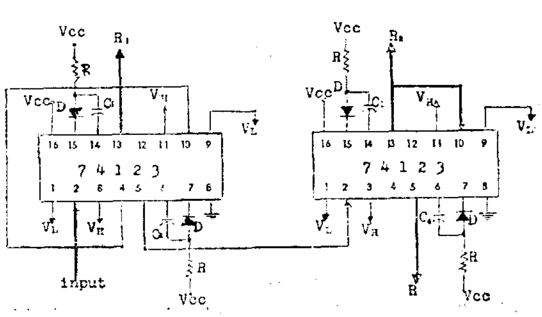


Fig. 5. Simulation of integral input difference control law

The comparator CP1 and the integrator I1 of the upper portion in Fig. 5 are to make the ma-



*note: $R=18K$
D=Switching Diode
V_H: High level
V_L: Low level

The values of capacitors to each sampling method.

Sampling metod	C_1	C_2	C_3	C_4
Integral input diff.	500PF	500PF	0.2μF	0.3μF
Amplitude sensitivity	0.15μF	0.02μF	0.2μF	0.3μF
Local sensitivity	0.15μF	0.02μF	0.2μF	0.3μF

Fig. 6. Schematic circuit diagram of monostable multi.

ximum sampling period Tmax. The monostable multivibrator shown in Fig. 5 is externally connected to the Analog/Hybrid computer for pulses to make the integrator reset and hold. The circuit diagram and the values of capacitors areshown in Fig. 6.

5. Simulation of amplitude sensitivity control law

For the simulation of this method, the first thing to do is to find the sensitivity function of the system. Then, the simulation is the direct decomposition of the solution with the Analog/Hybrid computer.

The sensitivity function of the illustrative example in Fig. 1. is

$$U_k(t)=70(t-t_k)^2 \qquad (15)$$

where $t_k \leq_k < t_{k+1}$

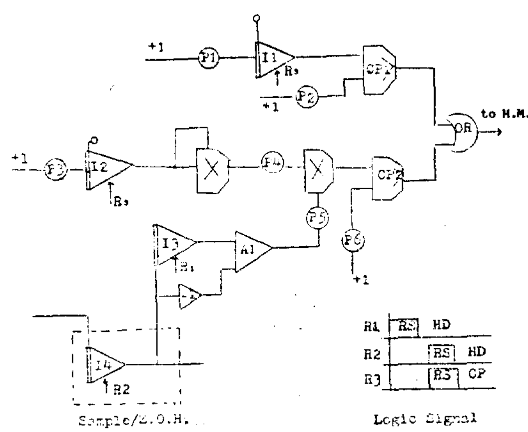


Fig. 7. Simulation of amplitude sensitivity control law

The adaptive sampling control law of eq. (9) and (10) is simulated in Fig. 7.

6. Simulation of local sensitivity control law

The simulation of this method also requires the solution of the discrete system as in the amplitude sensitivity method. But as stated in section Ⅱ 3. the sensitivity function U_k can be obtained from the input of last integrator. In Fig. 8. it is shown by an illustrative example.

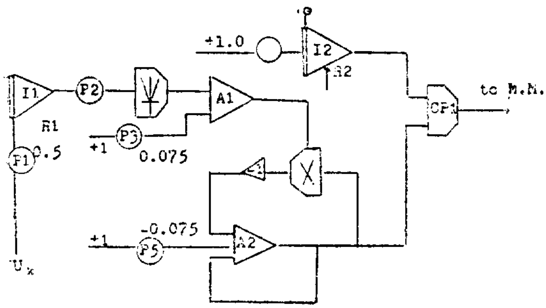


Fig. 8. Simulation of local sensitivity control law

Thus we have shown the simulalion techniques of adaptive sampling control systems. But there are some errors to be considered to simulate each component. These are the sampling error, hold error and timing error. All these errors should be minimized by time and magnitude scaling of the Analog/Hybrid computer.

Ⅳ. Results in an Illustrative Example

In section Ⅲ, simulation techniques of two hold devices and three adaptive sampling control laws are suggested. For the comparison of the performance index J or mean sampling period Tm, in each sampling criteria, it is necessary to compare each simulator into systems. Block diagram of such systems to be composed are shownin Fig. 9.

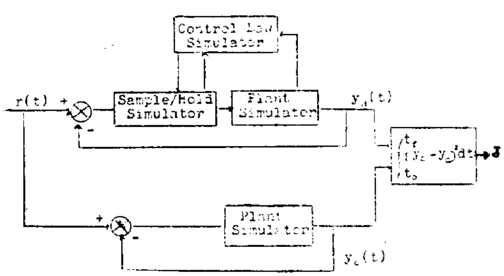


Fig. 9. Simulation of systems for performance index

The results of computed output in Fig. 9. with unit step input $r(t)$ is listed in Table 1. and is graphically shown in Fig. 10.

Table 1. Result of performance index;

Unit: $\times 10^{-3}$ c.u.

No. of Samples	Integral input diff. method	Amplitiude Sensitivity Method	Local Sensitivity Method
13	3.91	11.56	5.26
14	2.75/3.26*	7.29	3.60
15	2.45	5.25/6.06*	2.76
16	1.70	4.25	2.35
17	1.29/1.36*	3.38/3.84*	1.64/2.09*
18	1.21	2.74	1.29/1.83*
19	—	2.36/3.08*	1.08
20	0.66/0.85*	1.94/2.04*	1.00
21	—	1.86/2.46*	0.68/1.04*
22	0.5	1.88	0.54/0.91*
23	0.46	1.46	0.49
24	—	1.40	—
25	0.26/0.34*	1.03	—

Note: * : two values depend on the threshold constant
— : no value on the sampling number.

$$J=\int_0^{10}(y_c-y_d)^2\,dt$$

Table 1. shows that there can be two or more values of performance index at the same number of samples. It means that the sampling sequence is not Unique and cannot be controlled by the threshold constant only.

From the result of the computed output in Fig. 10, we can notice that the integral input difference method is the most efficient and stable.

The output waveforms of each sampling system are shown in Fig. 11, and 12.

The notation used in these figures are as follow,

y_c : output of continuous system.

y_d : output of sampled-data system

e : error (y_c-y_d).

$m(t)$: hold device output.

I_0 : output of ohe integrator for Tmax

$n(T_f)$: number of samples during $T_f=1.0$sec

$J(T_f)$: performance index during $T_f=1.0$sec

Comparing the responses in Fig. 11 and 12, we can find that there can be large error when the

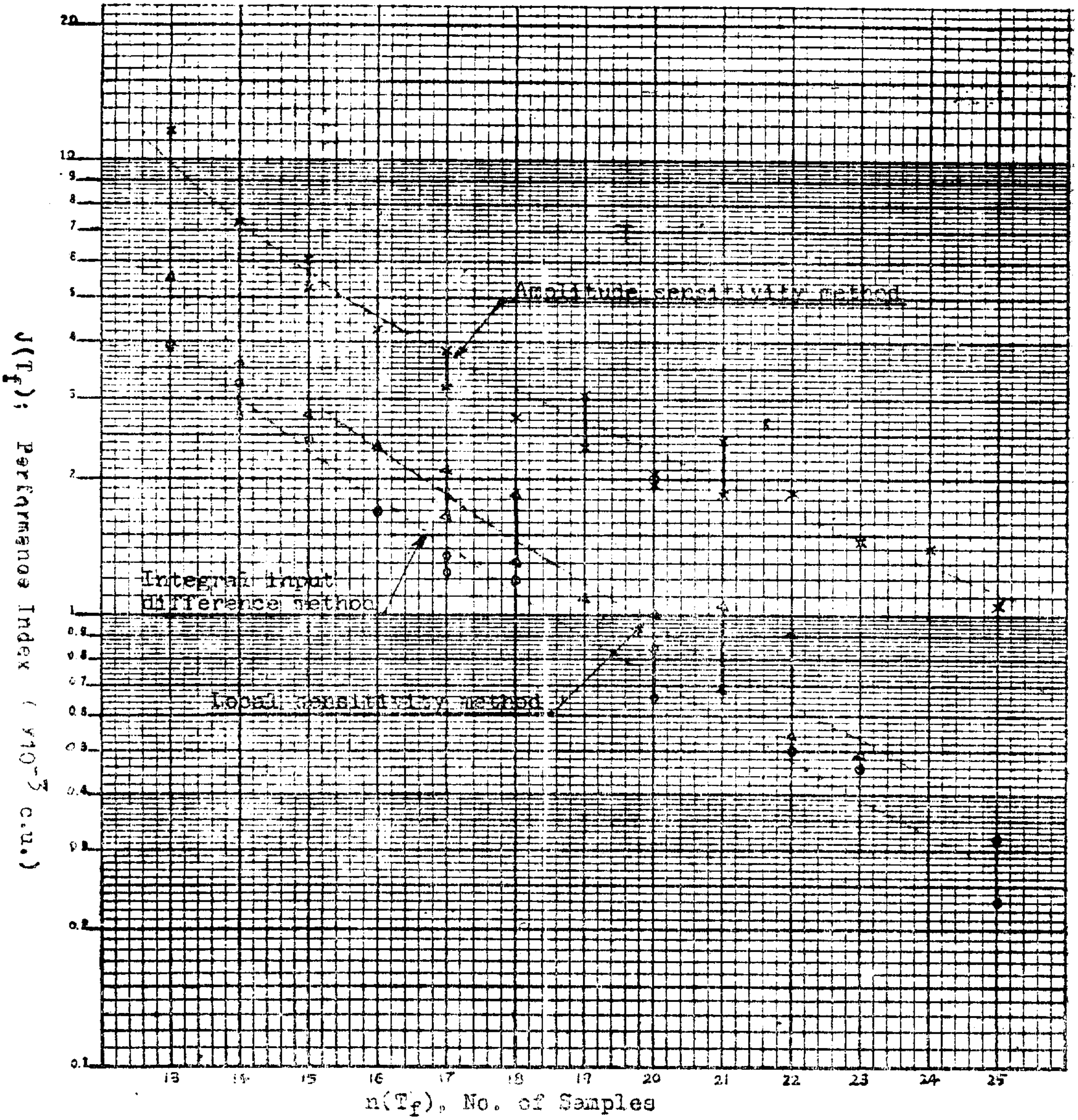
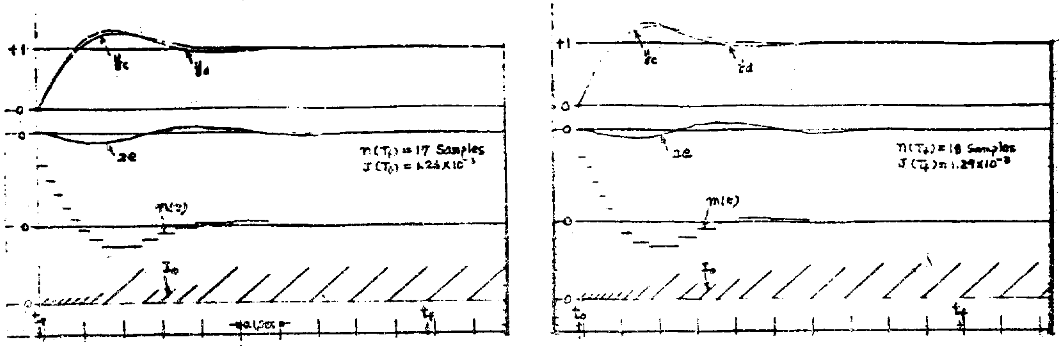


Fig. 10. Performance index in each control law.

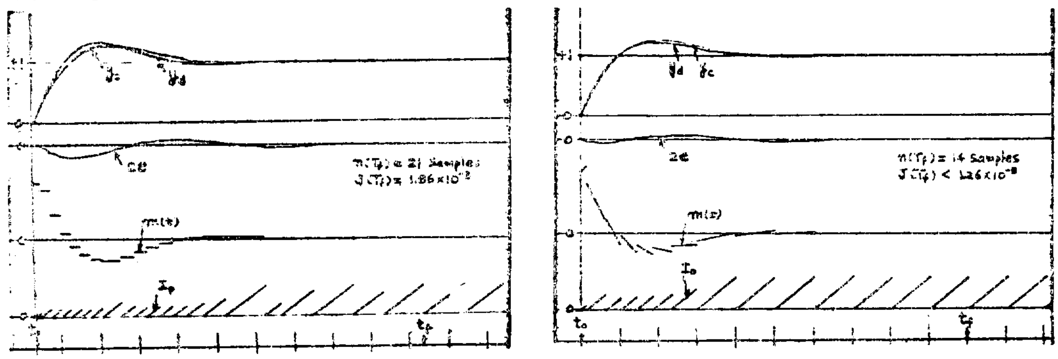
sampling instant meets by chance the characteristic point (where sensitivity is zero and the rate of it is not zero). Fig. 11. (d) and 12. (d) show that the performance index can be much reduced by the improvement of the hold device.

V. Conclusions

The analytical design method of the sampled data control system based on the continuous system with the constraint of performance index is described. The performance indices by the three adaptive sampling control laws are computed and compared. It shows that the most effective control law is the integral input difference method. And it is desirable to improve the hold device as shown in this paper from zero order hold to first order hold.

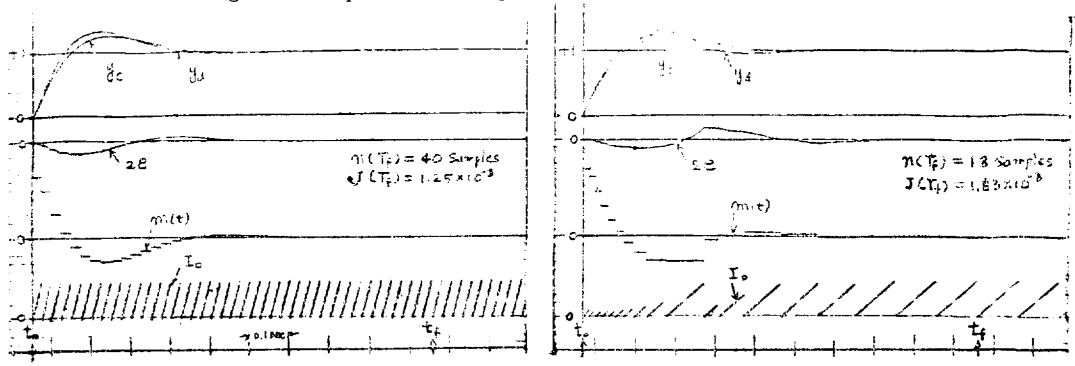


(a) Integral input difference method (Z.O.H.)　　(b) Local sensitivity method (Z.O.H.)

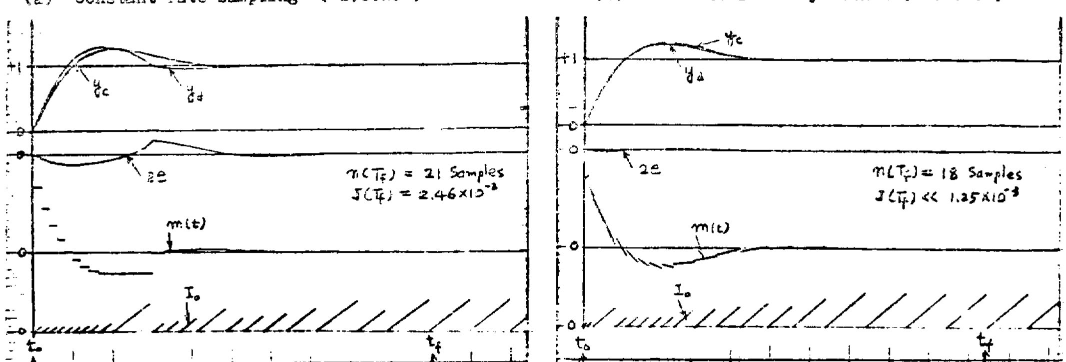


(c) Amplitude sensitivity method (Z.O.H.)　　(d) Integral input difference method (F.O.H.)

Fig. 11. Step function response of adaptive control systems.



(a) Constant rate sampling (Z.O.H.)　　(b) Local sensitivity method (Z.O.H.)



(c) Amplitude sensitivity method(Z.O.H.)　　(d) Integral inputdifference method(F.O.H.)

Fig. 12. Step function response of adaptive control systems.

The techniques of simulation by Analog/Hybrid computer are presented and the results of the output illustrate that the maxinum and minimum sampling interval can be applied to the time sharing of digital controller or computer.

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