A NETWORK MODEL FOR NAVAL OFFICER MANPOWER PLANNING*

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Abstract

For a large organization such as a military service which can foresee future growths in its manpower requirements, a systematic tool that can provide analysis of its present manpower structure and policies in terms of meeting the future requirements, is in order today. This paper proposes a network model for such a purpose. The ROK Navy officer corps manpower system is studied and formulated as a network model, which may be expressed as a linear programming problem of minimizing total cost. An appropriate cost concept is developed and the out-of-kilter algorithm of Ford and Fulkerson is computer programmed to be used as a solution procedure for this network problem. A case study is conducted with a set of hypothetical data on a possible Navy combat-line specialty manpower problem.

1. Introduction

In a military service which sees rapid growths in its force size in near future one would like to see how the present manpower policies can fit into the future state of affairs in terms of meeting the manpower requirements. Also the planner would desire to foresee any possible bottlenecks or trouble spots in his organization's manpower scene of a few years in the future. If such a foresight were available, proper coordinations in the areas of recruiting and retirements control could be made to alleviate such problems beforehand.

The model proposed in this paper is designed to work on the assumption that the future manpower requirements of the organization are quantitatively delineated and a set of promotion and retirements policies is clearly available.

2. Mathematical Development of Model

2.1 Characteristics of the Manpower System

The Navy officer corps can be said to be a family of specialties and each specialty a class of subsets each of which is characterized by specialty and a rank. An officer may have only one rank at a time. This implies that an officer may be in one subset at one time. As time passes an officer

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moves from one subset to another. Let us call these subsets 'career states' for our purpose.

The officer ranks in the ROK Navy are, from the lowest Ensign(ENS), Lieutenant Junior Grade (LTJG), Lieutenant(LT), Lieutenant Commander(LCDR), Commander(CDR), Captain(CAPT), and Admirals. We deal with only the first six here. Specialties are, for example, combat-line, engineering-line, supply, communications, construction, instructors, etc.

2.2 The Linear Network Model

If we represent each career state as a labeled node and the path of a transfer from one state to another as a directed arc from one node to another, we may represent the manpower system of the Navy officer corps as a network such as the one in Figure 1. In Figure 1 arcs from So (source node) to numbered nodes represent the present numbers of officers in ranks A,B,C, through F, or direct inputs to those ranks in Year 1, Year 2, through Year 5. The numbers in nodes represent years. Vertical paths represent promotions. Paths to Si (sink node) represent losses from career states to the general public due to discharge, deaths, retirements, etc. The figure does not depict flows between different specialties. This is eliminated for simplicity. Multiplicity of specialties will mean three-dimensional network system with each layer being one like Figure 1.

In such networks as so far described we associate three variables with each are flow. They are

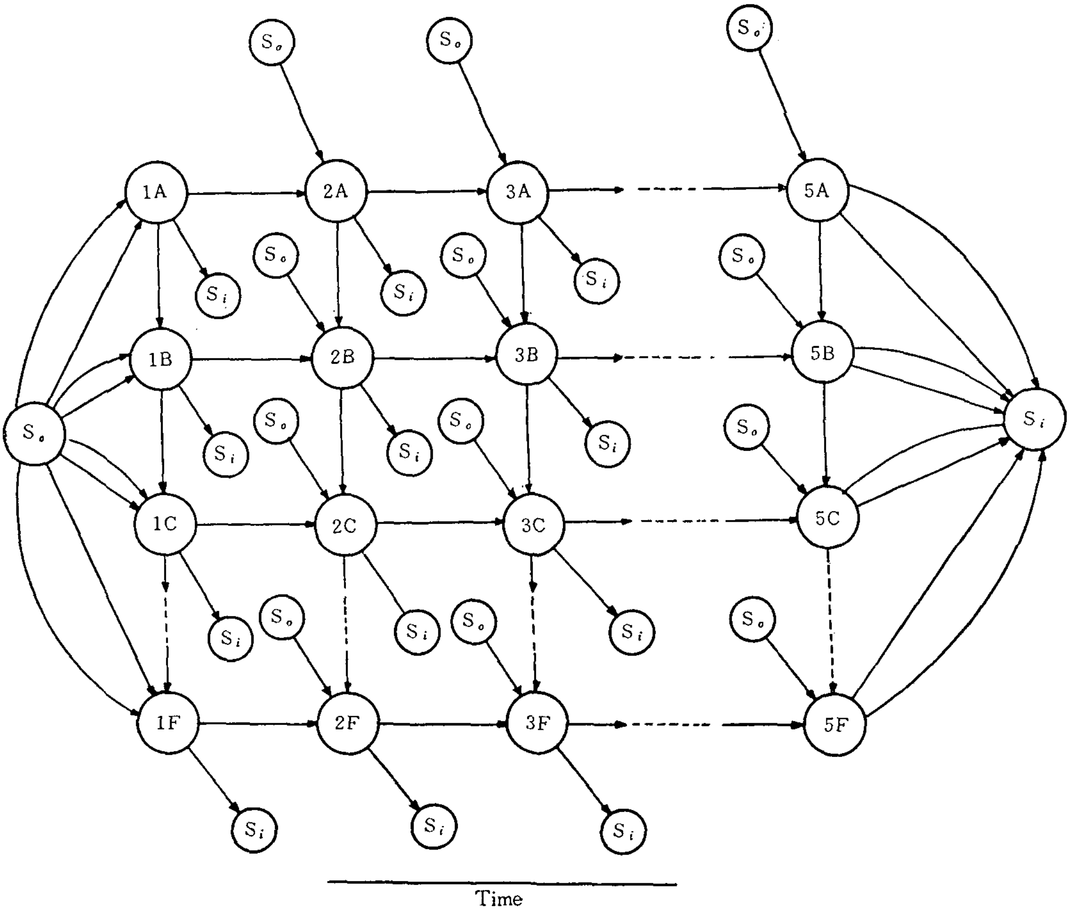

Figure 1. Network Representation of Officer Manpower System

lower bound (L_{ij}), upper flow bound (U_{ij}), and unit cost (C_{ij}). We seek the set of optimal flows throughout the network which satisfies the flow constraints and at the same time minimizes the total cost. Let X_{ij} denote the flow assigned to the arc from node i to node j. Ford and Fulkerson states that such a network optimization problem can be expressed in a linear progarm as following:(5)

$$\text{Minimize} \quad \sum_{i,j} C_{ij}\cdot X_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{k} X_{ik} - \sum_{k} X_{ki} = 0 \text{ for all } i, \tag{2}$$

$$\text{and} \quad L_{ij} \leqq X_{ij} \leqq U_{ij}. \tag{3}$$

Such lineac programming problems as depicted by equations (1), (2), and (3) can be efficiently solved by the out-of-kilter algorithm of Ford and Fulkerson.(5) Specific values for C_{ij}, L_{ij}, and U_{ij} for each arc in the network must be provided in integer for the solution to be possible.

3. MODEL STRUCTURE

3.1 Network Structure

Figure 1 depicts the general concept of manpower flows over the period of five years. It does not, however, include arcs representing distribution of manpower over units which constitute the Navy. As indicated earlier, the main interest in setting up the model is to see how well the present set of manpower policies and conditions will meet the estimated future manpower requirements of the units. Out of such necessity out model must include distribution arcs for each career state (combination of sqecialty and rank) in each year. A portion of the total network that depicts various flows for a single career state for a particular year period is shown in Figure 2.

Direct input are from the source to the ensign rank would mean the flow of newly commissioned officers into the force and to other ranks it would mean the flow of officers recommissioned out of the reserve force for the purpose of filling up the deficiencies that may exist in trying to meet the manpower requirements with the available active service officers.

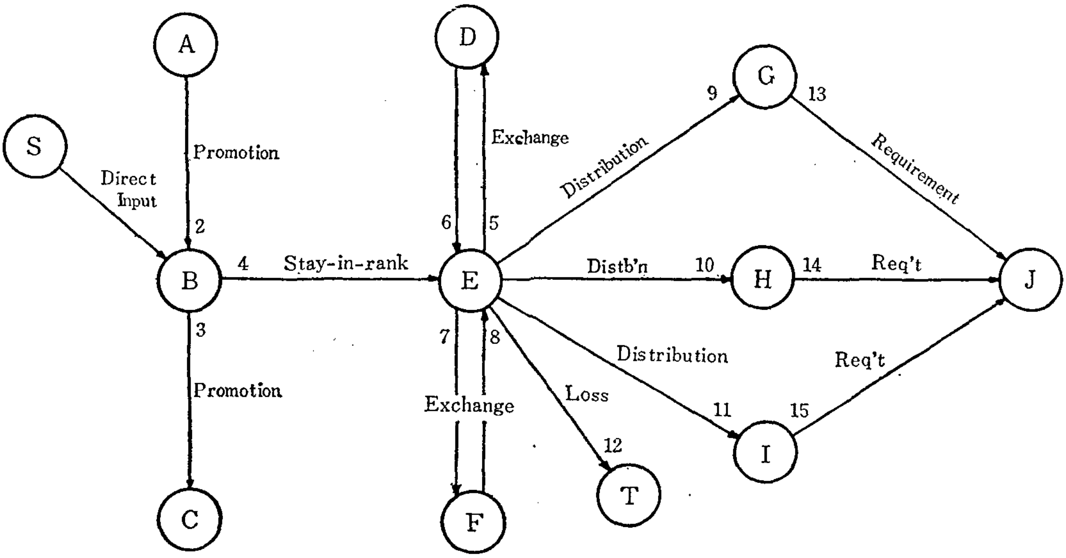

Figure 2. Network Structure for a Career State in a Year Period

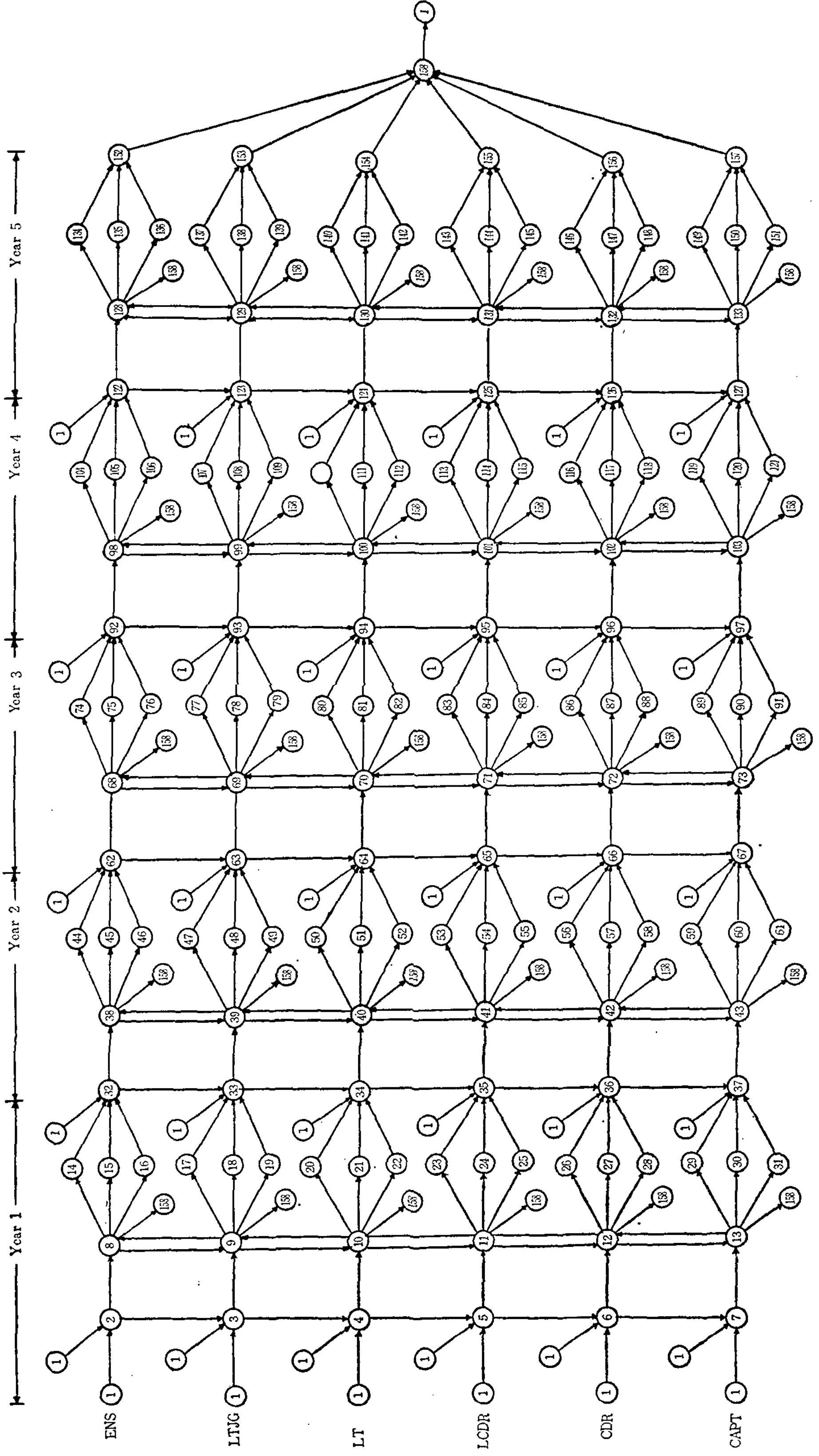

Figure 3. Total Network for Case Study Problem

In Figure 2, arc ＃2 is for the promotion flow from the neighboring lower rank and arc ＃3 to the next higher one. Arc ＃4 indicates the flow of officers who stay in the same rank and are subject to distribution to meet requirements in arcs ＃13, ＃14, and ＃15, and the actual assigned flows are represented by arcs ＃9,, ＃10, and ＃11 respectively to the units of the Navy. Sometimes a commander might be called upon to take on the job of a captain billet, or even a lower rank's, i.e., a lieutenant commander billet. Arcs ＃5, ＃6, ＃7, and ＃8 representing exchange flows between two neighboring ranks describe such situations.

If we frame a network in the manner described so far for the six ranks in a particular specialty over the period of five years, one like Figure 3 can be obtained. Note that a year's manpower consequence becomes input for the next year's scene.

3.2 Derivation of Costs

3.2.1 Costs for Distribution Arcs

The total effectiveness of the Navy would be a function of several contributing factors, which we denote as e_1, e_2, e_3, etc. This notion can be expressed as in equation(4).

$$E_t = f(e_1, e_2, e_3,) \tag{4}$$

Let E_{mp} denote the portion of the Navy's effectiveness(E_t) solely depended upon by its manpower status. Then there are several variables that affect E_{mp}. They are degree of requirement fill, morale, amount and quality of training given to men, health conditions, etc. In our study we consider the degree of fill as the major manpower variable.

We hypothesize, then, that E_{mp} can be expressed as a weighted linear combination of E_{mp}(j)'s, where E_{mp} (j) denotes manpower effectiveness index for the jth unit in the Navy. Equation (5) expresses this concept.

$$E_{mp} = \sum_j W_j \cdot E_{mp}(j) \quad \text{where} \quad \sum_j W_j = 1 \quad \text{and} \quad W_j \geq 0. \tag{5}$$

Let R_{ij} denote the number of officers of career state i required in unit j and X_{ij} the number of officers of career state i assigned to unit j. Let W_j of equation (5) be called activity weight, which is a quantitative measure of the degree with which an activity's (unit's) role contributes to the Navy's total effectiveness.

Let S_i denote rank significance, which is to represent the degree of capability which an officer of a rank has in carrying out his function in relation to officers in other ranks within a specialty. The case study illustrates an example of S_i's for the combat-line specialty. Let us define the unit effectiveness manpownr index(e_{ij}) by the portion of E_{mp} (j) increased by assigning an officer of career state i to activity j. Then the following relationship holds:

$$e_{ij} = \frac{W_j \cdot S_i}{R_{ij}} \tag{6}$$

Also E_{mp} can be expressed as follows:

$$E_{mp} = \sum_i \sum_j e_{ij} \cdot X_{ij} \tag{7}$$

We assume here that eah of the activities in the Navy requires some number of officers from each career state. This means that an officer can be assigned to any of the activities. Then the unit opportunity cost of assigning an officer of a career state to an activity is the effectiveness contribution foregone by assigning him to the particular activity. This is the difference between the greatest possible unit effectiveness index and that of the particular assignment. Therefore, the unit cost of a distribution arc ($c_{\text{dist}}(i,j)$) may be expressed as follows:

$$c_{\text{dist}}(i,j^*)=\underset{j}{\text{Max}}(e_{ij})-e_{ij}{}^* \tag{8}$$

or,

$$C_{\text{dist}}(i,j^*)=\underset{j}{\text{Max}}\left(\frac{W_j\cdot S_i}{R_{ij}}\right)-\left(\frac{W_j{}^*\cdot S_i}{R_{ij}{}^*}\right) \tag{9}$$

3.2.2 Costs for Loss Arcs

The concept of opportunity cost employed in determining costs for distribution arcs may similarly be applied to costs for loss arcs. Loss arcs represent flow of officers leaving a career state in a particular year period. Losing a person from a career state means loss of his capability to contribute to the effectiveness of the organization. Therefore, the unit cost for a loss arc is assigned with the maximum of unit costs for distribution of the career state. Let $C_{\text{loss}}(i)$ represent the unit cost for the loss arc pertaining to career state i. Then the following equation describes the above concept:

$$C_{\text{loss}}(i)=\underset{j}{\text{Max}}(C_{\text{dist}}(i,j)) \tag{10}$$

3.2.3 Costs for Exchange Arcs

Let $C_{\text{ex}}(i_1,i_2)$ denote the unit cost of assigning an officer of career state i_1 to a billet of career state i_2. When such an exchange occurs, a difference in effectiveness contribution occurs. The general manner that is employed in exchange assignments is either that an officer who is a junior in a rank takes the billet which requires a senior of the neighboring lower rank, or that an officer who is a senior in a rank takes the billet which requires a junior of the neighboring higher rank. A difference in effectiveness contribution occurs because either an officer cannot fully employ his capability and experiences, or that one has to carry out the jobs of a billet which requires a man of higher caliber and more experience.

Also we assume that the Navy does not desire the exchanges take place so much that requirements for officers of a career state have to be sacrificed. The notions described above are reflected in the following definition of unit cost of an exchange arc:

$$C_{\text{ex}}(i_1,i_2)=\text{Max}[\underset{j}{\text{Min}}(e_{i_2j})-\underset{j}{\text{Max}}(e_{i_1j}),\underset{j}{\text{Max}}(C_{\text{dist}}(i_1,j))]\text{ where }i_2>i_1. \tag{11}$$

Equation (11) pertains to the case when i_2 is a higher rank than i_1. A similar definition of the unit cost for the case of i_1 being higher than i_2 is as follows:

$$C_{\text{ex}}(i_1,i_2)=\text{Max}[\underset{j}{\text{Min}}(e_{i_1j})-\text{Max}(e_{i_2j}),\ \text{Max}(C_{\text{dist}}(i_1,j))]\text{ where }i_1>i_2. \tag{12}$$

The first term inside the bracket of each of the equations (11) and (12) must be positive, because otherwise it would mean that a person of a higher rank cannot contribute so much to the total effectiveness as one of a lower rank can. Values for S_i's should be appropriately determined to ensure this condition.

3.2.4 Costs for Direct Input Arcs

Direct input for the ensign rank means new commission, so this does not mean any cost to the Navy in the sense of opportunity cost. Commissioning of reserve officers for the purpose of filling up deficiencies in meeting requirements for ranks other than ensign means cost to the Navy in two ways: First, it requires capital layout to retrain them. Second, they would not have had the same amount of current active service experience as those officers who are already in the active service. This would mean difference in job capabilities between officers of these two types.

Now, we define the range of values for the unit cost of the direct input arc for career state i ($c_{di}(i)$) as the following, except for ensign rank for which the values are zero.

$$0 < C_{di}(i) < \underset{j}{\text{Max}}(C_{dist}(i,j)) \tag{13}$$

In the case study $C_{di}(i)$ is set to be $1/2 \underset{j}{\text{Max}}\, C_{dist}(i,j)$. This definition is compatible with the range defined by equation (13).

3.3 Flow Limits

Upper flow limit of a requirement arc is the number of officers required. Lower limit is set to be equal to 80% of the upper limit in order to reflect a hypothetical policy that the requirement of any activity for officers of a career state must be saitsfied at least by 80% so that it can function without causing too much disruption in the overall missions of the Navy.

Legal and organizational factors limit the number of officers who can be promoted to the higher rank each year. This limit becomes the upper limit for promotion arcs. The lower limits set equal to the upper limit because this ceiling is almost always 100% reached. Discussion on flow limits for other arcs will be defered till the case study.

4. Optimization Procedure--the Out-Of-Kilter Algorithm

References (5) and (10) contain detailed discussions on the out-of kilter algorithm and its applications. In solving a complex network involving several hundreds of arcs and nodes by a computerized out-of kilter routine it is important to minimize computer time. The labeling part is the key area where a good scheme can considerable total computer time. The FORTRAN program for the out-of-kilter algorithm developed by the author is listed below, and followed by a discussion on the labeling scheme employed in the program.

4.1 The FORTRAN Program for the Out-of-Kilter Algorithm

```
      PROGRAM KILTER (INPUT, OUTPUT, TAPE10=INPUT)
C     PROGRAM KILTER OBTAINS A MINIMAL COST FLOW ASSIGNMENT TO CAPA-
C     CITATED NETWORK BY MEANS OF THE OUT-OF-KILTER ALGORITHM.
C     PROGRAM BY DOUG WOON CHO, I.E. DEPT, KAIS, OCTOBER, 1975.
      LOGICAL BRKTHRU, KISS, HONEY
      IMTEGER OUTKT, AA, TERM, ORIGIN, P, S, A, EPS, EPS1, PI, SS,
      *T, C, U, X, R, UU, V, G, H, EJT, UUB, CC
      DIMENSION SS(500), T(500), C(500), U(500), L(500), X(500),
      *              PI(500), UU(500), V(500), G(500), H(500), LL(500),
      *              R(500), CC(500), STATE(500)
```

```
      DATA PI/500*0/, X/500*0/. EJT/0/, STATE 500*2HK/
C     VARIABLE DESCRIPTION
C     M IS NO. OF NODES
C     N IS NO. OF ARCS
C     SS IS SOURCE NODE
C     T IS SINK NODE
C     C IS UNIT COST OF EACH ARC
C     CC IS MODIFIED UNIT COST
C     PI IS NODE PRICE
C     U IS UPPER FLOW LIMIT
C     L IS LOWER FLOW LIMIT
C     READ IN AND CHECK INPUT NETWORK DATA
      READ 1,M,N
    1 FORMAT(2I10)
    2 FORMAT(6I10)
      DO 3 I=1,N
      READ 2, K, SS(I), T(I), CC(I), L(I), U(I)
      IF (I.EQ.K) GO TO 4
      PRINT 501, I
  501 FORMAT (//, ≠CARD ORDER MIXED UP FOR ARC≠.I4,/)
      STOP
    4 IF(L(I).LE.U(I)) GO TO 5
      PRINT 500, I
  500 FORMAT (//, ≠WRONG DATA--LOWER LIMIT GREATER THAN UPPER LIMIT
     *                IN ARC≠, 14,/)
      STOP
    5 IF (EOF(10).NE.O. AND. K.LT.N) 6,3
    6 PRINT 502
  502 FORMAT(//.≠INSUFFICIENT NUMBER OF INPUT DATA CARDS≠, /)
      STOP
    3 CONTINUE
C
C       COUNT ARCS BEGINNING ANDENDING AND ENDING AT NODES.
C
        M2=M+2
        DO 10 I=3, M2
        UU(I)=0
 10     V(I)=0
        DO 20 J=1, N
        JA=SS(J)+2
        JB=T(J)+2
        UU(JA)=UU(JA)+1
 20     V(JB)=V(JB)+1
```

```
C
C          CUMULATE COUNTS
C
           UU(1)=1 $ UU(2)=1
           V(1)=1 $ V(2)=1
           M1=M+1
           DO 30 I=3, M1
           UU(1)=UU(1)+UU(I-1)
  30       V(I)=V(I)+V(I-1)
C
C              SET UP ARC LOCATOR LISTS
C
               DO 40 J=1, N
               JA=SS(J)+1 $ JB=T(J)+1
               JA1=UU(JA) $ JB1=V(JB)
               G(JA1)=J $ H(JB1)=J
               UU(JA)=UU(JA)+1
  40           V(JB)=V(JB)+1
               DO 50 J=1, N
  50           C(J)=CC(J)
               JJ=1 $ AA=0 $ BRKTHRU=.TRUE. OUTKT=O
C
C   LOOK FOR AN OUT-OF-KILTER ARC
C
  60 IF (X(JJ) .LT. L(JJ)  .OR.C(JJ) .LT. O .AND. X(JJ) .LT. U(JJ)) 70, 80
C
C       ADDED REVERSE FLOW POSSIBLE
C
  70    TERM=SS(JJ)
        ORIGIN=T(JJ) $LABORG=JJ
        GO TO 821
  80 IF (X(JJ) .GT. U(JJ). OR. C(JJ) . GT. O. AND. X(JJ) .GT. L(JJ) 81, 82
C
C          ADDED FORWARD FLOW POSSIBLE
C
  81       TERM=T(JJ) $ ORIGIN=SS(JJ) $ LABORG=-JJ
 821       R(1)=ORIGIN
           IF(.NOT. BRKTHRU .AND. JJ .EQ. AA) 90, 91
C
C              ZERO OUT ALL PREVIOUS LABEIS
C
  91           DO 92 I=1, N
  92           LL(I)=0
               S=1
  90           P=1 $ AA=JJ $ BRKTHRU=.f.
               LL(ORIGIN)=LABORG
               GO TO 100
C
C                CHECK NEXT ARC
C
```

```
  82             JJ=JJ+1
                 IF(JJ .LE. N) 59,89
  59             PRINT 2222, JJ
 2222            FORMAT (5X, ≠JJ =≠, I6)
                 GO TO 60
C
C  PRINT ROUTINE
C
 89 PRINT 1234
 1234 FORMAT (1H1, ///, 5X,≠ARC NO≠. 3X,≠SOURCE≠, 3X,≠SINK≠, 3X,≠INI
      1          U/COST≠, 3X,≠FIN U/COST≠, 3X,≠LOWER LIM≠,
      2          3X,≠OPT FLOW≠, 5X,≠STATE≠,/)
      DO 333 I=1,N
      PRINT 1235, I,SS(I), T(I), CC(I), C(I), L(I), U(I), X(I), STATE(I)
 1235 FORMAT (5X, I4, 4X, I4, 5X, I7, 5X, I7, 6X, I7, 5X, I8, 5X, I6, 7X, A10,/)
      EJT=EJT+1
      IF(EJT .GE. 25) 301, 333
  301 EJT=0
      PRINT 1234
  333 CONTINUE
     PRINT 1111, OUTKT
 1111 FORMAT (//, 5X,≠THE NUMBER OF ARCS THAT CANNOT BE BROUGHT
      *                    INTO KILTER IS≠,I5)
      STOP 10
C
C     TRY TO LABEL THE FORWARD ARCS
C
  100 I=R(P)
      UUA=UU(I)  $ UUB=UU(I+1)-1
      DO 101 A=UUA, UUB
      J=G(A) $ K=T(J)
      KISS=X(J) .LT. L(J) .OR. C(J) .LE. O .AND. X(J) .LT.U(J)
      IF (LL(K) .EQ. O .AND. KISS) 1011, 101
 1011 LL(K)=J $ S=S+1 $ R(S)=K
  101 CONTINUE
C
C         TRY TO LABEL THE BACKWARD ARCS
C
          JA=V(I) $ JB=V(I+1)-1
          DO 102 A=JA, JB
          J=J(A)  $ K=SS(J)
          KISS=X(J)  .GT.U(J).OR.C(J).GE.O.AND.X(J).GT.L(J)
          IF (LL(K).EQ.O.AND.KISS) 1021, 102
 1021     LL(K)=-J $ S=S+1 $ R(S)=K
  102     CONTIINUE
```

```
C
C               TEST FOR THE TERMINAL LABELED
C
                IF(LL(TERM) .NE. O) GO TO 200
                P=P+1
C
C                   IF SCAN LIST EXHAUSTED, NON BREAKTHRU
C
                    IF (P .GT. S) 103, 100
C
C       FIND DELTA FOR NON-BREAKTHRU
C
  103 ESP1=1.E10
      DO 104 J=1, N
      JA=SS(J) $   JB=T(J)
      KISS=LL(JA) .NE. O .AND. LL(JB) .EQ. O .AND. X(J) .LE. U(J) .AND. C(J) .GT.O
      HONEY=LL(JA) .EQ. O .AND. LL(JB) .NE. O .AND. X(J) .GE.L(J) .AND. C(J) .LT.O
      IF(KISS .OR. HONEY) 1041, 104
 1041 EPS1=MINO (EPS1, IABS(C(J)))
  104 CONTINUE
C
C           TEST FOR CASE 2
C
            EPS=EPS1
            IF (EPS .NE. 1. E10) 110, 120
  120       JA=LL(ORIGIN)    $ JB=C(JJ)
            JAB=JAJB
            IF(C(JJ). EQ. O. OR. JAB. GE. O) 150, 121
  121       EPS=IABS(C(JJ))
C
C                   CHANGE UNIT COSTS
C
  110           DO 111 J=1, N
                JA=SS(J)    $ JB=T(J)
                IF (LL(JA) .EQ. O .AND. LL(JB) .NE. O) 112, 113
  112           C(J)=C(J)+EPS
                GO TO 111
  113           IF (LL(JA) .NE. O .AND. LL(JB) .EQ. O) 114, 111
  114           C(J)=C(J)-EPS
  111           CONTINUE
C
C                   CHANGE NODE PRICES
C
                    DO 131 I=1, M
                    IF(LL(I) .NE. O) GO TO 131
                    PI(I)=PI(I)+EPS
  131               CONTINUE
C
C                   CHECK TO SEE IF POSSIBLE TO BRING INTO KILTER EVER
C
```

```
  160          IF(EPS .EQ. EPS1 .OR. X(JJ) .EQ. L(JJ) .OR. X(JJ) .EQ. U(JJ)) 60, 150
  150          STATE (JJ)=2HNK
               OUTKT=OUTKT+1
               GO TO 82
C
C        FIND FLOW INCREMENT IN CYCLE
C
 200     EPS=1.E10
         BRKTHRU=.T.  $KT=TERM $J=1
C
C     BREAK-LOOP
C
 210 KQ=LL(KT)     $ KP=IABS(KQ)
     IF(KQ.GT.O) 220, 230
 220 KT=SS(KP)
     IF(C(KP).GT.O) GO TO 221
 222 EPS=MINO(EPS, IABS(U(KP)-X(KP)))
     GO TO 223
 230 KT=T(KP)
     IF(C(KP). GE.O) 221, 222
 221 EPS=MINO(EPS, IABS(L(KP)-X(KP)))
 223 R(J)=KQ
     IF(KT .EQ. TERM) GO TO 240
     J=J+1
     GO TO 210
C
C        INCREMENT FLOW
C
 240 DO 241 I=1, J
     IF (R(I).GT.O) GO TO 242
     JA=-R(I)
     X(JA)=X(JA)-EPS
     GO TO 241
 242 JA=R(I)
     X(JA)=X(JA)+EPS
 241 CONTINUE
     GO TO 60
     END
```

4.2 Computational Methods

4.2.1 Labeling

Clasen offers a method that requires less searching for arcs with exactly one node labeled. (1) We set up what is called a scan list, with which two indices are associated: *S*, the length of the length of the scan list, and *P*, the position of the scanner. Denote the list itself by $R(1)$, $R(2)$, ...

$R(S)$.

First, we set $P=1, S=1$, and $R(1)$ =node which is the labeling origin. Then, scan each arc for which the node $R(P)$ is its source node or its sink node. If the other node of any of these arcs can be labeled, wo do, increase 3 by 1, and set $R(S)$ to the node just labeled. If $R(S)$ is the terminal node, then labeling procedure is finished and a breakthrough has occurred. When all of the arcs joining node $R(P)$ have been scalled and no breakthrough has occurred, increase P by 1, then repeat the process for the new node R(P). If P>S, then the scan list is exhausted and a non-breakthrough situation has occurred.

When this procedure is used, each arc can be looked at most twice: once from each of its nodes. But, in order for this method to be more efficient than one that merely searches for arcs with one node labeled, lists must be set up of the arcs that join each particular node. The following section describes how this is done.

4.2.2 List Structure(1)

Let there be n arcs such that the jth arc has source node SS(j) and sink node T(j). Suppose that there are m nodes numbered from 1 through m and, therefore, that SS and T have values in this range.

Assuming that the arcs are in no particular order, it is necessary to set up four lists, say *UU, V, G,* and *H,* where *UU* and *V* are arrays of length $m+1$ and *G* and *H* are of length n. Let T_i be the number of arcs that have node i as their sink node, and let σ_i be the number of arcs that have node i as their source node. Either σ_i or T_i, but not both, may be zero. Then *UU* and *V* are defined recursively as follows:

$$UU_1=1$$

$$UU_{i+1}=UU_i+\sigma_i, \quad i=1,2,\dots,m$$

and

$$V_1=1$$

$$V_{i+1}=V_i+T_i, \quad i=1,2,\dots,m$$

Now, let $\rho_i=U_i$ for $i=1,2,\dots,m$. For each j from i through n, let $i=SS(j)$, $G(\rho_i)=j$, and then increase ρ_i by 1. When this is done, $G(U_i)$ through $G(U_{i+1}-1)$ is a list of the arcs with node i as their source node.

The same procedure is repeated for V and H with the sink nodes, giving a list of the arcs with the same sink node.

Using the FORTRAN program that incorporates above schemes, the execution time for the case study network which has 358 arcs on the CYBER 72 computer at KIST (Korea Institute of Science & Technology) took less than 9 seconds. Considering the complexity of the network this time seems to prove the efficiency of the program.

5. CASE STUDY

5.1 The Problem

Suppose that we are now at the beginning of a year and at present the Navy has the following number of men in each of the six ranks of combat-line specialty.

We assume here that the Navy knows its manpower requirements corresponding to its FMDP

No.	Rank	No. of Officers
1	ENS	4,000
2	LTJG	7,700
3	LT	9,100
4	LCDR	4,900
5	CDR	3,000
6	CAPT	1,300

Table 3. Present Manpower Structure

(Force Modernization & Development Plan), for each of the three activities which comprise the Navy.

For our purpose let us call the three activities as A, B, and C activities. We may categorize activity A as the operational units such as the fleet and radar sites, activity B as direct support units such as the shipyard, supply depots, fuel depots, etc., and activity C as other indirect support units. Table 4 lists the hypothetical manpower requirements of the three activities for officers of combat-line specialty over the next five years.

		Year 1	Year 2	Year 3	Year 4	Year 5
ENS	A	2,700	2,770	2,850	2,930	3,020
	B	900	940	960	990	1,010
	C	900	910	930	970	1,000
LTJG	A	4,620	4,740	4,870	5,010	5,140
	B	1,540	1,580	1,640	1,690	1,740
	C	1,500	1,540	1,610	1,650	1,690
LT	A	5,460	5,610	5,770	5,940	6,100
	B	1,820	1,880	1,940	1,990	2,050
	C	1,800	1,860	,1910	1,960	2,010
LCDR	A	3,000	3,090	3,180	3,270	3,360
	B	1,000	1,030	1,060	2,090	1,120
	C	1,000	1,000	1,030	1,060	1,090
CDR	A	1,920	1980	2,040	2,100	2,160
	B	640	660	680	700	720
	C	600	620	640	660	680
CAPT	A	8,40	870	900	930	960
	B	3,00	310	310	310	320
	C	2,80	290	300	310	310

Table 4. Yearly Manpower Requirementr Estimation

Promotions depend on ceilings imposed by the MDP(Manpower Development Plan). We assume that we have at our disposal an estimated figure for such a ceiling for each rank during the coming five years. Table 5 lists a set of such hypothetical ceilings.

	Year 1	Year 2	Year 3	Year 4	Year 5
ENS to LTJG	4,000	4,320	4,450	4,570	4,700
LTJG to LT	2,770	2,850	2,940	3,020	3,100
LT to LCDR	1,370	1,410	1,450	1,490	1,530
LCDR to CDR	780	800	830	850	8,70
CDR to CAPT	360	370	380	3,90	400

Table 5. Estimation of Yearly Promotion Ceilings

i	Rank	Significance(S_i)
1	ENS	2
2	LTJG	4
3	LT	6
4	LCDR	8
5	CDR	10
6	CAPT	12

Table 8. Rank significances

small fraction numbers. The out-of-kilter procedure, however, takes in integers only for input data. Without losing any significance we may modify the definition of e_{ij} as in equation(14) and call it e'_{ij}.

$$e'_{ij}=\frac{W_j \cdot S_i}{R_{ij}}\times 10,000 \tag{14}$$

Since there are three activities to distribute manpower to, $C_{\text{dist}}(i, j^*)$ is expressed as follows;

$$C_{\text{dist}}(i, j^*)=\text{Max}(e'_{i1}, e'_{i2}, e'_{3i})-e'_{ij^*} \tag{15}$$

Cost values for loss arcs, direct input arcs, and exchange arcs should be determined as discussed in section 3.2. An example of cost computation is shown below for Year 3, ranks LT and LCDR.

		LT($S_3=6$)			LCDR($S_4=8$)		
j	W_j	R_{ij}	e'_{ij}	$C_{\text{dist}}(i,j)$	R_{ij}	e'_{ij}	$C_{\text{dist}}(i,j)$
1	.65	577	68	0	318	164	0
2	.20	194	62	6	106	151	13
3	.15	191	47	21	103	117	47
		$C_{\text{loss}}(3)=21$ $C_{\text{di}}(3)=11$ $C_{\text{ex}}(3,4)=49$			$C_{\text{loss}}(4)=47$ $C_{\text{di}}(4)=24$ $C_{\text{ex}}(4,3)=49$		

Table 9. Example of Cost Computation

5.2.2 Flow Limits

Many of small portions of network like the one of Figure 2 compose our total network. One by one we will discuss how we may set up flow constraints for arcs that constitute this network. (Refer to Figure 2.)

For arc ♯1 (direct input) we set $L=O$, $U=$ a large number such as 10000. For arc ♯2 (promotion into) we set $L=U=$ the value obtained from Table 5. Costs are set zero for these arcs. Arc ♯3 (promotion out of) has the same cost and limits.

Arc ♯4 (stay in rank) should have $L=O$, $U=2000$ (a large value), and $C=O$. Arcs ♯5, ♯6, ♯7, and ♯8 all have $L=O$ and $U=1000$. Their cost values are as determined previously. Any policy decision may limit the flow along these arcs by setting U to be a certain number above which exchange assignments are not desired.

For arcs ♯9, ♯10, and ♯11 we set $L=O$, $U=1000$. (The only reason we choose the value of 1000 is that we know the flow along any of these arcs will not exceed the value in this case study

Promotion from CAPT to Admiral rank is not dealt with in Table 5 since it is considered as loss from the CAPT rank.

We assume the following trend of loss from each rank will most likely persist the next five years if no artificial control is exercised. Again, loss means outflow of personnel from a career state due to discharge, change of specialty, retirements, or promotion to Admiral rank in case of CAPT. Table 6 lists this trend estimation.

	1	2	3	4	5	6
Rank	ENS	LTJG	LT	LCDR	CDR	CAPT
Yearly Loss	00	850	1,360	590	450	350

Table 6. Yearly Loss Trend

Our objective is to see how the present manpower structure and policies will function in meeting the furture manpower requirements. The outcome will be in the form of recruit size, promotion sizes, numbers of men to be discharged from each rank, direct inputs required, numbers of exchange assignments, and degree of requirements satisfaction for each of the five years.

5.2. Formulation As a Network Problem

Now we need formulate the so-far-described problem as a minimal cost network problem. In other words, the full network must be structured, unit cost and flow constraints must be specified for each arc in the network, and then the problem is ready to be solved by the out-of-Kilter computer program.

5.2.1 Computation of Costs

In order to compute cost values according to the cost concept developed in Chapter 3, we first need to determine values of W_j's and S_i's. R_{ij}'sare obtained from Table 4.

An activity weight W_j is a coefficient representing relative degree of significance with which the activity contributes to the total Navy effectiveness. This coefficient is to be determined by the decision maker or maker group. Note that the sum of W_j's is to equal one. Table 7 lists an example of W_j's which we will use for our case study.

j	Activity	W_j
1	A	.65
2	B	.20
3	C	.15

Table 7. Activity Weights

Next, the rank significance(S_i) must be assigned with appropriate values. Several factors, such as the years of service, average men under control, etc. may be used to determine this relative significance of a rank in relation with other ranks. Further effort need to be expended to come up with an organized approach to determining values for this coefficient. Table 8 lists the values of S_i's which we suppose to have determined.

When we apply actual values into Equation (7) to calculate e_{ij}'s we find that the values are network.) Cost values previously determined are assigned to these arcs as well as to arc ♯12. But the lower limit of this loss arc is set to be equal to 80% of its upper limit. The reasoning was explained in Chapter 3. Table 5 determines the upper limit.

For arcs ♯13, ♯14, and ♯15 of Figure 2 we set $U=R_{ij}$, as listed in Table 4. The lower limits are set to be equal to 80% of upper limits because the Navy sees it essential that each activity has at least 80% of its manpower requirements met for each rank so that it may function properly. The cost values are set to be the negative of e'^{ij}s, as determined by equation (14). This is so because a requirement met means negative of cost, *i.e.*, contribution to the total effectiveness by as much as e'_{ij}.

6. Results & Conclusions

The case study network problem is solved with the out-of-kilter computer routine and a set of optimal flows is obtained. The flows imply deterministic values for yearly recruit sizes, promotions, losses and direct inputs for each rank. Some between-rank exchanges occur as a result. The requirements in all cases are fully satisfied as designed. Hypothetical data are used for this case study and therefore its results do not bear and real significance. But we come to the conclusion that the model developed here can be used as an effective tool for analyzing future effects of present manpower structure and policies because of what we were able to show through the case study analysis.

The complexity of the problem under consideration will not pose any additional difficulties in using the model and the solution procedure.

The proposed model and its solution can be easily employed to tackle similar manpower problems of other services as well as large non-military organizations whose manpower system characteristics are similar to those of military services. The out-of-kilter computer procedure can be conveniently used to solve other nework problems such as maximal flow, shortest distance, and minimal time and in addition, transportation problems can also be solved by using this algorithm. (See Ref. 10.)

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