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HYPERSURFACES OF A K-SPACE WITH CONSTANT CURVATURE

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Dedicated to Prof. Chung Ki Pahk on his 60th Birthday

0. Introduction

An almost Hermitian manifold is called a K-space if the associated form of an almost Hermitian structure tensor is a Killing 2-form. The orientable differentiable hypersurface in an almost Hermitian manifold admits an almost contact structure [7] [1]

The purpose of the present paper is to study the hypersurfaces of a K-space with constant curvature. In section 1, some preliminaries on the hypersurface of a K-space is given. In section 2, it is shown that, if the induced structure tensor and the second fundamental tensor are anti-commutative, then the hypersurface is totally geodesic. Section 3 is devoted to the study of the case that the induced structure tensor and the second fundamental tensor are commutative.

1. Preliminaries

Let $\tilde{M}$ be an almost Hermitian manifold of dimension $n(>2)$ with Hermitian structure $(F_\beta{}^\alpha,\ G_{\beta\alpha})$, i.e., with a complex structure tensor $F_\beta{}^\alpha$ and a positive definite Riemannian metric tensor $G_{\beta\alpha}$ satisfying

$$F_\beta{}^\lambda F_\lambda{}^\alpha = -\delta_\beta^\alpha,\ G_{\lambda\mu}F_\beta{}^\lambda F_\alpha{}^\mu = G_{\beta\alpha}.$$

Then, putting $F_{\beta\alpha} = F_\beta{}^\lambda G_{\lambda\alpha}$ we have $F_{\beta\alpha} = -F_{\alpha\beta}$.

If an almost Hermitian manifold $\tilde{M}$ satisfies

$$\nabla_\beta F_\alpha{}^\gamma + \nabla_\alpha F_\beta{}^\gamma = 0, \tag{1.1}$$

where ∇_β denotes the operator of covariant differentiation with respect to Christoffel symbols $\{{}^{\,\gamma}_{\beta\alpha}\}$ formed with $G_{\beta\alpha}$, then the manifold is called a *K-space, an almost Tachibana space* or *a nearly Kaehlerian manifold.*

Let M be an orientable hypersurface of a K-space $\tilde{M}$, then M is represented parametrically by the equation[2]

$$X^\lambda = X^\lambda(x^h),$$

1) Number in brackets refer to the references at the end of the paper.
2) Indices $i,\ j,\ \cdots$ run over $1,\ 2,\ \cdots,\ n-1$ and $\alpha,\ \beta,\ \cdots$ run over $1,\ 2,\ \cdots,\ n$.

where $\{x^\lambda\}$, $\{x^h\}$ are local coordinates of $\tilde{M}$ and M respectively.

If we put $B_j{}^\lambda=\partial_j x^\lambda$, $(\partial_j=\partial/\partial x^j)$, then $B_j{}^\lambda$ are linearly independent tangent vectors at each point of M. The induced Riemannian metric g_{ji} in M is given by $g_{ji}=G_{\beta\alpha}B_j{}^\beta B_i{}^\alpha$.

Choosing unit normal vector C^λ to the hypersurface M in such a way that C^λ and $B_j{}^\lambda$ form a frame of positive orientation of M, then we have

$$G_{\beta\alpha}B_j{}^\beta C^\alpha=0,\quad C_{\beta\alpha}C^\beta C^\alpha=1 \tag{1.2}$$
$$B_j{}^\lambda B_\lambda{}^i=\delta_j^i,\quad B_i^\alpha B_\beta{}^i=\delta_\beta^\alpha-C_\beta C^\alpha,$$

where $B^i{}_\beta=G_{\beta\lambda}B_j{}^\lambda g^{ji}$, $C_\lambda=G_{\lambda\mu}C^\mu$.

The transforms $F_\lambda{}^\mu B_j{}^\lambda$ and $F_\lambda{}^\mu C^\lambda$ can be expressed as linear combinations of $B_j{}^\lambda$ and C^λ. That is

$$F_\lambda{}^\mu B_j{}^\lambda=f_j{}^h B_h{}^\mu+u_j C^\mu, \tag{1.3}$$

$$F_\lambda{}^\mu C^\lambda=-u^h B_h{}^u, \tag{1.4}$$

where $u^h=g^{hi}u_i$, from which

$$f_j^h=B^h{}_\lambda F_\mu{}^\lambda B_j{}^\mu, \tag{1.5}$$

$$u_j=C_\lambda F_\mu{}^\lambda B_j{}^\mu=B_j{}^\mu F_{\mu\lambda}C^\lambda. \tag{1.6}$$

It is easily seen that the aggregate $(f_j{}^i,\ u^i,\ u_j,\ g_{ji})$ defines an almost contact metric structure in a hypersurface of $\tilde{M}$, i.e., the following relations are valid

$$u^i u_i=1,\qquad f_j{}^t f_t{}^i=-\delta_j{}^i+u_j u^i, \tag{1.7}$$
$$f_j{}^i u^j=0,\qquad f_j{}^i u_i=0,$$

and there exists a positive definite Riemannian metric g_{ji} such that

$$g_{ji}f_s{}^j f_t{}^i=g_{st}-u_s u_t.$$

If we put $f_{ji}=g_{ti}f_j{}^t$, we have $f_{ji}=-f_{ij}$.

Now denoting by the operator of covariant differentiation with respect to Christoffel symbols $\left\{ {h \atop j\ i} \right\}$ formed with g_{ji}, we see that the equations of Gauss are

$$\nabla_j B_i{}^\lambda=\partial_j B_i{}^\lambda+B_j{}^\beta B_i{}^\gamma\left\{ {\lambda \atop \beta\ \gamma} \right\}-B_h{}^\lambda\left\{ {h \atop j\ i} \right\}=H_{ji}C^\lambda, \tag{1.8}$$

where H_{ji} is the second fundamental tensor of the hypersurface, and the equations of Weingarten are

$$\nabla_j C^\lambda = \partial_j C^\lambda + B_j{}^\beta C^\gamma \left\{ {\lambda \atop \beta\gamma} \right\} = -H_j{}^i B_i{}^\lambda, \tag{1.9}$$

where $H_j{}^i = H_{jt} g^{ti}$.

Differentiating (1.3) covariantly along M and taking account of (1.8) and (1.9), we have

$$(\nabla_\mu F_\lambda{}^\nu) B_j{}^\mu B_i{}^\lambda - H_{ji} u^t B_t{}^\nu = (\nabla_j f_i{}^t) B_t{}^\nu + f_i{}^t H_{jt} C^\nu + (\nabla_j u_i) C^\nu - u_j H_i{}^t B_t{}^\nu, \tag{1.10}$$

similarly, operating ∇_i to $F_\mu{}^\nu B_j^\mu$ and adding the equation thus obtained with (1.10), then we get

$$(\nabla_\mu F_\lambda{}^\nu + \nabla_\lambda F_\mu{}^\nu) B_j{}^\mu B_i{}^\lambda = (\nabla_j f_i{}^t + \nabla_i f_j{}^t + 2u^t H_{ji} - u_j H_i{}^t - u_i H_j{}^t) B_t{}^\nu$$
$$+ (\nabla_j u_i + \nabla_i u_j + f_i{}^t H_{jt} + f_j{}^t H_{it}) C^\nu.$$

Since the left hand side of the equation above is vanish by (1.1) and since $B_j{}^\lambda$, C^λ are linearly independent, the following are consistent:

$$\nabla_j f_i{}^t + \nabla_i f_j{}^t = -2u^t H_{ji} + u_j H_i{}^t + u_i H_j{}^t, \tag{1.11}$$

$$\nabla_j u_i + \nabla_i u_j = -f_i{}^t H_{jt} - f_j{}^t H_{it}. \tag{1.12}$$

We consider an orientable hypersurface M of a K-space with constant curvature. In this case, the K-space $\tilde{M}$ is restricted to that of 6-dimension (cf. [4]).

The Riemannian curvature tensor K of $\tilde{M}$ takes the form

$$K_{\gamma\beta\alpha\lambda} = k(G_{\gamma\lambda} G_{\beta\alpha} - G_{\gamma\alpha} G_{\beta\lambda}), \tag{1.13}$$

k being a positive constant.

Substituting (1.13) into the Gauss and Codazzi equation:

$$R_{kjih} = B_k{}^\gamma B_j{}^\beta B_i{}^\alpha B_h{}^\lambda K_{\gamma\beta\alpha\lambda} + H_{kh} H_{ji} - H_{jh} H_{ki},$$

$$\nabla_k H_{ji} - \nabla_j H_{ki} = B_k{}^\gamma B_j{}^\beta B_i{}^\alpha C^\lambda K_{\gamma\beta\alpha\lambda},$$

where R_{kjih} is a Riemannian curvature tensor in the hypersurface M, we have

$$R_{kjih} = k(g_{kh} g_{ji} - g_{ki} g_{jh}) + H_{kh} H_{ji} - H_{jh} H_{ki}, \tag{1.14}$$

$$\nabla_k H_{ji} - \nabla_j H_{ki} = 0. \tag{1.15}$$

2. Case of $f_j{}^t H_{ti} = f_i{}^t H_{tj}$

We assume that two tensors $f_j{}^i$ and $H_j{}^i$ are anti-commutative, i.e.

(2.1) $$f_j{}^t H_{ti} - f_i{}^t H_{tj} = 0.$$

Transvecting (2.1) with $f_k{}^j$, we obtain

$$H_{ki} + u_k u^t H_{ti} - f_i{}^t H_{tj} f_k{}^j = 0,$$

and also transvecting this equation with u^i,

(2.2) $$H_{ki} u^i = \alpha u_k,$$

where α is a scalar field.

LEMMA 2.1. *In a hypersurface M of a K-space with constant curvature, if the structure tensor* $f_j{}^i$ *and the second fundamental tensor* $H_j{}^i$ *are anti-commutative, then* α *is constant.*

PROOF. Differentiating (2.2) covariantly along M, we have

$$(\nabla_k H_{jt}) u^t + H_{jt} \nabla_k u^t = (\nabla_k \alpha) u_j + \alpha \nabla_k u_j,$$

from which, taking the skew-symmetric parts with respect to the indices k and j, we have

(2.3) $$H_{jt} \nabla_k u^t - H_{kt} \nabla_j u^t = (\nabla_k \alpha) u_j - (\nabla_j \alpha) u_k + \alpha (\nabla_k u_j - \nabla_j u_k)$$

by virtue of (1.15).

On the other hand, we have from (1.12) and (2.1)

(2.4) $$\nabla_k u_j + \nabla_j u_k = -2 f_k{}^t H_{jt}.$$

Transvecting (2.3) with u^k and taking account of (2.4) and unity of u^k, we have

(2.5) $$\nabla_j \alpha = \beta u_j,$$

where $\beta = u^k \nabla_k \alpha$. Therefore (2.3) becomes

(2.6) $$H_{jt} \nabla_k u^t - H_{kt} \nabla_j u^t = \alpha (\nabla_k u_j - \nabla_j u_k).$$

Differentiating (2.5) covariantly, we have

$$\nabla_k \nabla_j \alpha = (\nabla_k \beta) u_j + \beta \nabla_k u_j,$$

from which, taking the skew-symmetric parts with respect to the indices k and j, we have

(2.7) $$(\nabla_k \beta) u_j - (\nabla_j \beta) u_k + \beta (\nabla_k u_j - \nabla_j u_k) = 0,$$

because $\nabla_j \alpha$ is a gradient vector.

Transvecting (2.7) with u^k and taking account of (2.4) and unity of u^k, we get

(2.8) $$\nabla_j\beta=\omega u_j,$$

where $\omega=u^k\nabla_k\beta$. Therefore, by the substitution of (2.8) into (2.6), we obtain

(2.9) $$\beta(\nabla_k u_j-\nabla_j u_k)=0.$$

Comparing (2.4) and (2.9), we have

(2.10) $$\beta\nabla_k u_j=-\beta f_k{}^t H_{jt}.$$

Substituting (2.9) and (2.10) into (2.6) and taking account of (2.1), we find

$$\beta H_{jt} f_k{}^s H_s{}^t=0.$$

Transvecting the equation above with $f_i{}^j$, we have

$$\beta(H_{jt}H_i{}^t-\alpha^2 u_j u_i)=0$$

by virtue of (1.7) and (2.2), which implies

(2.11) $$\beta(H_{jt}-\alpha u_j u_t)(H^{jt}-\alpha u^j u^t)=0.$$

Now denoting by M_1 the subset of the hypersurface M defined by $\{P\in M\,|\,\beta(P)\neq 0\}$, we obtain from (2.11)

(2.12) $$H_{ji}=\alpha u_j u_i,$$

from which

(2.13) $$R_{kjih}=k(f_{hk}g_{ji}-g_{ki}g_{jh})$$

on M_1.

The substitution of (2.12) into (2.4) gives

$$\nabla_k u_j+\nabla_j u_k=0,$$

or, comparing this equation with (2.9), we have

(2.14) $$\nabla_k u_j=0$$

on M_1.

Differentiating (2.12) covariantly, we have from (2.5) and (2.14)

$$\nabla_k H_{ji}=\beta u_k u_j u_i$$

on M_1. Furthermore differentiating the equation above covariantly, we have

$$\nabla_l\nabla_k H_{ji}=\omega u_l u_k u_j u_i$$

on M_1 because of (2.8) and (2.14).

By the Ricci identity and (2.12), we get

$$\alpha(R_{lkj}{}^t u_t u_i+R_{kli}{}^t u_j u_t)=0$$

on M_1. Transvecting the last equation with u^i and using (2.13), we obtain

(2.15) $$k(g_{kj}u_l - g_{lj}u_k) = 0$$

on M_1. Furthermore transvecting (2.15) with $g^{kj}u^l$, we have

$$4\alpha k = 0,$$

from which $\alpha = 0$ on M_1 because $k > 0$. Therefore, from (2.5) we have $\beta = 0$ on M_1. It contradicts the fact $\beta \neq 0$ on M_1. Thus M_1 is an empty set, and consequently we see from (2.5) that α is a constant on the hypersurface M. This completes the proof of Lemma 2.1.

Let λ be a principal curvature of $H_j{}^i$ and X^j the corresponding eigenvector to λ, i.e.,

$$H_j{}^i X^j = \lambda X^i.$$

Then we see from (2.1)

$$H_t{}^i (f_j{}^t X^j) = -\lambda f_t{}^i X^t.$$

Thus, using (2.2), $(H_j{}^i)$ has the following form

(2.16) $$(H_j{}^i) = \begin{bmatrix} \alpha & & & & \\ & \lambda_1 & & 0 & \\ & & -\lambda_1 & & \\ & 0 & & \lambda_2 & \\ & & & & -\lambda_2 \end{bmatrix}$$

for suitable orthonormal frame at each point M, thereby we have

(2.17) $$H_t{}^t = \alpha.$$

Differentiating (2.1) covariantly, we have

$$(\nabla_k f_j{}^t)H_{ti} + f_j{}^t \nabla_k H_{it} - (\nabla_k f_i{}^t)H_{jt} - f_i{}^t \nabla_k H_{jt} = 0.$$

If we add this equation to the equation obtained by interchanging the indices k and j in this

$$-2\alpha u_i H_{kj} + u_k H_j{}^t H_{it} + u_j H_k{}^t H_{it} + f_j{}^t \nabla_k H_{it} + f_k{}^t \nabla_j H_{it}$$
$$-(\nabla_k f_i{}^t)H_{jt} - 2f_i{}^t \nabla_j H_{kt} = 0$$

by virtue of (1.11), (1.15) and (2.2).

Transvecting this equation above with $g^{ki}u^j$ and taking account of symmetry of H_{ji} and skew-symmetry of f_{ji} with respect to the indices, we find

(2.18) $$-\alpha^2+H_{kt}H^{kt}-\alpha(\nabla_k f^{kt})u_t=0$$

because of (1.7) and (2.2).

On the other hand, (1.11) leads to

$$\nabla_k f^{kt}=-u^t H_k{}^k+\alpha u^t.$$

If substitute the equation above into (2.18), then

(2.19) $$H_{kt}H^{kt}=2\alpha^2-\alpha H_t{}^t.$$

Comparing (2.17) and (2.19), we have

$$H_{kt}H^{kt}=\alpha^2.$$

Hence we get from this equation and (2.16)

$$\alpha^2+2\lambda_1^2+2\lambda_2^2=\alpha^2,$$

and consequently $\lambda_1=\lambda_2=0$. Thus (2.16) has the form

$$(H_j{}^i)=\begin{bmatrix} \alpha & & & & \\ & 0 & & & 0 \\ & & 0 & & \\ & 0 & & 0 & \\ & & & & 0 \end{bmatrix}$$

for suitable orthonormal frame at each point of M.

Now we assume that $\alpha\neq 0$.

Using Cartan's lemma with respect to the hypersurface with constant principal curvature of a space of constant curvature (cf. [1]),

$$5\frac{k+\alpha 0}{\alpha-0}=0,$$

where k is a constant curvature of K-space, therefore $k=0$. It contradicts to $k>0$. Hence $\alpha=0$, i.e., $H_{ji}=0$. Thus we have

THEOREM 2.2 *In a hypersurface M of a K-space with constant curvature, if the structure tensor* $f_j{}^i$ *and the second fundamental tensor* $H_j{}^i$ *are anti-commutative, then hypersurface M is totally geodesic.*

3. Case of $f_j{}^t H_{ti}=-f_i{}^t H_{tj}$

In this section we assume that two tensors $f_j{}^i$ and $H_j{}^i$ are commutative, i.e.,

(3.1) $$f_j{}^t H_{ti}+f_i{}^t H_{tj}=0.$$

In this case, the vector field u^j is a Killing vector field, i.e.,

(3.2) $$\nabla_k u_j + \nabla_j u_k = 0.$$

Transvecting (3.1) with $f_k{}^j u^i$, we have

(3.3) $$H_{ji} u^i = \alpha u_j.$$

LEMMA. 3.1. *In a hypersurface M of a K-space with constant curvature, if the structure tensor $f_j{}^i$ and the second fundamental tensor $H_j{}^i$ are commutative, then α is a constant.*

PROOF. Differentiating (3.3) covariantly along M and taking the skew-symmetric parts of the equation obtained thus, we have

$$H_{jt}\nabla_k u^t - H_{kt}\nabla_j u^t = (\nabla_k\alpha)u_j - (\nabla_j\alpha)u_k + \alpha(\nabla_k u_j - \nabla_j u_k).$$

Transvecting this equation with u^k and taking account of (3.2), we find

(3.4) $$\nabla_j\alpha = \beta u_j,$$

where $\beta = u^t\nabla_t\alpha$. Differentiating (3.4) covariantly and taking the skew-symmetric parts of the equation obtained thus, we get

$$(\nabla_k\beta)u_j - (\nabla_j\beta)u_k + \beta(\nabla_k u_j - \nabla_j u_k) = 0.$$

Transvecting the equation above with u^k, we obtain

(3.5) $$\nabla_j\beta = \omega u_j,$$

where $\omega = u^k\nabla_k\beta$.

The last two equations imply that

(3.6) $$\beta(\nabla_k u_j - \nabla_j u_k) = 0.$$

Comparing (3.2) and (3.6), we see that

(3.7) $$\beta\nabla_k u_j = 0.$$

We will denote M_1 the subset of the hypersurface M defined by $\{P\in M\} \mid \beta(P)\neq 0\}$. Then we have from (3.7)

(3.8) $$\nabla_j u_i = 0$$

on M_1.

If we differentiate both sides of the equation $f_j{}^i u_i = 0$, then

(3.9) $$(\nabla_k f_j{}^i)u_i = 0$$

on M_1 by virtue of (3.8).

Transvecting (1.11) with u_t and taking account of (3.3) and (3.9), we have

$$H_{ji}=\alpha u_j u_i, \tag{3.10}$$

which implies

$$R_{kjih}=k(g_{kh}g_{ji}-g_{ki}g_{jh}) \tag{3.11}$$

on M_1.

From (3.4), (3.5), (3.8) and (3.10), we obtain

$$\nabla_l\nabla_k H_{ji}=\omega u_l u_k u_j u_i$$

on M_1.

By the Ricci identity and (3.10), we have

$$\alpha(R_{lkj}{}^t u_t u_i+R_{kli}{}^t u_j u_t)=0$$

on M_1. Transvecting the last equation with u^i, we find

$$\alpha R_{lkj}{}^t u_t=0,$$

or, using (3.11)

$$\alpha k(g_{kj}u_l-g_{lj}u_k)=0 \tag{3.12}$$

on M_1. Furthermore transvecting (3.12) with $g^{kj}u^l$, we have $4\alpha k=0$, from which $\alpha=0$ on M_1. Therefoer from (3.4) we have $\beta=0$ on M_1. It contradicts the fact that $\beta\neq 0$ on M_1. Thus M_1 is an empty set, and consequently we see from (3.4) that α is a constant on the hypersurface M. This completes the proof of Lemma 3.1

THEOREM 3.2. *In a hypersurface M of a K-space with constant curvature, if the structure tensor* $f_j{}^i$ *and the second fundamental tensor* $H_j{}^i$ *are commutative, then the hypersurface M is totally umbilical.*

PROOF. Differentiating (3.1) covariantly, we find

$$(\nabla_k H_{jt})f_i{}^t+H_{jt}\nabla_k f_i{}^t+(\nabla_k H_{it})f_j{}^t+H_{it}\nabla_k f_j{}^t=0. \tag{3.13}$$

Adding (3.13) to the equation which obtained thus by interchanging of the indices j and k in (3.13), we find

$$2(\nabla_j H_{kt})f_i{}^t+H_{jt}\nabla_k f_i{}^t+H_{kt}\nabla_j f_i{}^t+(\nabla_k H_{it})f_j{}^t+(\nabla_j H_{it})f_k{}^t$$
$$-2\alpha u_i H_{kj}+u_k H_{it}H_j{}^t+u_j H_{it}H_k{}^t=0$$

by virtue of (1.11) (1.15) and (3.3).

Transvecting this equation with g^{ki}, we have

(3.14) $$(H_{ki}H^{kt}-\alpha H_k{}^k)u_j+(\nabla_t H_k{}^k)f_j{}^t=0$$

because of (1.11), (1.15) and (3.3).

Also transvecting (3.14) with u^j, we obtain

(3.15) $$H_{ki}H^{ki}=\alpha H_t{}^t.$$

The substitution of (3.15) into (3.14) gives

$$(\nabla_t H_k{}^k)f_j{}^t=0.$$

Transvecting this equation with $f_i{}^j$ and using (1.7), we find

(3.16) $$\nabla_i H_k{}^k-(\nabla_t H_k{}^k)u_i u^t=0.$$

On the other hand, if we differentiate (3.3) covariantly, then

$$(\nabla_k H_{ji})u^i+H_{ji}\nabla_k u^i=\alpha\nabla_k u_j.$$

Transvecting this equation with g^{kj}, we obtain

(3.17) $$u^i(\nabla_i H_t{}^t)=0$$

by virtue of (1.15) and (3.2).

Comparing (3.16) and (3.17), we see that

$$\nabla_j H_k{}^k=0,$$

which implies that $H_k{}^k$ is a constant.

Now putting λ_k the principal curvature of $H_j{}^i$ distinct to α, we easily see from (3.1) and (3.3) that $(H_j{}^i)$ has the form

(3.18) $$(H_j{}^i)=\begin{bmatrix} \alpha & & & & \\ & \lambda_1 & & 0 & \\ & & \lambda_1 & & \\ & 0 & & \lambda_2 & \\ & & & & \lambda_2 \end{bmatrix}$$

for suitable orthonormal frame at each point of M. Thereby we have

(3.19) $$H_t{}^t=\alpha+2\lambda_1+2\lambda_2,$$

from which

(3.20) $$\lambda_1+\lambda_2=c;\ \text{constant}$$

because $H_t{}^t$ and α are constants.

From (3.15) and (3.18), we find

$$\lambda_1^2+\lambda_2^2=\alpha(\lambda_1+\lambda_2), \tag{3.21}$$

which is a non-negative constant. Since

$$(\lambda_1+\lambda_2)^2-(\lambda_1^2+\lambda_2^2)=2\lambda_1\lambda_2,$$

we see

$$\lambda_1\lambda_2=\text{constant}. \tag{3.22}$$

The substitution of (3.20) into (3.22) gives

$$c\lambda_1-\lambda_1^2=\text{constant}.$$

Differentiating this equation covariantly, we get

$$\nabla_j\lambda_1(c-2\lambda_1)=0,$$

which implies that λ_1 is a constant, therefore λ_2 is a also constant because of (3.20).

Now we assume that $\alpha<\lambda_1<\lambda_2$. Then

$$2\alpha-\lambda_1-\lambda_2<\alpha-\lambda_2<\alpha-\lambda_1<0. \tag{3.23}$$

Using Cartan's lemma with respect to the hypersurface with constant principal curvature of a space of constant curvature (cf. [1]),

$$\frac{k+\alpha\lambda_1}{\alpha-\lambda_1}+\frac{k+\alpha\lambda_2}{\alpha-\lambda_2}=0, \tag{3.24}$$

where k is a positive constant.

The equation (3.24) is rewritten as

$$k+\alpha\lambda_1=-\frac{\alpha-\lambda_1}{\alpha-\lambda_2}(k+\alpha\lambda_2), \tag{3.25}$$

from which

$$(k+\alpha\lambda_1)(k+\alpha\lambda_2)<0$$

by virtue of (3.23). Thus either

$$k+\alpha\lambda_1<0 \text{ or } k+\alpha\lambda_2<0, \tag{3.26}$$

which implies

$$\alpha\lambda_1<-k<0 \text{ or } \alpha\lambda_2<-k<0. \tag{3.27}$$

On the other hand, the equation (3.24) can be written as

$$\alpha k(2\alpha-\lambda_1-\lambda_2)+\alpha^2(\lambda_1-\lambda_2)^2=0,$$

from which

(3.28) $$\alpha(2\alpha-\lambda_1-\lambda_2)=0,\ \alpha^2(\lambda_1-\lambda_2)^2=0$$

because of (3.23), (3.27) and $\alpha<\lambda_1<\lambda_2$.

Therefore, from (3.28) we have $\alpha=0$, which implies $k<0$ from (3.26). It contradicts the fact that the positive constancy of k.

Hence $\lambda_1=\lambda_2=\alpha$ on the hypersurface M, therefore we find $H_{ji}=\alpha g_{ji}$. It implies that the hypersurface M is totally umbilical.

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REFERENCES

[1] E. Cartan, *Familles de surfaces isoparamétrques dans les espaces à courbure constante*, Oeuvres completes, (1938).

[2] S. Kobayashi and K. Nomizu, *Foundations of differential geometry*, Vol. I, II. Intersc-ience publishers, (1963).

[3] M. Okumura, *Certain almost contact hypersurfaces in Kaehlerian manifolds of constant holomorphic sectional curvature*, Tôhoku Math. J., 16(1964), 270—284.

[4] S. Tachibana, *On infinitesimal conformal and projective trasformations of compact K-space*, Tôhoku Math. J., 13(1961), 386—392.

[5] K. Takamatsu, *Some properties of 6-dimensional K-spaces*, Kōdai Math. Sem. Rep., 23(1971), 215—232.

[6] K. Takamatsu and H. S. Park, *On certain hypersurfaces of a 6-dimensional K-space*. Memo. Fac. Tech. Kanazawa Univ., 15(1963), 401—408.

[7] Y. Tashiro, *On contact structure of hypersurfaces in complex manifolds I*, Tôhoku Math. J., 15(1963), 62-78.