J. Korean Math. Soc.
Vol. 12, No. 1, 1975

COMPACT HYPERSURFACES WITH ANTINORMAL (f, g, u, v, λ)-STRUCTURE IN AN ODD-DIMENSIONAL SPHERE

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Dedicated to professor Chung-Ki Pahk on his sixeth birthday

0. Introduction.

It is well known that a hypersurface of an almost contact metric manifold or of an odd-dimensional sphere with canonical contact structure admits an (f, g, u, v, λ)-structure, that is, there exist a set of a tensor field $f_j{}^h$ of type (1, 1), a Riemannian metric tensor g_{ji}, two 1-forms u_i and v_i (or two vector fields $u^h=u_i g^{ih}$, $v^h=v_i g^{ih}$) and a function λ which satisfy

$$
(0.1)\qquad
\begin{aligned}
&f_t{}^h h_j{}^t=-\delta_j{}^h+u_j u^h+v_j v^h,\\
&f_j{}^t f_i{}^s g_{ts}=g_{ji}-u_j u_i-v_j v_i,\\
&f_j{}^t u_t=\lambda v_j,\quad f_j{}^t v_t=-\lambda u_j,\\
&u_t u^t=v_t v^t=1-\lambda^2,\quad u^t v_t=0
\end{aligned}
$$

([8], [9]).

In studing the manifold structure of hypersurfaces of the sphere admitting the induced (f, g, u, v, λ)-structure many authors have found several results (cf. [1], [3], [4], [5], [7] and [9]).

Recently Blair, Ludden and Yano in their paper [1] proved that

THEOREM A. *Let M be a complete orientable hypersurface in $S^{2n+1}(1)$ such that induced (f, g, u, v, λ)-structures satisfy*

$$
(0.2)\qquad f_j{}^t h_t{}^h+h_j{}^t f_t{}^h=0,
$$

$h_j{}^h$ being second fundamental tensor of M.

If the scalar curvature K of M is constant and $\lambda(1-\lambda^2)$ is non-zero almost everywhere function on M, then M is isometric to S^{2n} or $S^n\times S^n$.

The main purpose of the present paper is reformation of Theorm A. We shall prove this theorem under the condition that the hypersurface is compact and the sectional curvature of the section spanned by two vectors u and v has semi-definite sign, but without constancy of scalar curvature.

In section 1 and 2, we prepare fundamental properties of the hypersurface with (0.2) in an odd-dimensional sphere. In the last section 3, we study the compact hypersurface

Received by the editors Feb. 1, 1975.

under the condition stated above by using the same method as in the paper [5] and results of Simons and of Chern-do Carmo-Kobayashi.

1. Preliminaries.

Let $S^{2n+1}(1)$ be a $(2n+1)$-dimensional sphere of radius 1 in Euclidean $(2n+2)$-space E^{2n+2}. As is well known, $S^{2n+1}(1)$ admits a canonical contact metric structure (ϕ, ξ, η, g), which is induced from the natural Kaehlerian structure equipped on E^{2n+2}. Throughout this paper, manifolds, functions, vector fields and other geometric objects we discuss are assumed to be differentiable and of class C^∞.

Let M be a $2n$-dimensional orientable and connected hypersurface in $S^{2n+1}(1)$ covered by a system of coordinate neighborhood $\{U: x^h\}$, where here and in the sequel, indices $h, i, j, k, \ldots$ run over the range $\{1, 2, \ldots, 2n\}$ and the summation convention will be used with respect to these indices.

It is well known that the (f, g, u, v, λ)-structure induced on a hypersurface M of $S^{2n+1}(1)$ with induced Riemannian metric tensor and the second fundamental tensor h_{ji} satisfy

$$\nabla_j f_i^{\ h} = -g_{ji}u^h + \delta_j^{\ h}u_i - h_{ji}v^h + h_j^{\ h}v_i, \tag{1.1}$$

$$\nabla_j u_i = f_{ji} - \lambda h_{ji}, \tag{1.2}$$

$$\nabla_j v_i = -h_{jt}f_i^{\ t} + \lambda g_{ji}, \tag{1.3}$$

$$\nabla_j \lambda = h_{jt}u^t - v_j \tag{1.4}$$

(cf. [7], [9]), where $h_j^{\ i} = h_{jt}g^{ti}$, $(g^{ji}) = (g_{ji})^{-1}$, $f_i^{\ h}$, u_i and v_i are components of f, u and v respectively, ∇_j being the operator of covariant differentiation with respect to g_{ji}.

Moreover, the structure equations of the submanifold M, i.e. the equations of Gauss and Codazzi are given by

$$K_{kji}^{\ \ \ h} = \delta_k^{\ h}g_{ji} - \delta_j^{\ h}g_{ki} + h_k^{\ h}h_{ji} - h_j^{\ h}h_{ki}, \tag{1.5}$$

$K_{kji}^{\ \ \ h}$ being components of the curvature tensor of M, and

$$\nabla_k h_{ji} - \nabla_j h_{ki} = 0 \tag{1.6}$$

respectively.

From (1.5), the scalar curvature K of M is written in the form

$$K = 2n(2n-1) + (h_t^{\ t})^2 - h_{ji}h^{ji} \tag{1.7}$$

in terms of the second fundamental form.

As a matter of convenience, in the sequel, we denote by $N_0 = \{P \in M \mid \lambda(P) = 0\}$, $N_1 = \{P \in M \mid \lambda^2(P) = 1\}$ and $N = M - (N_0 \cup N_1)$.

First we note that (1.2) implies the set N_1 is bordered (cf. [3], [4]).

We assume on the submanifold M of $S^{2n+1}(1)$ the (f, g, u, v, λ)-structure is *antinormal*, that is,

$$S_{ji}^{\ \ h} = 2v_j(\nabla_i v^h - \lambda\delta_i^{\ h}) - 2v_i(\nabla_j v^h - \lambda\delta_j^{\ h}), \tag{1.8}$$

or, equivalently

(1.9) $$h_{jt}f_i^t - h_{it}f_j^t = 0$$

because N_1 is a bordered set (See [9]).

REMARK. If the hypersurface M in $S^{2n+1}(1)$ satisfies

(1.10) $$h_{jt}f_i^t - h_{it}f_j^t = 2\phi f_{ij},$$

or, equivalently

(1.11) $$\nabla_j v_i - \nabla_i v_j = 2\phi f_{ji}$$

by virtue of (1.3), ϕ being a function on M. We know that ϕ is a constant on M if $n > 2$ (See [5]).

If we put

(1.12) $$T_{ji} = h_{ji} - \phi g_{ji},$$

then (1.6) and (1.10) become respectively

(1.13) $$\nabla_k T_{ji} - \nabla_j T_{ki} = 0,$$

(1.14) $$T_{jt}f_i^t - T_{it}f_j^t = 0.$$

Comparing (1.6) and (1.9) with (1.10)-(1.14), the condition (1.9) seems to be more essential than (1.10).

2. Antinormal (f, g, u, v, λ)-structures induced on $S^{2n+1}(1)$.

We first prove

LEMMA 2.1. *If M is a hypersurface with antinormal (f, g, u, v, λ)-structure $(n>1)$, then the set N_0 is a bordered set.*

Proof. If there exists a connected open kernel W of N_0,

(2.1) $$h_{jt}u^t = v_j \quad \text{on } W$$

by virtue of (1.4).

Transvecting (1.9) with $f_k^i v^j$ and using (0.1) and (2.1), we find

(2.2) $$h_{kt}v^t = u_k + \alpha v_k \quad \text{on } W,$$

where we have put $\alpha = h_{ji}v^j v^i$.

Differentiating (2.2) covariantly and substituting (1.2) and (1.3) with $\lambda=0$, we have

$$(\nabla_k h_{jt})v^t - h_j^t h_{ks} f_t^s = f_{kj} + (\nabla_k \alpha)v_j - \alpha h_{kt} f_k^t \quad \text{on } W,$$

from which, taking skew-symmetric parts and using (1.6),

$$-2h_j^t h_{ks} f_t^s = 2f_{kj} + (\nabla_k \alpha)v_j - (\nabla_j \alpha)v_k + \alpha(h_{jt}f_k^t - h_{kt}f_j^t),$$

or, using (1.9)

$$2h_j^t h_{ts} f_k^s = 2f_{jk} + (\nabla_k \alpha)v_j - (\nabla_j \alpha)v_k \quad \text{on } W.$$

Transvecting the last equation with v^k and taking account of (2.2), we find $\nabla_j \alpha$

$=(v^t\nabla_t\alpha)v_j$ on W. Thus we have $h_j^t h_{ts} f_k^s = f_{jk}$ on W.
Transvecting this with f_i^k and using (0.1), (2.1) and (2,2), we obtain

$$h_j^t h_{it} = -g_{ji} + 2u_j u_i + (2+\alpha^2)v_j v_i + \alpha(v_j u_i + u_j v_i) \tag{2.3}$$

on W.

On the other hand, when one gives attention to equation (2.1) and (2.2), it is seen that there exists, at any point of W, two eigenvectors of the second fundamental tensor of M belong to the plane section $P(u,v)$ spanned by u^h and v^h. Let σ_1 and σ_2 be eigenvalues corresponding to the eigenvectors. Then the eigenvalues are roots of the quadratic equation: $\sigma^2-\alpha\sigma-1=0$, and consequently $\sigma_1=\frac{1}{2}(\alpha+\sqrt{\alpha^2+4})$, $\sigma_2=\frac{1}{2}(\alpha-\sqrt{\alpha^2+4})$.

For an eigenvector X corresponding to the eigenvalues σ_1, (1.9) implies $h_j^h(f_t^j X^t) = -\sigma_1(f_t^h X^t)$. This means that the transforms $f_t^h X^t$ of the vector X by the linear transformation f is also an eigenvector, of which eigenvalue is equal to $-\sigma_1$.

From (2.3) we see that there does not exist eigenvalue corresponding eigenvector Y orthogonal to the plane section $P(u,v)$.

Thus we have $\sigma_1=-\sigma_2$ and consequently $\alpha=0$. Therefore, (2.3) becomes

$$h_j^t h_{it} = -g_{ji} + 2(u_j u_i + v_j v_i) \quad \text{on } W,$$

from which, $0 \leq h_{ji}h^{ji} = -2(n-2)$ on W, and consequently $h_{ji}=0$ on W. But h_{ji} can not be zero on W by virtue of (2.1). So W is empty. This completes the proof of Lemma 2.1.

LEMMA 2.2. *Under the same assumptions as those stated in Lemma 2.1, we have*

$$h_{jt}u^t = \beta v_j, \tag{2.4}$$

$$h_{jt}v^t = \beta u_j, \tag{2.5}$$

$$h_t^t = 0 \tag{2.6}$$

and

$$\nabla_j \lambda = (\beta-1)v_j \tag{2.7}$$

in $N \cup N_0$, *where* β *is a function in* $N \cup N_0$ *defined by* $\beta = \frac{1}{1-\lambda^2} h_{st}u^s v^t$.

Proof. Transvecting (1.9) with f_k^i, we find

$$h_{st} f_j^t f_k^s + h_{jk} - (h_{jt}u^t)u_k - (h_{jt}v^t)v_k = 0,$$

from which, taking skew-symmetric parts,

$$(h_{jt}u^t)u_k - (h_{kt}u^t)u_j + (h_{jt}v^t)v_k - (h_{kt}v^t)v_j = 0.$$

Since u_i and v_i do not vanish in $N \cup N_0$, transvecting the equation above with u^k and v^k and using (0.1), we have

$$h_{jt}u^t = \alpha u_j + \beta v_j, \tag{2.8}$$

$$h_{jt}v^t = \beta u_j + \gamma v_j \tag{2.9}$$

respectively in $N\cup N_0$, where α and γ are defined by

$$\alpha=\frac{1}{1-\lambda^2}h_{st}u^s u^t, \qquad \gamma=\frac{1}{1-\lambda^2}h_{st}v^s v^t.$$

Differentiating (2.8) covariently, we have

$$(\nabla_k h_{jt})u^t+h_j{}^t(\nabla_k u_t)=(\nabla_k\alpha)u_j+(\nabla_k\beta)v_j+\alpha\nabla_k u_j+\beta\nabla_k v_j$$

in $N\cup N_0$. Taking skew-symmetric parts and using (1.2), (1.3), (1.6) and (1.9), we obtain

$$(\nabla_k\alpha)u_j-(\nabla_j\alpha)u_k+(\nabla_k\beta)v_j-(\nabla_j\beta)v_k=2\alpha f_{jk}, \tag{2.10}$$

from which, we see that $\nabla_j\alpha$ and $\nabla_j\beta$ are linear combinations of u_j and v_j, that is,

$$(1-\lambda^2)\nabla_j\alpha=(u^t\nabla_t\alpha)u_j+(u^t\nabla_t\beta)v_j,$$

$$(1-\lambda^2)\nabla_j\beta=(v^t\nabla_t\alpha)u_j+(v^t\nabla_t\beta)v_j.$$

Thus (2.10) reduced to $\frac{1}{1-\lambda^2}(v^t\nabla_t\alpha-u^t\nabla_t\beta)(u_k v_j-u_j v_k)=2\alpha f_{jk}$ in $N\cup N_0$.

On the other hand, f_{ji} is of rank $2n-2\geq 2$ in $N\cup N_0$ by assumption $\dim M\geq 2$. Thus we have, from the equation above, $\alpha=0$ and consequently $u^t\nabla_t\beta=0$ in $N\cup N_0$. From this fact (2.10) becomes

$$\nabla_j\beta=\frac{1}{1-\lambda^2}(v^t\nabla_t\beta)v_j. \tag{2.11}$$

Transvecting (1.9) with $u^j v^i$ and using (2.8) and $\alpha=0$, we find $\lambda(1-\lambda^2)\gamma=0$, i.e. $\lambda\gamma=0$ in $N\cup N_0$. But, since N_0 is bordered set and $N\cup N_0$ is open, $\gamma=0$ in $N\cup N_0$. Thus (2.4) and (2.5) are obtained.

Transvecting (1.9) with f^{ji} and taking account of $\alpha=\gamma=0$, we get (2.6). Therefore, Lemma 2.2 is proved.

LEMMA 2.3. *Under the conditions as those stated in Lemma* 2.1, *the equations*

$$(1-\lambda^2)\nabla_j\beta=-2\lambda\beta(\beta+1)v_j \tag{2.12}$$

and

$$(1-\lambda^2)(h_{jt}h_i{}^t+\beta g_{ji})=\beta(\beta+1)(u_j u_i+v_j v_i) \tag{2.13}$$

hold in $N\cup N_0$.

Proof. Differentiating (2.5) covariantly and using (1.2) and (1.3), we have in $N\cup N_0$

$$(\nabla_k h_{jt})v^t+h_j{}^t(-h_{ks}f_t{}^s+\lambda g_{kt})=(\nabla_k\beta)u_j+\beta(f_{kj}-\lambda h_{kj}),$$

from which, taking skew-symmetric parts and taking account of (1.6), (1.9) and (2.11),

$$2h_j{}^t h_{ts}f_k{}^s=2\beta f_{jk}+\frac{1}{1-\lambda^2}v^t\nabla_t\beta(u_k v_j-u_j v_k). \tag{2.14}$$

Transvecting (2.14) with u^k and using (2.4) and (2.5), we see that

(2.15) $$v^t\nabla_t\beta=-2\lambda\beta(\beta+1) \quad \text{in } N\cup N_0.$$

Thus (2.12) is gotten from (2.11) and (2.15).

Substituting (2.15) into (2.14), we find in $N\cup N_0$

$$h_j{}^t h_{ts} f_k{}^s=\beta f_{jk}+\frac{\lambda}{1-\lambda^2}\beta(\beta+1)(u_j v_k-u_k v_j),$$

from which, transvecting $f_i{}^k$ and using (2.4) and (2.5), we find (2.13).

3. Compact hypersurfaces with antinormal (f, g, u, v, λ)-structure.

In this section, we assume that M is a compact hypersurface with antinormal (f, g, u, v, λ)-structure in $S^{2n+1}(1)$, $(n\geq 1)$.

From (2.4), (2.5) and (2.13) we get at most four distinct principal curvatures $\pm\sqrt{-\beta}$, $\pm\beta$ at each point in $N\cup N_0$, and hence N_0 and N_1 being bordered sets, also in M because of the continuity of principal curvatures. Under the condition (1.9) we see easily that the multiplicities of β and $-\beta$ are 1 and those of $\sqrt{-\beta}$ and $-\sqrt{-\beta}$ are $n-1$.

We have from (2.13)

(3.1) $$h_{ji}h^{ji}=2\beta(\beta-n+1) \quad \text{in } N\cup N_0.$$

Thus, by (1.7), (2.6) and (3.1), the scalar curvature K is given by

(3.2) $$K=2(\beta+n)(2n-1-\beta) \quad \text{in } N\cup N_0.$$

Since β is non-positive, solving the quadratic equation above with respect to β, we have $\beta=(n-1-\sqrt{\theta})/2$ in $N\cup N_0$, where $\theta=(n-1)^2-4\{K/2-n(2n-1)\}\geq 0$ in $N\cup N_0$. We have $\theta>0$ in M, because N_1 is bordered. Hence we can define a function $\bar{\beta}$ in M by

(3.3) $$\bar{\beta}=(n-1-\sqrt{\theta})/2.$$

Then, the function $\bar{\beta}$ thus defined is an extension of the function β defined only in $N\cup N_0$ and differentiable on M. Thus all differential equations involving β established in $N\cup N_0$ are valid about $\bar{\beta}$. Without fear of confusion, we denote the extension $\bar{\beta}$ by the same letter β. Then $\beta(x)$ is equal to 0 or -1 at each point x in N_1 (See [5]).

For a symmetric matrix H of degree $2n$, it is well known that

(3.4) $$2n\operatorname{tr} H^2-(\operatorname{tr} H^2)^2-(\operatorname{tr} H)^2-(\operatorname{tr} H)(\operatorname{tr} H)^3 =\sum_{i<j}(\sigma_i-\sigma_j)^2(1+\sigma_i\sigma_j),$$

where σ_i are eigenvalues of H, (cf. [6]).

We have already known that the principal curvatures at each point of M are $\sigma_1=\beta$, $\sigma_2=-\beta$, $\sigma_3=+\sqrt{-\beta}$ and $\sigma^4=-\sqrt{-\beta}$, and their multiplicities are respectively 1 and $n-1$.

In [5], a formula of Simon's type was computed without consideration of these multiplicities. But if we consider these multiplicities, then

$$\sum_{i<j}(\sigma_i-\sigma_j)^2(1+\sigma_i\sigma_j)$$

$$=(\sigma_1-\sigma_2)^2(1+\sigma_1\sigma_2)+(n-1)(\sigma_1-\sigma_3)^2(1+\sigma_1\sigma_3)$$

$$+(n-1)(\sigma_2-\sigma_3)^2(1+\sigma_2\sigma_3)+(n-1)^2(\sigma_3-\sigma_4)^2(1+\sigma_3\sigma_4)$$
$$=4\beta(\beta+1)(\beta-n+1)(n-\beta).$$

Of course, from (3.4), calculating the left hand side of (3.4) by using (2.6) and (3.1), we also get the same result.

Now, using this result, if we calculate a formula of Simon's type for the hypersurface of constant mean curvature in a sphere [6], then

$$\frac{1}{2}\Delta(h_{ji}h^{ji})=(\nabla_k h_{ji})(\nabla^k h^{ji})+4\beta(\beta+1)(\beta-n+1)(n-\beta). \tag{3.6}$$

On the other hand, differentiating both sides of (2.12), we find

$$-2\lambda(\nabla_k\lambda)\nabla_j\beta+(1-\lambda^2)\nabla_k\nabla_j\beta$$
$$=-2(\nabla_k\lambda)\beta(\beta+1)v_j-\lambda(2\beta+1)(\nabla_k\beta)v_j-2\lambda\beta(\beta+1)\nabla_k v_j \quad \text{on } M,$$

from which, using (1.3), (2.7) and (2.12),

$$\Delta\beta=\frac{2\beta(\beta+1)}{1-\lambda^2}\{\lambda^2(5\beta-2n+1)+2\lambda(1-\beta)+1-\beta\} \quad \text{on } M. \tag{3.7}$$

From (3.1) and (3.2), we find

$$\Delta(h_{ji}h^{ji})=4(\nabla^j\beta)(\nabla_j\beta)+2(2\beta-n+1)\Delta\beta,$$

from which, substituting (2.12) and (3.7),

$$\Delta(h_{ji}h^{ji})=\frac{4\beta(\beta+1)}{1-\lambda^2}[4\lambda^2\beta(\beta+1)+(2\beta-n+1) \times\{2\lambda^2(\beta-n+2)+(1-\beta)(1-\lambda^2)\}]. \tag{3.8}$$

Comparing (3.6) and (3.8), we obtain

$$(\nabla_k h_{ji})(\nabla^k h^{ji})=\frac{2\beta(\beta+1)}{1-\lambda^2}[\lambda^2(\beta+1)(8\beta-3n+3)+ (n-1)(2n-3\beta-1)],$$

from which $\beta(\beta+1)\geq 0$ at every point in N_0 since β is nonpositive.

On the other hand, from (0.1), (1.5), (2.4) and (2.5), we have

$$K_{kjih}u^k v^j u^i v^h=-(1-\beta^2)(1-\lambda^2)^2,$$

from which sectional curvature of the section $P(u,v)$ is $1-\beta^2$.

Now we assume that the sectional curvature is semi-definite sign in $N\cup N_0$, that is, $1-\beta^2$ is semi-definite sign.

At first we consider the case of $1-\beta^2\geq 0$. Then, since β is non-positive, $-1\leq\beta\leq 0$ in $N\cup N_0$.

As we have already shown that $\beta(\beta+1)=0$ at every point in N_1, $\beta(\beta+1)\leq 0$ on the whole space M. Hence from (3.1)

$$\operatorname{tr} h^2=h_{ji}h^{ji}\leq 2n.$$

Next, in the case of $1-\beta^2 \leq 0$ in $N \cup N_0$, we also find $\beta(\beta+1) \geq 0$ at every point in M by the same reason. Then, by continuity of β, it follows that β is not greater than -1. But, from the compactness of the space M, we can prove that $\beta=-1$ on M by the same method in the last part of the paper [5]. Hence in both cases we have from (3.1) that $\mathrm{tr}\, h^2 \leq 2n$. Since M is compact, by the result of Simons, $\mathrm{tr}\, h^2$ can take only two values 0 and $2n$. So, combining Chern-do Carmo-Kobayashi's result we have

THEOREM. *Let M be a compact hypersurface with antinormal (f, g, u, v, λ)-structure in an odd-dimensional sphere $S^{2n+1}(1)$. If sectional curvature of the section spanned by mutually orthogonal vectors u and v is semi-definite sign, then M is a great sphere S^{2n} or $S^n \times S^n$.*

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