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A NOTE ON C^*-ALGEBRAS

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1. Introduction.

Let A be a commutative C^*-algebra with involution $+$, written $+(x)=x^+$ for all $x\in A$, and let $\Delta(A)$ be the set of all homomorphisms of A onto the field C of complex numbers. Then $\Delta(A)$ is isomorphic to the set $\{\mu^{-1}(0) : \mu\in\Delta(A)\}$ whose element is a regular Maximal ideal in A ([2]). $\Delta(A)$ with the Gelfand topology is called the regular maximal ideal space of A. Let $\hat{A}$ be the set of all Gelfand transforms of A.

Some works on ideals of a C^*-algebra are found in [1] and [4]. The purpose of this paper is to define a sub-C^*-algebra of A (Definition 2.1) and to prove some properties of sub-C^*-algebras (Theorem 2.4 and Proposition 2.3 and 2.5), in particular, that every sub-C^*-algebra of A is an intersection of some regular maximal ideals of A (Theorem 2.6). Furthermore, it will be proved that $\mu(A)=C(\Delta(A))$ under some conditions (Theorem 3.2), where $\mu(A)$ is the normed algebra of all bounded continuous functions ϕ on $\Delta(A)$ with $\phi\hat{A}\subset\hat{A}$, and $C(\Delta(A))$ is the normed algebra of all bounded continous functions from $\Delta(A)$ to C.

2. Definition and preliminary theorems.

In the sequel, we assume that A is a commutative C^*-algebra with a minimal approximate identity ([3]). Then A is semi-simple, self-adjoint and $A=C_0(\Delta(A))$, where $C_0(\Delta(A))$ is the normal algebra consisting of all bounded continuous complex-valued functions on $\Delta(A)$ vanishing at infinity ([2]). Let $Cc(\Delta(A))$ be the algebra with supremum norm consisting of all bounded continuous complex-valued functions on $\Delta(A)$ with compact supports. Then $Cc(\Delta(A))\subset\hat{A}$ ([2]). Moreover, if we denote A_0 the derived algebra of A, then $A=A_0$([2]).

DEFINITON 2.1 (Sub-C^*-algebra) Let $\mathfrak{A}$ be an ideal of A. If $\mathfrak{A}$ satisfies the conditions:
(i) $\mathfrak{A}$ is closed under involution operation ($a\in\mathfrak{A}\Rightarrow a^+\in\mathfrak{A}$),
(ii) $\mathfrak{A}$ is a Banach algebra under the same norm given to A, then it is called a sub-C^*-algebra of A.

For each $\mu_0\in\Delta(\mathfrak{A})$, let $m_0=\mu_0{}^{-1}(0)$ and define $\mu(x)=\mu_0(ax)/\mu_0(a)$, where $x\in A$ and $a\in\mathfrak{A}$, $a\notin m_0$. (Note that $\mathfrak{A}$ is a C^*-algebra).

Then μ is well-defined since $\mathfrak{A}$ is an ideal of A. $\mu(x)$ is independent of the choice of a, because for $a, b\notin m_0$

$$\mu_0(xab)=\mu_0(xa)\cdot\mu_0(b)=\mu_0(bx)\cdot\mu_0(a),$$

and so

$$\mu_0(ax)/\mu_0(a)=\mu_0(bx)/\mu_0(b).$$

PROPOSITION 2.2. *μ is in $\Delta(A)$.*

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Proof. For $x, y \in A$ $\mu(x+y)=\mu(x)+\mu(y)$ is obvious.

For $a \notin m_0$,
$$\mu(xy)=\mu_0(a^2xy)/\mu_0(a^2)$$
$$=(\mu_0(ax)/\mu_0(a))\cdot(\mu_0(ay)/\mu_0(a))$$
$$=\mu(x)\cdot\mu(y).$$

Since $\mu_0 : \mathfrak{A} \to C$ is an epimorphism, $\mu : A \to C$ is also an epimorphism. We define

$$H(\mathfrak{A})=\bigcap_{a\in\mathfrak{A}}\{\mu\in\varDelta(A) : \mu(a)=0\}.$$

Then $H(\mathfrak{A})$ is the hull of $\mathfrak{A}$ in A, and $H(\mathfrak{A})$ is a closed subset of $\varDelta(A)$ in the weak* and the hull-kernel topologies ([2]). Thus, $\varDelta(A)\sim H(\mathfrak{A})$ is an open subset of $\varDelta(A)$.

PROPOSITION 2.3. *The correspondence* $\mu_0 \to \mu$ *defines a bijective mapping from* $\varDelta(\mathfrak{A})$ *to* $\varDelta(A)\sim H(\mathfrak{A})$.

Proof. That this mapping is continous in the weak* and in the hull-kurnel topologies ([3]) is clear from the above definition. For $\mu \notin H(\mathfrak{A})$ (i.e., there is at least an element $a(\neq 0)$ in $\mathfrak{A}$ such that $\mu(a)\neq 0$) define

$$\mu_0 : \mathfrak{A}\to C \quad \text{such that } \mu_0(a)=\mu(a).$$

Then, for $x\in A$, $\mu(xa)=\mu(x)\cdot\mu(a)=\mu(x)\cdot\mu_0(a)=\mu_0(ax)$, and so $\mu(x)=\mu_0(xa)/\mu_0(a)$. This implies that our mapping is bijective.

If we define $\varDelta^0(\mathfrak{A})=\varDelta(A)\sim H(\mathfrak{A})$, $\varDelta^\circ(\mathfrak{A})$ is homeomorphic to $\varDelta(\mathfrak{A})$ in the weak* and in the hull-kernel topologies, and

$$\varDelta(A)=\varDelta^\circ(\mathfrak{A})\cup H(\mathfrak{A}).$$

Then we can prove that $\varDelta^\circ(\mathfrak{A})$ is dense in $\varDelta(A)$ with the hull-kernel topology ([2]). We summarize the above results as follows

THEOREM 2.4.

$$\varDelta(A)=H(\mathfrak{A})\cup\varDelta^\circ(\mathfrak{A}),$$

where $H(\mathfrak{A})$ *is the hull of* $\mathfrak{A}$ *in* A *and* $\varDelta^\circ(\mathfrak{A})$ *is an open subset of* $\varDelta(A)$ *which is homeomorphic to* $\varDelta(\mathfrak{A})$ *in the weak* and in the hullkernel topologies. Moreover, in the hull-kernel topology* $\varDelta^\circ(\mathfrak{A})$ *is dense in* $\varDelta(A)$.

If we put $\mathfrak{A}^\circ=\{a^0 : a\in\mathfrak{A}\}$, where $a^0(m_0)=\mu_0(a)=\hat{a}(m)=\mu(a)$ for $\mu_0\to\mu$, $\mu_0{}^{-1}(0)=m_0$ and $\mu^{-1}(0)=m$, then we have the following.

PROPOSITION 2.5. *If* A *has a minimal approximate identity, then following holds.*

(i) $\mathfrak{A}$ *is semi-simple and self adjoint,*

(ii) $\mathfrak{A}$ *is regular,*

(iii) *The mapping* $\dot{a}\to\hat{a}$ *is an isometric isomorphism of* $\mathfrak{A}^\circ$ *into* A *in the usual supremum norms on* $\mathfrak{A}^\circ$ *and* $\hat{A}$.

Proof. By the definition $\mathfrak{A}$ has an involution, i.e., $a\in\mathfrak{A}$ implies that $a^+\in\mathfrak{A}$. For $\mu_0\to\mu$, $m_0=\mu_0^{-1}(0)$ and $m=\mu^{-1}(0)$, since $a^0(m_0)=\hat{a}(m)$ and A is self-adjoint,

$$\overline{a^0(m_0)}=\overline{\hat{a}(m)} \text{ and } \hat{a}^+(m)=a^{0+}(m_0)$$

which implies that $\mathfrak{A}$ is self-adjoint. Assume $a^0(\Delta(\mathfrak{A}))=0$. Then, for $\mu_0\in\Delta(\mathfrak{A})$ and $\mu\in\Delta^\circ(\mathfrak{A})$, where $\mu_0\to\mu$, we have $\mu_0(a)=\mu(a)$. Of course, for each $\nu\in H(\mathfrak{A})$, $\nu(a)=0$. Thus $\hat{a}(\Delta(A))=0$, and it follows that $a=0$ (by the semi-simplicity of A). Thus $\mathfrak{A}$ is semi-simple.

Note that for a locally compact Hausdorff space S, $C_0(S)$ is a regular commutative Banach algebra and $\Delta(C_0(S))=S$([3]). Since $\Delta(\mathfrak{A})$ is a locally compact Hausdorff space ([2]), $C_0(\Delta(\mathfrak{A}))$ is regular which means that the weak* topology and the hull-kernel topology on $\Delta(C_0(\Delta(\mathfrak{A})))=\Delta(\mathfrak{A})$ coincides. Therefore $\mathfrak{A}$ is regular.

Finally, for $a\in\mathfrak{A}$

$$\|\mathfrak{A}^0\|_\infty=\sup_{m_0\in\Delta(\mathfrak{A})}|\mathfrak{A}^\circ(m_0)|=\sup_{m\in\Delta^0(\mathfrak{A})}|\hat{a}(m)|$$

$$=\sup_{m\in\Delta(A)}\|\hat{a}(m)\|=\|\hat{a}\|_\infty.$$

THEOREM 2.6. *If A has a minimal approximate identity, then the Kernel $K(H(\mathfrak{A}))$ of $H(\mathfrak{A})$ in $\Delta(A)$ is equal to $\mathfrak{A}$.*

Proof. Obviously $\mathfrak{A}\subset K(H(\mathfrak{A}))$. Take $x\in K(H(\mathfrak{A}))$. Then for each $m'\in H(\mathfrak{A})$, $\hat{x}(m')=0$. We shall define $\phi\in C_0(\Delta(\mathfrak{A}))$ by $\phi(m_0)=\hat{x}(m)$ for $m_0=\mu_0^{-1}(0)$ and $m=\mu(m)$, where

$$\Delta(\mathfrak{A})\ni\mu_0\longrightarrow\mu\in\Delta^0(\mathfrak{A})\subset\Delta(A),$$

Since $a^0=C_0(\Delta(\mathfrak{A}))$ (Note: $\mathfrak{A}$ is a commutative C^*-algebra) there exists an element b in $\mathfrak{A}$ such that $b^0=\phi$. Noting $b^0(m_0)=\hat{b}(m)$ we have $\hat{b}=\hat{x}$, and by the semi-simplicity $b=x$. Thus, $K(H(\mathfrak{A}))\subset\mathfrak{A}$.

EXAMPLE 2.7. Let $\mathfrak{M}$ be a regular maximal ideal of A. Since A is semi-simple and self-adjoint, for $x\in\mathfrak{M}$, $\hat{x}^+(m)=\hat{x}(m)=0$. Therefore $x^+\in\mathfrak{M}$ implies that $\mathfrak{M}$ is closed under involution operation. If A has a minimal approximate identity and $\mathfrak{M}$ is complete with the same norm as one of A, then $\mathfrak{M}$ is a sub-C^*-algebra of A, $H(\mathfrak{M})=\mathfrak{M}$, and $\Delta(\mathfrak{M})=C_0(\Delta(\mathfrak{M}))$ ($\cong\mathfrak{M}$) is isometric and isomorphic to $\Delta(A)\sim\{\mathfrak{M}\}$ in the weak* and hull-kernel topologies. In particular, by the previous theorem, every sub-C^*-algebra is an intersection of some regular maximal ideals.

Recall that A^* is the dual space of A and that $\mu(A)$ is the normed algebra of all bounded continuous functions ϕ on $\Delta(A)$ such that $\phi\hat{A}\subset\hat{A}$. We denote by $C(\Delta(A))$ the normed algebra consisting of all bounded continuous complex-valued functions on $\Delta(A)$. Then it is obvious that $\mu(A)\subset C(\Delta(A))$.

3. The main theorem.

We define $\langle x, x^*\rangle=x^*(x)$, where $x\in A$ and $x^*\in A^*$

LEMMA. 3.1. *If $x^*\in A^*$ is a continuous linear functional, there exists a unique complex-valued regular Borel measure μ_{x^*} on $\Delta(A)$ such that*

$$<xy, x^*>=\int_{\Delta(A)}\hat{x}\hat{y}(m)d\mu_{x^*}(m).$$

Furthermore, if $\hat{x}$ has a compact support, then

$$<x, x^*>=\int_{\Delta(A)}\hat{x}(m)d\mu_{x^*}(m).$$

Proof. For $x, y\in A$ $\|xy\|\leqslant\|x\|\cdot\|y\|=\|\hat{y}\|_\infty\|x\|$.

Thus $\quad |<xy, x^*>|\leqslant\|x^*\|\cdot\|xy\|\leqslant\|x\|^*\|x\|\cdot\|\hat{y}\|_\infty,$

and hence, for a fixed element $x^*\in A^*$ each $x\in A$ defines a continuous linear functional on $\hat{A}$ whose value at $\hat{y}$ is $<xy, x^*>$.

However, since $\hat{A}=C_0(\Delta(A))$ this functional is defined on $C_0(\Delta(A))$.

If we denote by $M(\Delta(A))$ the linear space of all regular complex valued Borel measures on $\Delta(A)$ which have finite total mass [3], then there exists a unique $\mu_x\in M(\Delta(A))$ ([3]) such that

$$<xy, x^*>=\int_{\Delta(A)}\hat{y}(m)d\mu_x(m) \tag{3.1}$$

and $\|\mu_x\|\leq\|x^*\|\|x\|$.

Also for $x, z\in A$ and all $y\in A$, it is clear that

$$\int_{\Delta(A)}\hat{y}\hat{z}(m)d\mu_x(m)=<xyz, x^*>=<zyx, x^*>$$
$$=\int_{\Delta(A)}\hat{y}\hat{x}(m)d\mu_z(m).$$

By the uniqueness of measures, for each $x, z\in A$ $\hat{x}\mu_z=\hat{z}\mu_x$.

Let $S_x=\{\mathfrak{M}\in\Delta(A) : \hat{x}(\mathfrak{M})\neq 0\}$ for each $x\in A$.

Then $S_x=\Delta(A)-\{\mathfrak{M}\}$, where $x\in\mathfrak{M}$, and since $\Delta(A)$ is Hausdroff, $\{\mathfrak{M}\}$ is closed in A. Thus S_x is open in $\Delta(A)$, and S_x is a locally compact subspace of $\Delta(A)$. Therefore the following integral is well-defined.

$$\int_{S_x}\phi(m)/\hat{x}(m)d\mu_x(m), \tag{3.2}$$

where $\phi\in C_c(S_x)$. If K is a compact subset of S_x, we denote all these functions in $C_c(S_x)$ whose support lies in K by $C_c^K(S_x)$. For each compact set K the above (3.2) defines a continuous linear functional in the inductive limit topology on $C_c(S_x)$ ([3]) Hence there exists a unique complex-valued regular Borel measure μ_x^x* on ([3]) such that

$$\int_{S_x}\phi(m)/\hat{x}(m)\ d\mu_x(m)=\int_{S_x}\phi(m)d\mu_x^x*(m)$$

for $\phi\in C_c(S_x)$. $\{S_x : x\in A\}$ is an open covering of $\Delta(A)$, and $\hat{x}\mu_z=\hat{z}\mu_x(x, z\in A)$ says that $\mu_z^x*=\mu_z^z*$ on $S_x\cap S_z$. Hence there exists a unique complex-valued regular measure μ_{x^*} on $\Delta(A)$ such that

$$\mu_{x^*}|S_x=\mu_x^x*.$$

From the above (3.1)

$$<xx, x^*>=\int_{\Delta(A)}\hat{x}(m)d\mu_x(m)=\int_{S_x}\hat{x}(m)d\mu_x(m),$$

and by (3.2)

$$\int_{S_x}\phi(m)\hat{x}(m)d\mu_{x^*}(m)=\int_{S_x}\phi(m)d\mu_x(m), \tag{3.3}$$

where $x\in A$ and $\phi\in C_c(S_x)$.

By the definition of S_x it is clear that $\hat{x}$ is the uniform limit of a sequence $\{\phi_n\}\subset C_c(S_x)$, and thus

$$\begin{aligned}<xx, x^*>&=\int_{S_x}\hat{x}(m)d\mu_x(m)=\lim_n\int_{S_x}\phi(m)d\mu_{x^*}(m)\\&=\lim_n\int_{S_x}\phi_n(m)\hat{x}(m)d\mu_{x^*}(m)\\&=\int_{S_x}\hat{x}\hat{x}(m)d\mu_{x^*}(m)\\&=\int_{\Delta(A)}\hat{x}\hat{x}(m)d\mu_{x^*}(m).\end{aligned}$$

With the above equation and the identity

$$4xy=(x+y)^2-(x-y)^2 \quad \textit{for } x, y\in A,$$

we can easily conclude the following:

$$<xy, x^*>=\int_{\Delta(A)}\hat{x}\hat{y}(m)d\mu_{x^*}(m) \quad \textit{for } x, y\in A.$$

If the support of $\hat{x}$ is compact, then we can find a $y\in A$ such that $\hat{x}=\hat{x}\hat{y}$ ([2]). In this case, by the semi-simplicity of A, $x=xy$. So

$$<x, x^*>=<xy, x^*>=\int_{\Delta(A)}\hat{x}\hat{y}(m)d\mu_x(m)=\int_{\Delta(A)}\hat{x}(m)d\mu_{x^*}(m).$$

The fact that $C_c(\Delta(A))\subset\hat{A}$ ([2]) and the preceding equation is valid for all $x\in A$ such that $\hat{x}$ has compact support implies that the measure μ_{x^*} constructed for each $x^*\in A^*$ is unique.

THEOREM 3.2. *If the linear span of A^2 is norm dense in A, then*

$$\mu(A)=C(\Delta(A)).$$

Proof. We shall prove thisby using the previous lemma. At first we prove that the set of all $z\in A$ such that z has compact support is dense in A. We have already proved that $C_c(\Delta(A))\subset\hat{A}$.

For $x\in A$ and $\varepsilon>0$ there exists an element $y\in A$ such that $\hat{y}\in C_c(\Delta(A))$ and $\|\hat{x}-\hat{y}\|\leqslant\varepsilon/\|x\|$, since $\hat{A}=C_0(\Delta(A))$. Since $\|xy-x^2\|\leqslant\|x\|\|y-x\|=\|x\|\|\hat{x}-\hat{y}\|<\varepsilon$, the support of $\hat{x}\hat{y}$ is compact, $4xy=(x+y)^2-(x-y)^2$ and A^2 is dense in A. Thus our assertion is true.

In order to prove our theorem it suffices to verify that $\hat{A}$ is an ideal of $C(\Delta(A))$, since $\mu(A)\subset C(\Delta(A))$. According to this reason we have to prove that for $\phi\in C(\Delta(A))$ and each $x\in A$ $\phi\hat{x}\in\hat{A}$.

For $x^*\in A^*$ and $y\in A$ with $\hat{y}\in C_c(\Delta(A))$ we define

$$<y, \beta(x^*)>=\int_{\Delta(A)}\phi\hat{y}(m)d\mu_{x^*}(m),$$

where μ_{x^*} is the regular Borel measure constructed in the proof of the preceding lemma. Since $C_c(\Delta(A))\subset\hat{A}$ and A is self-adjoint, if $U(y)\supset K(y)$ is an open set with compact closure, then there exists a $z\in A$ with $z\in C_c(\Delta(A))$, $\hat{z}\equiv 1$ on $U(y)$ and $\|\hat{z}\|_\infty=1$, where $K(y)$ is the compact support of y. Then

$$\hat{y}=\hat{z}\hat{y} \quad (z\equiv 1 \text{ on } K(y)).$$

Since $\phi\hat{z}$ has compact support, $\phi\hat{z}\in C_c(\Delta(A))$. Thus there exists

$$z_\phi\in A \text{ such that } \hat{z}_\phi=\hat{z}\phi.$$

By the above lemma and our definition

$$(3.4) \qquad <y, \beta(x^*)>=\int_{\Delta(A)}\hat{y}\phi(m)d\mu_{x^*}(m)=\int_{\Delta(A)}\hat{y}\hat{z}_\phi(m)d\mu_{x^*}(m)$$
$$=<yz_\phi, x^*>=\int_{\Delta(A)}\hat{z}_\phi(m)d\mu_y(m).$$

Therefore we have

$$|<y, \beta(x^*)>|\leqq\|\hat{z}\phi\|_\infty\|\mu_y\|\leqq\|\phi\|_\infty\|x^*\|\|y\|$$

for each $y\in A$ with $\hat{y}\in C_c(\Delta(A))$ ((3.1) in the proof of Lemma 3.1).

Thus, by the definition of $\beta(x^*)$ and the above inequality (3.4) it is clear that $\beta(x^*)$ is a bounded linear functional on $y\in A$ with $\hat{y}\in C_c(\Delta(A))$.

By the remark at the begining of our proof it follows that $\beta(x^*)$ can be uniquely extended to A without increasing norm. By the lemma 3.1

$$<xy, x_1^*>+<xy, x_2^*>=<xy, x_1^*+x_2^*>$$
$$=\int_{\Delta(A)}\hat{x}\hat{y}(m)d\mu^{x_1*}(m)+\int_{\Delta(A)}\hat{x}\hat{y}(m)d\mu_{x_2^*}(m)$$
$$=\int_{\Delta(A)}\hat{x}\hat{y}(m)d(\mu_{x_1^*}+\mu_{x_2^*})(m)$$
$$=\int_{\Delta(A)}\hat{x}\hat{y}(m)d\mu_{x_1^*+x_2^*}(m),$$

and thus $\mu_{x_1^*}+\mu_{x_2^*}=\mu_{x_1^*+x_2^*}$, where $x_1^*, x_2^*\in A^*$ and $x_1, x_2\in A$. This implies that $\beta: A^*\longrightarrow A^*$ is linear.

Let $\lim_n\|x_n^*-x^*\|=0$ and $\lim_n\|\beta(x_n^*)-z^*\|=0$. Then by the above (3.4) for any $y\in A$ with $\hat{y}\in C_c(\Delta(A))$ we have

$$|<y, \beta(x_n^*)>-<y, \beta(x^*)>|=|<y, \beta(x_n^*-x^*)>|$$
$$=|<y_\phi, x_n^*-x^*>| \quad (y_\phi=z_\phi y)$$

$$\leq \|y_\phi\| \|x_n^* - x^*\|,$$

and $|\langle y, \beta(x_n^*)\rangle - \langle y, z^*\rangle| = |\langle y, \beta(x_n^*) - z^*\rangle| \leq \|y\| \|\beta(x_n^*) - z^*\|$.

Thus $\lim_n \langle y, \beta(x_n^*)\rangle = \langle y, \beta(x^*)\rangle$ and $\lim_n \langle y, \beta(x_n^*)\rangle = \langle y, z^*\rangle$

for all $y \in A$ such that $\hat{y} \in C_c(\Delta(A))$. In consequence $\langle y, \beta(x^*)\rangle = \langle y, z^*\rangle$ for all such y and so $\beta(x^*) = z^*$. By the closed graph theorem this fact implies that β is a continuous mapping.

Assume β^* is the continuous adjoint mapping of A^{**} to A^{**}. We may consider A as isometrically embedded in A^{**} in the canonical manner [2].

Let $x \in A$ and choose a sequence $\{x_n\} \subset A$ such that $\lim_n \|x_n - x\| = 0$ and $\{\hat{x}_n\} \subset C_c(\Delta(A))$. Take $z_n \in A$ such that $\hat{z}_n = \hat{x}_n \phi$. Then for each $x^* \in A^*$

$$\langle \beta^*(x_n), x^*\rangle = \langle x_n, \beta(x^*)\rangle = \langle z_n, x^*\rangle.$$

Therefore $\beta^*(x_n) = z_n$.

Since $\lim_n \|x_n - x\| = 0$ and β^* is continuous there exists an element $z \in A$ such that

$$\lim_n z_n = \lim_n \beta^*(x_n) = \beta(x) = z.$$

Then it is clear that $z = \hat{x}\phi$, and so $\hat{x}\phi \in A$, i.e., $\hat{A}$ is an ideal of $C(\Delta(A))$. Thus

$$\mu(A) = C(\Delta(A)).$$

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