Streamline Tracing of Marine Propeller Blade

——A Formulation of an Indirect Problem——

by

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Abstract

An analytical formulation of obtaining propeller sections for a given vortex system of radial and chordwise distribution is given as an indirect problem of tracing the propeller surface.

The formulation satisfies the boundary condition of potential flow exactly rather than previous approximate use of induced streamline curvatures at the zero camber line.

General Remarks

The crux of an analytical method of propeller design is to obtain a circulation distribution for a given geometry such as section shapes, camber, etc., assuming Kutta's condition at the trailing edge [1]. Once the circulation distribution is found, induced velocities and section lifts can be readily obtained by Biot-Savart's and Kutta-Joukowsky's laws, thus leading to required pitch and thrust and torque distribution.

To account for the width of blade, in lifting surface theory, a chordwise as well as radial distribution of circulation has to be considered. Viscous correction follows as usual.

Historically, the earlier approach, e.g. in Ludwig and Ginzel [2], was to give a circular arc profile with a circular arc camber on the geometrical side and to assume an elliptical chordwise distribution on the circulation side. Then from the curvature at the $\frac{1}{2}$-chord point computed from the induced velocity at that point and from the given geometrical camber, a "camber correction factor" is derived to adjust eventually the geometrical pitch.

Later the same approach was followed for other profiles such as aerofoil sections. Also an improvement was made using Weissinger's lifting surface theory in determining circulation at $\frac{1}{4}$-chord instead of the given elliptical distribution, with the boundary condition satisfying at $\frac{3}{4}$-chord [3].

In Pien's work [4], the radial distribution of circulation is determined from the given propeller performance, and then the radial distribution is decomposed into an assumed chordwise distribution to bring about a lifting surface effect. From the chordwise distribution, the chordwise downwash distribution is computed from which meanline of the blade section camberline is obtained by integrating the induced streamline curvature on the helical surface or at the zero camberline (as we are considering at a particular section only). On this meanline then is superposed a blade section of some thickness distribution.

Later works by Kerwin and others [5] improved this theory by distribution of sources and sinks to simulate the displacement of streamlines due to finite thickness of a blade section.

The approach taken here will be similar to Ref. [4] in that the chordwise distribution of downawsh must be computed first, but the meanline of the blade section, or the blade shape if thickness is to be considered, is to be treated by streamline tracing technique, which is now well practiced in wave resistance theory.

This indirect formulation satisfied the boundary condition of potential flow exactly, or it can be made

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so with the original distribution surface of circulation adjusted, whereas use of induced stramline curvatures at the zero camber line is an approximation [6], which will be more apparent for high lift, high cambered, and wide blades. Also this should be significant with today's multibladed propellers where cascade effects are more apparent.

In the present exposition, only the formulation of the problem will be given.

Assumptions

1. Fluid is frictionless, incompressible, free of cavitation and infinite in extent.
2. The propeller is considered to be operating in an axially directed stream whose magnitude is a function of radius only, and the flow is steady, irrotational relative to a coordinate system rotating with the propeller.
3. The propeller has K symmetrically spaced identical blades. These, however, may easily be modified for non-symmetrical cases.
4. The propeller blade thickness is thin or in fact of zero thickness so that the disturbances to the flow caused by the propeller is small. This, however, may easily be modified.
5. Bound circulation of both radial and chordwise distribution is given on the surface of a blade outline on the reference helical surface on the zero camber line initially. This may be improved after the actual camber line is obtained and by recycling the formulation.
6. Rather than assuming Kutta's condition at the trailing edge, the condition is imposed that the normal component of velocity to the blade section is to be zero.
7. Trailing vortex sheet forms helical surface (of not necessarily a constant pitch) with respect to the shedding point.
8. An orthogonal curviliner system on helical surface is assumed. This is true of a constant pitch propeller only.

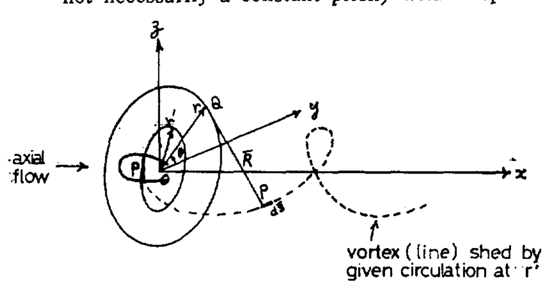

Fig. 1. Coordinate System

Coordinate System

Q : A general point at radius r and where the expression for induced velocity is sought.

P : A point on the vortex, 1) bound on the blade, 2) free on the blade and 3) free on sheet trailing behind the blade, all on radius r'

$\bar{R}$: distance vector between two points above; $R=|\bar{R}|$

$d\bar{s}$: vortex line element on r'

(x, y, z) : Cartesian coordinate system fixed on the propeller and rotating at the same angular velocity (constant). Origin of the $\theta(x,y,z)$ is contained in the plane perpendicular to x-axis and passing through the control point.

x : axis of revolution, positive down stream

y : selected to pass through a control point in the first blade

z : complete the right hand system

(x, r, θ) : Cylindrical coordinate system expressing (x, y, z), and x coinciding. θ is measured clockwise starting from y-axis looking aft.

$$\begin{cases} y=r\cos\theta & r=\sqrt{y^2+z^2} \\ z=r\sin\theta & \theta=\tan^{-1}(z/y) \end{cases} \qquad (1)$$

δ_k : θ coordnate shift designating the corresponding points on each of the K blades (all blades are assumed to be the same here).

If blades are symmetrically arranged,

$$\delta_k=\frac{2\pi(k-1)}{K};\ k=1, 2, \cdots\cdots, K \qquad (2)$$

When the effect of all blade is considered, k is to be accounted for.

(ξ, η, r) : An orthogonal curvilinear coordinate system for each blade at a particular radius r. The ξ coordinate is formed by the intersection of an axial cylinder of radius r and the line element on the helical surface

$$H_k(x, r, \theta)=x-\lambda(r)(\theta+\delta_k)=0 \qquad (3)$$

where

$$\left.\begin{aligned} \lambda(r)&=r\cdot\tan\phi(r) \\ \phi(r)&=\frac{1}{2\pi}P(r) \end{aligned}\right\} \qquad (4)$$

$\phi(r)$ being the geometric pitch angle and $P(r)$ the pitch.

The origin of (ξ, η, r) is taken to pass through a point of intersection of y and $r\theta$. When rake and skew back is zero, as assumed here, ξ is a helical line parallel to and/or passing through the nose tail line or the reference helical line for the blade section at the radius r, η coordinate is a helical line similar to ξ but perpendicular to it at the intersection of y and $r\theta$. Note that on this coordinate system the blade element is treated as an airfoil section in a uniform flow.

[See Figs, 1, 2]

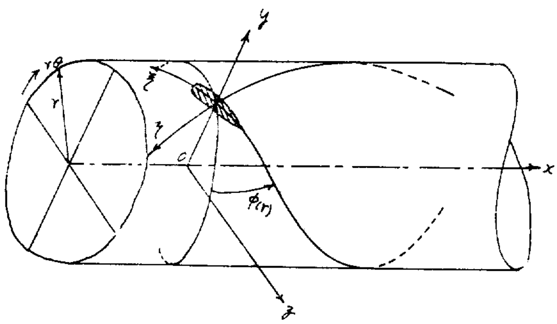

Fig. 2-a.
Curvilinear axes (ξ, η, r) on r=constant.

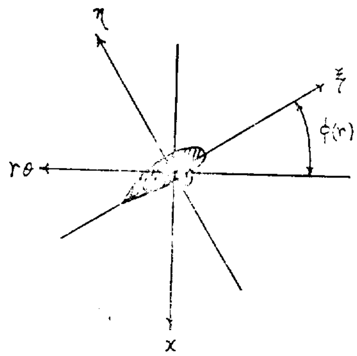

Fig. 2-b, Expanded View of (ξ, η, r) on r=constant.

With the above coodinate systems, Q may be expressed either $Q(x, y, z)$, $Q(x, r, \theta)$, or $Q(\xi, \eta, r)$ and r=const. Considering the orgin referring to Fig. 3-a and 3-b,

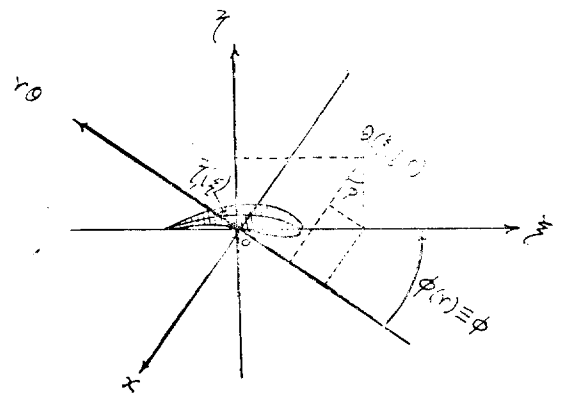

Fig. 3-a.

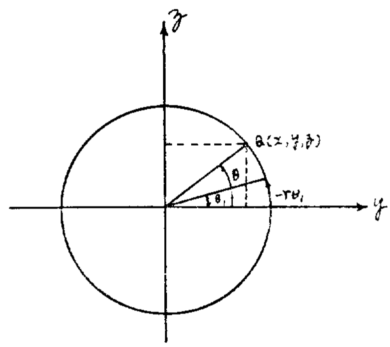

Fig. 3-b.

where :

$$\left.\begin{aligned} r\theta &= -\xi\cos\phi+\eta\sin\phi \\ x &= -\xi\sin\phi+\eta\cos\phi \end{aligned}\right\} \quad (5)$$

$$\left.\begin{aligned} \xi &= -r\theta\cos\phi-x\sin\phi \\ \eta &= -x\cos\phi+r\theta\sin\phi \end{aligned}\right\} \quad (6)$$

$$Q\left(\begin{aligned} x &= \lambda(r)(\theta+\delta_k)-(\eta\cos\phi-\xi\sin\phi) \\ y &= r\cos\left[\theta+\delta_k+\frac{1}{r}\ (\eta\sin\phi-\xi\cos\phi)\right] \\ z &= r\sin\left[\theta+\delta_k+\frac{1}{r}\ (\eta\sin\phi-\xi\cos\phi)\right] \end{aligned}\right) \quad (7)$$

with r=const, $\phi\equiv\phi(r)$,
$k=1, 2, \cdots\cdots, K$

Similarily P may be expressed as:

On the propeller blades:

$$P\left(\begin{aligned} x' &= \lambda(r')(\theta'+\delta_k')-(\eta_1'\cos\phi-\xi'\sin\phi) \\ y' &= r'\cos\left[\theta'+\delta_k'+\frac{1}{r'}(\eta_1'\sin\phi-\xi'\cos\phi)\right] \\ z' &= r'\sin\left[\theta'+\delta_k'+\frac{1}{r'}(\eta_1'\sin\phi-\xi'\cos\phi)\right] \end{aligned}\right) \quad (8)$$

with $\eta_1\equiv\eta_1(\xi)$ is the surface of vortex distribution as shown on Fig. 3-a

On the trailing vortex sheets :

$$P\begin{cases} x'=\lambda(r')(\theta'+\delta_k')+\xi'\sin\phi \\ y'=r'\cos(\theta'+\delta_k'-\frac{1}{r'}\xi'\cos\phi) \\ z'=r'\sin(\theta'+\delta_k'-\frac{1}{r'}\xi'\cos\phi) \\ \text{and } \theta'\geqq\theta(r')_T \end{cases} \tag{9}$$

where :

$\theta(r)_T=\frac{1}{r}\ \frac{c(r)}{2}\cos\phi$ or trailing edge

$c(r)$: chord

Vortex Distribution

Assume a distribution of vortex both radially and chordwise as shown in Fig. 4-a and b on the curvilinear coordinate system (r,ξ,η), where $\eta_1(\xi)$ the camber arc defined between the leading edge $\xi(r)_L$ and the trailing edge $\xi(r)_T$, and $\eta_1(\xi)$ is to be found for an indirect problem.

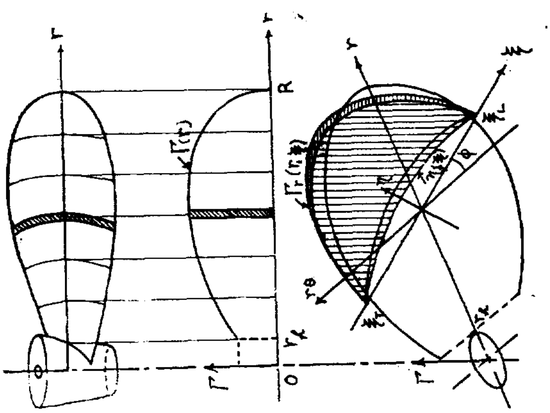

Fig. 4-a

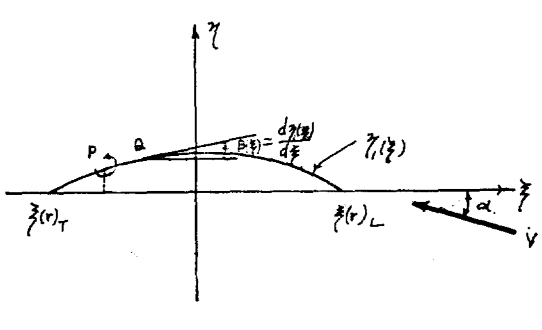

Fig. 4-b

With the above definition,

$$\Gamma(r)=\int_{\xi(r)_T}^{\xi(r)_L}\Gamma_r(r,\xi)\,d\xi \tag{10}$$

where $\Gamma(r)$: radial distribution

$\Gamma_r(r,\xi)$: chordwise disıribution

It is assumed that $\Gamma_r(r,\xi)$ is given.

Occasionally the same will begiven in the from of

$$\Gamma(r')=\int_{\theta(r)_T}^{\theta(r)_L}\Gamma_r(r,\theta)\,rd\theta \tag{10-1}$$

Boundary Condition

Let us assume an inflow into the section V ($V=\sqrt{V_a^2+4\pi^2n^2r^2}$; V_a being speed of advance and n, rps) at an angle of atlack α, then at Q

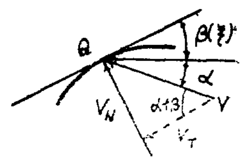

Fig. 5.

V_T : Tangential component of V

$$=V\cos[\alpha+\beta(\xi)]$$

$$=\sqrt{V_a^2+4\pi^2n^2r^2}\cos[\alpha+\beta(\xi)] \tag{11}$$

V_N : Normal component of V

$$=V\sin[\alpha+\beta(\xi)]$$

$$=\sqrt{V_a^2+4\pi^2n^2r^2}\sin[\alpha+\beta(\xi)] \tag{12}$$

$$\simeq\sqrt{V_a^2+4\pi^2n^2r^2}[\alpha+\beta(\xi)] \tag{13}$$

Then the exact boundary condition at Q is that the sum of all the velocity components normal to Q must disappear and the flow must be completely along the tangential line. Therefore what is to be done is to obtain all the induced velocities at Q due to the sum of the following sources :

(1) bound vortex distribution on the blade

(2) free vortex distribution on the blade

(3) trailing vortex distribution aft of trailing edge

(4) If exisits and not neglected, hub vortex

(5) inflow into the section, V,

and then the flow velocity is to be decomposed into normal and tangential components, and finally normal component of the sum is equated to zero.

And the resulting integro-differential equation is to be salved for $\eta_1(\xi)$.

Induced Velocities Due to Vortex Distribution

To obtain induced velocity Biot-Savart's law is to be applied. Sinc the the detail is given in [4], it's derivation will be limited to essence here.

(1) Due to the Free Vortex Trailing Aft of Blade

$$d\bar{w}_{f.1}^{(1)} = \frac{1}{4\pi}\int_{\theta(r')_T}^{\infty} \frac{d\Gamma(r')}{dr'} dr' \frac{d\bar{s}_1 \wedge \bar{R}_1}{|R_1|^3} d\theta' \tag{14}$$

where :

$$\Gamma(r') = \int_{\theta(r')_T}^{\theta(r')_L} \Gamma_r(r', \theta') r' d\theta'$$

For N-blades

$$d\bar{w}_{f.1}^{(N)} = \frac{1}{4\pi}\sum_{k=0}^{N-1} \frac{d\Gamma(r')}{dr'} dr' \int_{\theta(r')_T}^{\infty} \frac{d\bar{s}_1 \wedge \bar{R}_1}{|R_1|^3} d\theta' \tag{15}$$

with proper δ_k considered.

In the above

$$\bar{R}_1 = \left[\frac{x'-x}{|R_1|}\right]\bar{i} + \left[\frac{y'-y}{|R_1|}\right]\bar{j} + \left[\frac{z'-z}{|R_1|}\right]\bar{k}$$

$$|R_1|^2 = (x'-x)^2 + (y'-y)^2 + (z'-z)^2$$

where (x', y', z') is from (9) and (x, y, z) is from (7).

$$d\bar{s}_1 = \frac{dx'}{|ds_1|}\bar{i} + \frac{dy'}{|ds_1|}\bar{j} + \frac{dz'}{|ds_1|}\bar{k}$$

$$|ds_1|^2 = (dx')^2 + (dy')^2 + (dz')^2$$

$$dx' = \frac{1}{1+\sin^2\phi}\left[(\theta'+\delta_k') \cdot \frac{d\lambda(r')}{dr'} dr' - 2\theta' \times \sin\phi dr' + \lambda(r') d\theta' - 2r' \sin\phi d\theta'\right]$$

$$dy' = \cos\left[\theta' + \delta_k' + \theta' \cos\phi + \frac{x}{r'} \sin\phi\right]\left(-\frac{x}{r'^2}\sin\phi\right) \times dr' - r' \sin\left[\theta' + \delta_k' + \theta' \cos\phi + \frac{x}{r'} \sin\phi\right] \times \left(1 + \cos\phi - \frac{x}{r'^2}\sin\phi\right) d\theta'$$

$$dz' = \sin\left[\theta' + \delta_k' + \theta' \cos\phi + \frac{x}{r'} \sin\phi\right] \times \left(-\frac{x}{r'^2}\sin\phi\right) dr' - r' \sin\left[\theta' + \delta_k' + \theta' \cos\phi + \frac{x}{r'} \sin\phi\right] \times \left(1 + \cos\phi - \frac{x}{r'^2}\sin\phi\right) d\theta'$$

Likewise

(2) Due to the Free Vortex In the Region of Blade

$$d\bar{w}_{f.2}^{(N)} = \frac{1}{4\pi}\sum_{k=0}^{N-1} \frac{d}{dr}\int_{\theta(r')_L}^{\theta'} \Gamma_r(r', \theta') r' d\theta' dr' \times \int_{\theta'}^{\theta(r')_T} \frac{d\bar{s}_2 \wedge \bar{R}_1}{|R_2|^3} d\theta' \tag{16}$$

(3) Due to the Bound Vortex

$$d\bar{w}_b^{(N)} = \frac{1}{4\pi}\sum_{k=0}^{N-1} \Gamma_r(r', \theta') dr' \int_{\theta(r')_L}^{\theta(r')_T} \frac{d\bar{s}_3 \wedge \bar{R}_2}{|R_2|^3} d\theta' \tag{17}$$

(4) Due to the Hub Vortex (if any)

$$\bar{w}_h^{(1)} = \frac{\Gamma_h}{2\pi r'}(-\sin\theta' \bar{j} + \cos\theta' \bar{k}) \tag{18}$$

where $\Gamma_h = 2\pi r_h v_h = 4\pi^2 n r_h^2$

and n : prop · revolution

Induced velocity at Q is the vector sum of these integrated over the whole radii. Thus

$$\bar{w}_{x,y,z} = \bar{w}_{f.1} + \bar{w}_{f.2} + \bar{w}_b + \bar{w}_h \tag{19}$$

with

$$\bar{w}_{f.1} = \frac{1}{4\pi}\sum_{k=0}^{N-1}\int_{r_h}^{1.0} \frac{d\Gamma(r')}{dr'} \int_{\theta(r')_T}^{\infty} \frac{d\bar{s}_1 \wedge \bar{R}_1}{|R_1|^3} d\theta' dr' \tag{20}$$

$$\bar{w}_{f.2} = \frac{1}{4\pi}\sum_{k=0}^{N-1}\int_{r_h}^{1.0} \frac{d}{dr'} \int_{\theta(r')_L}^{\theta'} \Gamma_r(r', \theta') r' d\theta' \times \int_{\theta'}^{\theta(r')_T} \frac{d\bar{s}_2 \wedge \bar{R}_2}{|R_2|^3} d\theta' dr' \tag{21}$$

$$\bar{w}_b = \frac{1}{4\pi}\sum_{k=0}^{N-1}\int_{r_h}^{1.0} \Gamma_r(r', \theta') \int_{\theta(r')_L}^{\theta(r')_T} \frac{d\bar{s}_3 \wedge \bar{R}_2}{|R_2|^3} d\theta' dr' \tag{22}$$

$$\bar{w}_h = \frac{\Gamma_h}{2\pi r} \tag{23}$$

Volocities in

Equation (19) is given in (x, y, z) components and in order to apply the boundary condition the following transform will prove to be helpful.

$$\left.\begin{aligned} w_\xi &= w_x \cdot \frac{d\xi}{dx} + w_y \cdot \frac{d\xi}{dy} + w_z \frac{d\xi}{dz} \\ w_\eta &= w_x \cdot \frac{d\eta}{dx} + w_y \cdot \frac{d\eta}{dy} + w_z \frac{d\eta}{dz} \end{aligned}\right\} \tag{24}$$

where $(w_x, w_y, w_z) = \bar{w}_{x,y,z}$

and derivatives are obtainable from (1) and (6).

Formulation of the Problem as an Integro-Differential Equation

Applying boundary condition requires as in Fig. 6, Sum of normal components of velocities is

$$\bar{w}_n = w_\xi \sin\beta(\xi) + w_\eta \cos\beta(\xi) + \sqrt{V_a^2 + 4\pi^2 n^2 r^2} \times \sin[\alpha + \beta(\xi)] = 0 \tag{25}$$

where

$\beta(\xi) = \dfrac{d\eta_1(\xi)}{dr}$ and the Equation (25) is to be searched for $\eta_1(\xi)$ at a particular radius where Q is being located.

And such procedure is repeated for varius radii r to obtain a blade outline. Equation (25), not fully develop here is somewhat complex but it is to be noted for a flat thin 2-D airfoil in a straight flow

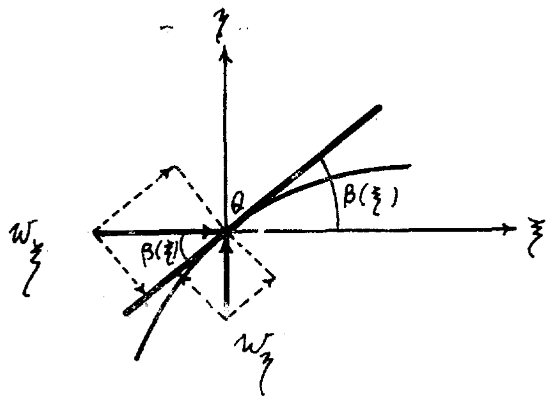

Fig. 6.

becomes.

$$\frac{1}{2\pi}\int_{-\xi_T}^{\xi_L}\Gamma(\xi)\frac{\sin\beta(x)}{x-\xi}d\xi \simeq -V[\alpha+\beta(x)] \quad (26)$$

with the first and the last terms being retained in (25). Equation (26) can be solved by expanding $\Gamma(x)$ function in trigonometric series. Equation (25) required a use of computer.

References

1. Eckhardt, M.K. and W.B. Morgan: "A Propeller Design Method," *Transactions of Society of Naval Architects and Marine Engineers*, Vol. 63, 1955.
2. Ginzel and Ludwieg, "On the Theory of Airscrew with Large-Chord Blades," *DTMB Reports and Translations*, No. 812, April 15, 1947.
3. Lerbs, H.W. "Propeller Pitch Correction Arising from Lifting Surface Effect," *TMB Report* 942, 1955.
4. Pien, P.C. "The Calculation of Marine Propellers Based on Lifting-Surface Theory," *Journal of Ship Researh*, Vol. 5, No. 2.
5. Kerwin, J.E. "The Solution of Propeller Lifting-Surface Problems by Vortex Lattice Methods," *MIT, Department of Naval Architectare*, June 1961.
6. Von Mises. *Theory of Flight*, Dover Publication, Section on Thin Foil Theory.