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# ON BIRECURRENT KÄHLER AND ALMOST TACHIBANA MANIFOLDS

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## 1. Introduction

We consider a differentiable manifold $V_{2n}$ of class $C^{\infty}$ endowed with a vector valued linear function $F$ such that for an arbitrary vector field $X$

(1.1) a) $\overline{\overline{X}}+X=0$, b) $\overline{X}\overset{\text{def}}{=\!=}F(X)$,

then $F$ is said to give an almost complex structure to $V_{2n}$ and $V_{2n}$ is called an almost complex manifold.

AGREEMENT. All the equations which follow hold for arbitrary vector fields $X,Y,Z,T,W,\cdots\cdots$ etc.

Let the almost complex manifold $V_{2n}'$ be also endowed with the Hermitian metric $g$:

(1.2) $$g(\overline{X},\overline{Y})=g(X,Y)$$

then $V_{2n}$ is called an almost Hermite manifold with the structure $(F,g)$.

Let us put

(1.3) $$'F(X,Y)=g(\overline{X},Y).$$

Then $'F$ is skew symmetric. An almost Hermite manifold $V_{2n}$ for which

(1.4) $$(\nabla F)\ (Y,X)=0,$$

or

(1.5) $$(\nabla F)(Y,X)+(\nabla F)(X,Y)=0,$$

where

(1.6) $$(\nabla F)(Y,X)\overset{\text{def}}{=\!=}(D_XF)(Y),$$

is called a Kähler and an almost Tachibana manifold respectively with a Riemannian connexion $D$.

Let $K$ be the curvature tensor of $V_{2n}$ with respect to $D$. Then for a Kähler manifold, we have

(1.7) a $$K(\overline{X},Y,Z)+K(X,\overline{Y},Z)=0,$$

(1.7) b $$K(X,Y,\overline{Z})=\overline{K(X,Y,Z)},$$

(1.7) c $$K(\overline{X},\overline{Y},Z)=K(X,Y,Z).$$

Let Ric be the Ricci tensor in $V_{2n}$. Then we have

(1.8) a $\qquad \mathrm{Ric}(\overline{X},\overline{Y})=\mathrm{Ric}(X,Y),$

(1.8) b $\qquad \mathrm{Ric}(\overline{X},Y)+\mathrm{Ric}(X,\overline{Y})=0,$

(1.8) c $\qquad \gamma(\overline{X})=\overline{\gamma(X)}$

where

(1.8) d $\qquad \mathrm{Ric}(X,Y)\overset{\mathrm{def}}{=\!=}g(\gamma(X),Y).$

From (1.6), we have

(1.9) $(D_XD_YK)\ (Z,T,W)-(D_{D_xY}K)(Z,T,W)=(\nabla\nabla K)(Z,T,W,Y,X)$

The projective curvature tensor $W^*$ in $V_{2n}$ is given by

(1.10) $W^*(Z,T,W)=K(Z,T,W)-\frac{1}{(2n-1)}\,[\mathrm{Ric}(T,W)Z-\mathrm{Ric}(Z,W)T]$

In $V_{2n}$, the Ricci identities are given by

(1.11) $(\nabla\nabla F)(Z,X,Y)-(\nabla\nabla F)(Z,Y,X)=K(Y,X,\overline{Z})-\overline{K(Y,X,Z)},$

(1.12) $(\nabla\nabla P)(Z,T,W,X,Y)-(\nabla\nabla P)(Z,T,W,Y,X)=$
$K(Y,X,P(Z,T,W))-P(K(Y,X,Z),T,W))$
$-P(Z,K(Y,X,T),W)-P(Z,T,K(Y,X,W)),$

where $P$ is $K$ or $W^*$.

The manifold $V_{2n}$ is said to be $P$-recurrent manifold if

(1.13) $\qquad (\nabla P)(Z,T,W,Z)=\alpha(X)P(Z,T,W),$

where $\alpha$ is the recurrence 1-form.

The manifold $V_{2n}$ is called a $P$-birecurrent manifold if

(1.14) $\qquad (\nabla\nabla P)(Z,T,W,Y,X)=a(Y,X)P(Z,T,W),$

where $a$ is a birecurrance 2-form.

The manifold $V_{2n}$ is called a Ricci birecurrent manifold if

(1.15) $\qquad (\nabla\nabla\mathrm{Ric})(T,W,Y,X)=a(Y,X)\ \mathrm{Ric}(T,W).$

From the above equation, we have

(1.16) $\qquad (\nabla\nabla\gamma)(T,Y,X)=a(Y,X)\ \gamma(T).$

An almost Hermite manifold $V_{2n}$ is said to be [1]

I. $P(1)$ birecurrent manifold (first order and second kind recurrent) if

(1.17) $(\nabla\nabla P)\ (\overline{Z},T,W,X,Y)+(\nabla P)\ ((\nabla F)(Z,Y),\ T,W,X)$
$+(\nabla P)((\nabla F)(Z,X),\ T,W,Y)+P((\nabla\nabla F)(Z,X,Y),T,W)$
$=a(X,Y)P(\overline{Z},T,W),$

II. $P(1,2)$ birecurrent manifold if

(1.18) $(\nabla\nabla P)(\overline{Z},\overline{T},W,X,Y)+(\nabla P)((D_YF)(Z),\overline{T},W,X)$
$+(\nabla P)(\overline{Z},(D_YF)(T),W,X)+(\nabla P)((D_XF)(Z),\overline{T},W,Y)$

$$+(\nabla P)(\overline{Z},(D_XF)(T),W,Y)+P((D_XF)(Z),(D_YF)(T),\overline{W})$$
$$+P((\nabla\nabla F)(Z),X,Y,\overline{T},W)+P((D_YF)(Z),(D_XF)(T),W)$$
$$+P(\overline{Z},(\nabla\nabla F)(T,X,Y),\overline{W})=a(X,Y)P(\overline{Z},\overline{T},\overline{W}),$$

III. $P(1,2,3)$ birecurrent manifold if

(1.19)
$$(\nabla\nabla P)(\overline{Z},\overline{T},\overline{W},X,Y)+(\nabla P)((D_YF)(Z),\overline{T},\overline{W},X)$$
$$+(\nabla P)((D_XF)(Z),\ \overline{T},\overline{W},Y)+(\nabla P)(\overline{Z},(D_YF)(T),\overline{W},X)$$
$$+(\nabla P)(\overline{Z},\overline{T},(D_YF)(W),X)+P((\nabla\nabla F)(Z,X,Y),\overline{T},\overline{W})$$
$$+P((D_XF)(Z),(D_YF)(T),\overline{W})+P((D_YF)(Z),(D_XF)(T),\overline{W})$$
$$+P(\overline{Z},(D_XF)(T),\ (D_YF)(W))+(\nabla P)(\overline{Z},\overline{T},(D_XF)(W),Y)$$
$$+P((D_XF)(Z),\ \overline{T},\ (D_YF)(W))+(\nabla P)(\overline{Z},(D_XF)(T),\overline{W},Y)$$
$$+P(\overline{Z},(\nabla\nabla F)(T,X,\ Y),\overline{W})+P(\overline{Z},(D_YF)(T),\ (D_XF)(W))$$
$$+P((D_YF)(Z),\overline{T},(D_XF)(W))+P(\overline{Z},\overline{T},(\nabla\nabla F)(W,X,Y))$$
$$=a(X,Y)\ P(\overline{Z},\overline{T},\overline{W}).$$

## 2. Birecurrent almost Tachibana manifold

THEOREM 2.1. *For a $P(1)$-birecurrent almost Tachibana manifold, we have*

(2.1)
$$(\nabla\nabla P)(\overline{X},T,W,Z,Y)+(\nabla\nabla P)(\overline{Y},T,W,X,Z)$$
$$+(\nabla\nabla P)(\overline{Z},T,W,X,Y)+P((\nabla\nabla F)(Y,X,Z),T,W)$$
$$=a(Z,Y)P(\overline{X},T,W)+a(X,Z)P(\overline{Y},T,W)+a(X,Y)P(\overline{Z},T,W).$$

PROOF. From (1.5), we have

(2.2)
$$(\nabla\nabla F)(Y,X,Z)+(\nabla\nabla F)(X,Y,Z)=0.$$

Interchanging $Y$ and $Z$ in (1.17) and then $X$ and $Z$ in (1.17) and adding these two equations thus obtained with (1.17) itself and making use of (1.5) and (2.2), we get (2.1).

COROLLARY 2.1. *For an (1) birecurrent almost Tachibana manifold, we have*

(2.3) a
$$(\nabla\nabla K)(\overline{X},T,W,Z,Y)+(\nabla\nabla K)(\overline{Y},T,W,X,Z)$$
$$+(\nabla\nabla K)(\overline{Z},T,W,X,Y)+K((\nabla\nabla F)(Y,X,Z),T,W)$$
$$=a(X,Y)K(\overline{Z},T,W)+a(X,Z)K(\overline{Y},T,W)+a(Z,Y)K(\overline{X},T,W),$$

(2.3) b
$$(\nabla\nabla \mathrm{Ric})(\overline{X},W,Z,Y)+(\nabla\nabla \mathrm{Ric})(\overline{Y},W,X,Z)$$
$$+(\nabla\nabla \mathrm{Ric})(\overline{Z},W,X,Y)+\mathrm{Ric}((\nabla\nabla F)(Y,X,Z),W)$$
$$=a(X,Y)\mathrm{Ric}(\overline{Z},W)+a(X,Z)\mathrm{Ric}(\overline{Y},W)+a(Z,Y)\ \mathrm{Ric}(\overline{X},W),$$

(2.3) c
$$(\nabla\nabla\gamma)(\overline{Z},X,Y)+(\nabla\nabla\gamma)(\overline{Y},X,Z)+(\nabla\nabla\gamma)(\overline{Z},X,Y)$$

$$+\gamma((\nabla\nabla F)(Y), X, Z)$$
$$=a(X,Y)\ \gamma(\overline{Z})+a(X,Z)\ \gamma(\overline{Y})+a(Z,Y)\ \gamma(\overline{X}).$$

PROOF. Putting $K$ for $P$ in (2.1) and contracting, the proof follows.

THEOREM 2.2. *If an almost Tachibana manifold is projective* (1) *birecurrent manifold and* (1) *birecurrent manifold for the same recurrence 2-form a, then it is a Ricci birecurrent manifold provided*

(2.4) a $$(\nabla\nabla F)(Y, X, Z)=0,$$

*or*

(2.4) b $$K(Z, X, \overline{Y})=\overline{K(Z, X, Y)}.$$

PROOF. From (1.10), we have

(2.5) $$W^*(\overline{Z}, T, W)=K(\overline{Z}, T, W)-\frac{1}{(2n-1)}[\mathrm{Ric}(T,W)\overline{Z}-\mathrm{Ric}(\overline{Z},W)T].$$

From (2.5), we have

(2.6) $$\begin{aligned}&(\nabla\nabla W^*)(\overline{Z}, T, W, X, Y)+(\nabla W^*)((\nabla F)(Z,Y),\ T, W, X)\\ &+(\nabla W^*)((\nabla F)(Z,X),\ T, W, Y)+W^*((\nabla\nabla F)(Z,X,Y),\ T, W)\\ &=(\nabla\nabla K)(\overline{Z}, T, W, X, Y)+(\nabla K)((\nabla F)(Z,Y),\ T, W, X)\\ &+(\nabla K)((\nabla F)(Z,X),\ T, W, Y)+K((\nabla\nabla F)(Z,X,Y),\ T, W)\\ &=\frac{1}{(2n-1)}[(\nabla\nabla F)(Z,X,Y)\mathrm{Ric}(T,W)+(\nabla F)(Z,X)((\nabla\mathrm{Ric})(T,W,Y))\\ &+(\nabla F)(Z,Y)(\nabla\mathrm{Ric})(T,W,X)+(\nabla\nabla\mathrm{Ric})(T,W,X,Y)\overline{Z}\\ &-((\nabla\nabla\mathrm{Ric})(\overline{Z},W,X,Y)+(\nabla\mathrm{Ric})((\nabla F)(Z,Y),W,X)\\ &+(\nabla\mathrm{Ric})((\nabla F)(Z,X),W,Y)+\mathrm{Ric}((\nabla\nabla F)(Z,X,Y),W))T].\end{aligned}$$

From (1.5), (2.2) and (2.6), in an almost Tachibana manifold, we have

(2.7) $$\begin{aligned}&(\nabla\nabla W^*)(\overline{Z}, T, W, X, Y)+(\nabla\nabla W^*)(\overline{X}, T, W, Z, Y)\\ &+(\nabla\nabla W^*)(\overline{Y}, T, W, X, Z)+W^*((\nabla\nabla F)(Y,X,Z), T, W)\\ &=(\nabla\nabla K)(\overline{Z}, T, W, X, Y)+(\nabla\nabla K)(\overline{X}, T, W, Z, Y)\\ &+(\nabla\nabla K)(\overline{Y}, T, W, X, Z)+K((\nabla\nabla F)(Y,X,Z), T, W)\\ &-\frac{1}{(2n-1)}[(\nabla\nabla\mathrm{Ric})(T,W,X,Y,Z)\ \overline{Z}+(\nabla\nabla\mathrm{Ric})(T,W,X,Z)\overline{Y}\\ &+(\nabla\nabla\mathrm{Ric})(T,W,Z,Y)\ \overline{X}+(\nabla\nabla F)(Y,X,Z)\ \mathrm{Ric}(T,W)\\ &-((\nabla\nabla\mathrm{Ric})(\overline{Z},W,X,Y)+(\nabla\nabla\mathrm{Ric})(\overline{X},W,Z,Y)\\ &+(\nabla\nabla\mathrm{Ric})(\overline{Y},W,X,Z)+\mathrm{Ric}((\nabla\nabla F)(Y,X,Z),W))\ T].\end{aligned}$$

Let the manifold $V_{2n}$ be projective (1) birecurrent and (1) birecurrent manifold. Then from (2.1), (2.3), (2.4) and (2.7), we have

(2.8) $$\begin{aligned}&((\nabla\nabla\mathrm{Ric})(T,W,X,Y)-a(X,Y)\ \mathrm{Ric}(T,W)\ \overline{Z}\\ &+((\nabla\nabla\mathrm{Ric})(T,W,X,Z)-a(X,Z)\ \mathrm{Ric}(T,W)\ \overline{Y}\end{aligned}$$

$$+((\nabla\nabla \mathrm{Ric})(T,W,Z,Y)-a(Z,Y)\ \mathrm{Ric}(T,W))\ \overline{X}=0.$$

Since the equations hold for arbitrary vector fields $X, Y, Z, T, W$ etc. Hence from (2.8), we get

(2.9) $$(\nabla\nabla \mathrm{Ric})(T,W,X,Y)=a(X,Y)\ \mathrm{Ric}(T,W),$$

which proves the statement.

THEOREM 2.3. *If an almost Hermite manifold is $P(1)$ birecurrent manifold, then we have*

(2.10) a $$K(Y,X,P(\overline{Z},T,W))-P(\overline{Z},K(Y,X,T),\ W)$$
$$-P(\overline{Z},T,K(Y,X,W))-P(\overline{K(Y,X,Z)},\ T,W)$$
$$=A(X,Y)P(\overline{Z},T,W),$$

*where*

(2·10) b $$A(X,Y)\overset{\mathrm{def}}{=\!=}a(X,Y)-a(Y,X).$$

PROOF. Interchanging $X$ and $Y$ in(1.17) and subtracting the resulting equation thus obtained from (1.17) and making use of Ricci identities (1.11) and (1.12), we get (2.10) a.

THEOREM 2.4. *In order that $P(1)$ birecurrent manifold be $P$-birecurrent manifold, we must have*

(2.11) $$P(\overline{K(Y,X,\overline{Z})}+K(Y,X,Z),T,W)=0$$

PROOF. Barring $Z$ in (1.17) and using (1.1), we have

(2.12) $$(\nabla\nabla P)(Z,T,W,X,Y)-(\nabla P)((\nabla F)(\overline{Z},Y),\ T,W,X)$$
$$-(\nabla P)((\nabla F)(\overline{Z},X),T,W,Y)-P((\nabla\nabla F)(\overline{Z},X,Y),\ T,W)$$
$$=a(X,Y)P(Z,T,W).$$

Let the manifold be $P$ birecurrent, Then from (1.14) and (2.12), we have

(2.13) $$P\ ((\nabla\nabla F)(\overline{Z},X,Y),\ T,W)+(\nabla P)((\nabla F)(\overline{Z},Y),\ T,W,X)$$
$$+(\nabla P)((\nabla F)(\overline{Z},X),\ T,W,Y)=0.$$

Interchanging $X$ and $Y$ in (2.13) and subtracting the resulting equation thus obtained from (2.13) and making use of (1.11), we get (2.11), which proves the statement.

THEOREM 2.5. *If an almost Hermite manifold is $P(1,2)$ birecurrent manifold, then we have*

(2.14) $$K(Y,X,P(\overline{Z},\overline{T},W))-P(\overline{Z},\overline{T},K(Y,X,W))$$
$$-P(\overline{K(Y,X,Z)},\ \overline{T},W)-P(\overline{Z},\overline{K(Y,X,T)},\ W)$$
$$=A(X,Y)P(\overline{Z},\overline{T},W).$$

THEOREM 2.6. *If an almost Hermite manifold is $P(1,2,3)$ birecurrent manifold, then we have*

$$K(Y,X,P(\overline{Z},\overline{T},\overline{W}))-P(\overline{K(Y,X,Z)},\ \overline{T},\overline{W}) - P(\overline{Z},\overline{K(Y,X,T)},\ \overline{W})-P(\overline{Z},\overline{T},\overline{K(Y,X,W)}) = A(X,Y)P(\overline{Z},\overline{T},\overline{W}). \tag{2.15}$$

The proof of theorems 2.5 and 2.6 follow from the proof of theorem 2.3 by making use of (2.18) and (2.19).

THEOREM 2.7. *In order that $P(1,2)$ birecurrent manifold be P-birecurrent manifold, we must have*

$$P(\overline{K(Y,X,\overline{Z})}+K(Y,X,Z),\ T,W) + P(Z,\overline{K(Y,X,\overline{T})}+K(Y,X,T),\ W)=0. \tag{2.16}$$

THEOREM 2.8. *In order that $P(1,2,3)$ birecurrent manifold be P-birecurrent manifold, we must have*

$$P(\overline{K(Y,X,\overline{Z})}+K(Y,X,Z),\ T,W) + P(Z,\overline{K(Y,X,\overline{T})}+K(Y,X,T),\ W) + P(Z,T,\overline{K(Y,X,\overline{W})}+K(Y,X,W))=0. \tag{2.17}$$

THEOREM 2.9. *In order that $P(1.2)$ birecurrent manifold be $P(1)$ birecurrent manifold, we must have*

$$P(Z,K(Y,X,T)+\overline{K(Y,X,\overline{T})},W)=0 \tag{2.18}$$

THEOREM 2.10. *In order that $P(1,2,3)$ birecurrent manifold be $P(1,2)$ birecurrent manifold, we must have*

$$P(Z,T,\overline{K(Y,X,\overline{W}},+K(Y,X,W))=0 \tag{2.19}$$

The proof of theorem 2.7 to 2.10 is similar to the proof of theorem 2.4.

## 3. Birecurrent Kähler manifold

THEOREM 3.1. *A birecurrent Kähler manifold of any order* ((1), (1,2), (1, 2,3)) *is a birecurrent manifold.*

PROOF. From (1.7) b and theorems 2.4, 2.7 and 2.8, the statement follows:

THEOREM 3.2. *Every recurrent Kähler manifold with the 1-form $\alpha$ satisfying*

$$(\nabla\alpha)(X,Y)+\alpha(X)\alpha(Y)\neq 0 \tag{3.1}$$

*is a birecurrent Kähler manifold but the converse is not true necessarily.*

PROOF. From (1.13), we have

(3.2) $$(\nabla\nabla P)(Z,T,W,X,Y)=((\nabla\alpha)(X,Y)+\alpha(X)\alpha(Y))P(Z,T,W).$$

Comparing (3.2) and (1.14), we get

(3.3) $$a(X,Y)=(\nabla\alpha)(X,Y)+\alpha(X)\alpha(Y),$$

which proves the statement.

THEOREM 3.3. *The 2-form a is symmetric in a birecurrent Kähler manifold with non-vanishing scalar curvature R.*

PROOF. Let the manifold $V_{2n}$ be a birecurrent Kähler manifold. Then from (1.14), we have

(3.4) $$(\nabla\nabla K)(Z,T,W,X,Y)=a(X,Y)\ K(Z,T,W).$$

From (3.4), we have

(3.5) $$(\nabla\nabla \mathrm{Ric})(T,W,X,Y)=a(X,Y)\ \mathrm{Ric}(T,W).$$

From(3.5), we have

(3.6) $$(\nabla\nabla\gamma)(T,X,Y)=a(X,Y)\ \gamma(T)$$

or

(3.7) $$(D_Y D_X\gamma)(T)-(D_{D_Y X}\gamma)(T)=a(X,Y)\ \gamma(T).$$

From (3.7), we have

(3.8) a $$Y(X,R)-X(Y,R)-([Y,X],R)=(a(X,Y)-a(Y,X))\ R,$$

where

(3.8) b $$(C_1^1\gamma)\overset{\text{def}}{=\!=}R.$$

From (3.8)a, we get

(3.9) $$a(X,Y)=a(Y,X),$$

which proves the statement.

THEOREM 3.4. *In a birecurrent Kähler manifold, the 2-form A is hybrid in both the slots:*

(3.10) $$A(\overline{X},\overline{Y})=A(X,Y).$$

PROOF. Interchanging $X$ and $Y$ in (3.6) and using Ricci identity, we have

(3.11) $$K(Y,X,\ \gamma(T))-\gamma(K(Y,X,T))=A(X,Y)\ \gamma(T).$$

Barring $X$ and $Y$ in (3.11) and using (1.7)c, we have

(3.12) $$K(Y,X,\ \gamma(T))-\gamma(K(Y,X,T))=A(\overline{X},\overline{Y})\ \gamma(T).$$

Comparing (3.11) and (3.12) we get

(3.13) $$A(\overline{X},\overline{Y})-A(X,Y)=0$$

Since

$$\gamma(T)\neq 0,$$

which proves the statement.

THEOREM 3.5. *Let us define*

(3.14) $$'A(X,Y)=A(\overline{X},Y).$$

*Then the tensor field 'A is hybrid.*

(3.15) $$'A(\overline{X},\overline{Y})='A(X,Y),$$

*and 'A is symmetric.*

(3.13) $$'A(X,Y)='A(Y,X).$$

PROOF. Barring $X$ and $Y$ in (3.14) and using (1.1), we get,

(3.17) $$'A(\overline{X},\overline{Y})=-A(X,\overline{Y})=A(\overline{X},Y)='A(X,Y).$$

From (3.17), we have

(3.18) $$'A(X,Y)=-A(X,\overline{Y})=A(\overline{Y},X)='A(Y,X),$$

which proves the result.

THEOREM 3.6. *In a birecurrent Kähler manifold, 1-form $\alpha$ and 2-form $A$ satisfies the relation*:

(3.19) $$(\nabla A)(X,Y,Z)=\alpha(Z)\ A(X,Y).$$

PROOF. Form (3.11), we have

(3.20) $$\begin{aligned}&(\nabla K)(Y,X,\ \gamma(T),Z)+K(Y,X,(\nabla\gamma)(T,Z))\\&-(\nabla\gamma)(K(Y,X,T),\ Z)-\gamma((\nabla K)(Y,X,T,Z))\\&=(\nabla A)(X,Y,Z)\ \gamma(T)+A(X,Y)(\nabla\gamma)(T,Z).\end{aligned}$$

Let us put

(3.21) $$(\nabla\gamma)(T,W)=\alpha(W)\ \gamma(T).$$

Then from (1.13), (3.21) and (3.20), we have

(3.22) $$((\nabla A)(X,Y,Z)-\alpha(Z)A(X,Y))\gamma(T)=0,$$

which yields (3.19), since $\gamma(T)\neq 0$. Hence proof.

THEOREM 3.7. *In a birecurrent Kähler manifold, we have*

(3.23) $$\alpha(\overline{X})A(Y,Z)+\alpha(\overline{Y})A(Z,X)+\alpha(\overline{Z})\ A(X,Y)=0.$$

PROOF. From (3.3), we have

(3.24) $$A(X,Y)=(\nabla\alpha)(X,Y)-(\nabla\alpha)(-Y,X).$$

From (3.24), we have

(3.25) $$(\nabla A)(X,Y,Z)=(\nabla\nabla\alpha)(X,Y,Z)-(\nabla\nabla\alpha)(Y,X,Z).$$

Adding two other equations by taking cyclic permutation of $X, Y$ and $Z$ in (3.25), we get

(3.26) $$(\nabla A)(X,Y,Z)+(\nabla A)(Y,Z,X)+(\nabla A)(Z,X,Y)=0.$$

From (3.19) and (3.26), we have

(3.27) $$\alpha(Z)A(X,Y)+\alpha(X)A(Y,Z)+\alpha(Y)\ A(Z,X)=0.$$

Barring $X, Y$ and $Z$ in (3.27) and using (3.10), we get (3.23). Hence proof.

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