Kyungpook Math. J.
Volume 14, Number 1
June, 1974

A THEOREM ON JOIN VARIETIES OF GROUPS

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The join of two f. b.* varieties of groups need not be f.b. is a well-known fact, but no example is known so far to testify this.

Bryant has, however, characterized that,

(i) If $\mathscr{U}$ is a variety of groups and $\mathscr{V}$ is a nilpotent variety, then the join variety $\mathscr{U}\vee\mathscr{V}$ is f.b. iff $\mathscr{U}$ is f.b. see [1]

(ii) Let $\mathscr{U}$ be a f.b. variety and $\mathscr{V}$, a vaughan Lee variety (a subvariety of $\mathscr{N}_c\mathfrak{A}<\mathfrak{A}\mathscr{N}_d$). Then the join variety $\mathscr{U}\vee\mathscr{V}$ is f.b. (see [2]).

Denoting varieties by doubly underlined Roman Capitals and using the notations of [2], we give the following general characterization of the join of two varieties. This includes (i).

THEOREM. *If V is a variety in which an identity* $[[x_1,\cdots,x_m],\ [x_{m+1},x_{m+2}]]$ *is satisfied and* $\mathscr{U}$ *is arbitrary then* $\mathscr{U}\vee\mathscr{V}$ *is f.b. iff* $\mathscr{U}$ *is f.b.*

PROOF. ($\Rightarrow$) $\mathscr{U}\vee\mathscr{V}$ is f.b. by assumption. Moreover, since $\mathscr{V}$ is f.b. (see [3]), $\mathscr{U}\wedge\mathscr{V}$ is f.b. because as subvariety it again satisfies the law $[[x_1,\cdots,x_m],[x_{m+1}, x_{m+2}]]$. Hence by Lemma 4 of [1] $\mathscr{U}$ is f.b.

($\Leftarrow$) Conversely for $m\geq 2$, the laws,

(a) $[[x_1, y_1]\cdots, [x_{m+1}, y_{m+1}]]$

(b) $[[x_1,\cdots,x_{m+1}],[y_1,\cdots,y_{m+1}]]$,

can easily be seen to be the consequences of the law $[[x_1,\cdots,x_m],[x_{m+1},x_{m+2}]]$. Hence the set of laws $\mathscr{V}$ defining the variety $\mathscr{V}$ includes $\gamma_{m+1}(X')$. $\gamma_{m+1}(X)'$ where $X, \gamma_{m+1}(X'), \gamma_{m+1}(X)'$ have their usual meanings as in [2]. In other words $\mathscr{V}$ is the subvariety of $\mathscr{N}_m\mathfrak{A}\wedge\mathfrak{A}\mathscr{N}_m$. Since $\mathscr{U}$ is f.b. by assumption in this case, therefore, in particular by (ii) $\mathscr{U}\vee\mathscr{V}$ is f.b.

The author is grateful to his supervisor prof. M.A. Kazim for encouragement and help in the preparation of this note.

* *f.b. = Finitely based.*

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