論 文

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Maximum Principle을 利用한 原子爐의 時間 最適制御
(Time Optimal Control of the Nuclear Reactor Using the Maximum Principle)

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要 約

Pontryagin의 最大原理의 놀라운 方式을 써서 原子爐에 있어서 最適制御 switching 時間과 最適制惟 switching 點을 求하였다. 그리고 原子爐의 初期狀態에서 그의 目標狀態로 出力을 變換시킬 때의 制御軌跡을 時間最適制御 方式을 利用하여 주어진 原子爐의 parameter 값과 piecewise constant 入力값에 따라 最適化 시켰다.

Abstract

The maximum principle of Pontryagin provides the celebrated method to obtain the optimum control switching time and switching points on the nuclear reactor.

The control trajectories transfered from its initial state to the target state are optimized based on time optioptimal control method with the given reactor parameters and the piecewise constant input values.

1. Introduction

The appearance of computer made possible the application of the various techniques developed in modern control theory such as the maximum principle[1] and dynamic programming[2]. In the light of these possiblilities, it certainly is worthwhile to study their applications in order to analyze, synthesize and design the control systems in more accurate and practical use. In this connection, optimal control of a certain physical process has many merits in economics and its pratical applications to complex systems. Concerning this minimization control, the minimum time optimal is also of primary importance in the reactor control system due to the more efficient fuel management and minimum time requirement for reactor start-up or change in the power level.

The minimum time optimal introduced in this paper for nuclear reactor is principally based on the maximum principle of Pontryagin[3] and the attempts are being made to use the computer to perform the calculation of switching points and its corresponding times as this control system is so complex that more elaborate control and accurate calculations are required.

In such cases, the maximum principle seems to offer the useful techniques in synthesizing and analyzing the complex multivariable system through the state space approach.

2. Reactor kinetics

The general aspect of reactor kinetics, common to all types of fission reators, is the mutual relations

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among the reactivity, neutron generation time and delayed neutron[4].

Reactivity ρ is the rate of neutron growth factor k from unity. When the reactor is not very far from critical in the point reactor model, k is used in the neighborhood of unity.

The neutron generation time l is the mean time between neutron cycles for neutron reproduction in multiplying assemblies.

In general, when k is limited to the near unity, the difference between the neutron life time and neutron generation time are treated to be negligible.

When the reactor is under the neighborhood of the critical, in the regime where the neutron chain reaction is not sufficiently sustained only with the prompt neutrons, the delayed neutrons are primarily dominant.

For the convenience of synthesis for the reactor kinetics, taking the delayed neutrons into account in the fission process, the rate of neutron population change and the dayed neutrons can be easily described in a set of the normalized form, according to the small perturbation theory[5]

$$\frac{dn}{dt}=\frac{\rho-\beta}{l}n+\frac{\beta}{l}c \tag{1}$$

$$\frac{dc}{dt}=\lambda c \quad -\lambda c \tag{2}$$

for a single group model without loss in generality.[6] Where $\rho=\frac{k-1}{k}$ is reactivity, $\beta=\sum\beta i$ the total delayed neutron fraction and λc represents the population of precursors.

3. Time optimal control

Time optimal control is to control the nuclear reactor system from its initial state to the desired target state within the allowable minimum time.[6]

The magnitude of admissible control is, of course, restricted as piecewise constant function. The n the reactivity for the reactor kinetics is symmetrically introduced in the manner of positive limits or negative limits during the control process in figure 1.

The optimal trajectory for the system is uniquely determined on the phase plane without any phsical constraint on the trajectories.

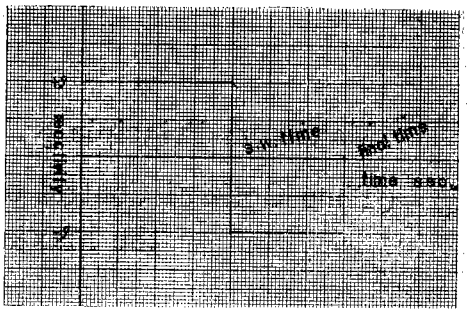

Figure 1. Piecewise constant input fun ction in reactivity

The set of equations (1) and (2) is replaced in matrix form;

$$\dot{X}=AX(t_0) \tag{3}$$

It is then desired to transfer the system from the initial state $X(t_0)$ to the target state $X(t_f)$ with the given reactivity restricted. Therefore, the performance function for time optimal control is given as; [7]

$$J=\int_{t0}^{tf} dt=t_f-t_0 \tag{4}$$

where t_f may be free and the reachable value of the reactivity is constrained as

$$|\rho|\leqq\xi\beta \tag{5}$$

where ξ is a constant depending on the delayed neutrons.

The additional state variable x_0 is then

$$x_0=t_f-t_0 \tag{6}$$

and the matrix with the added state variable is replaced by

$$\dot{X}=AX(t_0) \tag{7}$$

And the corresponding costate function for the adjoint system is defined by

$$\dot{P}=-[X]^TP(t_0) \tag{8}$$

The inner product of these two equations is of the form of Hamiltonian equation[8];

$$H=\langle X\cdot P\rangle=x_0p_0+\langle X\cdot P\rangle \tag{9}$$

The necessary condition for optimality is that $P_0=-1$ and the zeroth term(x_0, p_0) is superflous. It can be written as;

$$\begin{aligned}H&=\langle X\cdot P\rangle\\&=\frac{\rho-\beta}{l}x_1p_1+\frac{\beta}{l}x_2p_1+\lambda x_1p_2-\lambda x_2p_2\\&=M\end{aligned} \tag{10}$$

where M is the maximum value.

This equation implies that the control must be

maximum value lain on the allowable boundary of the closed set in order that Hamiltonian H maintains its maximum at

$$\rho=\xi\beta sgnp_1 \tag{11}$$

This shows that H is maximum and its maximum value is constant over the whole entire range of the time $t\in[t_0, t_f]$.

The corresponding canonical equations are;

$$\dot{X}=\frac{\partial H}{\partial P}=AX \tag{12}$$

$$\dot{P}=-\frac{\partial H}{\partial X}=-A^TP \tag{13}$$

with boundary condition $X(t_0)$, $X(t_f)$, $P(t_0)$ and $P(t_0)$. The solution of these canonical equations are described in more detail in the appendix.

If the time t is eliminated in the equation(A—9), its solution on the phase plain is readily obtained by

$$\left[\frac{a_{11}x_1-c_{12}x_2}{a_{21}x_{10}-a_{22}x_{20}}\right]^{1/\alpha_1}=\left[\frac{b_{11}x_1-b_{12}x_2}{b_{21}x_{10}-b_{22}x_{20}}\right]^{1/\alpha_2} \tag{14}$$

where $a_{11}=\lambda x_{10}-(\alpha_1+\lambda)x_{20}$

$$a_{12}=\left(\frac{\rho-\beta}{l}-\alpha_1\right)x_{10}+\frac{\beta}{l}x_{20}$$

$$a_{21}=\lambda x_{10}-\lambda x_{20}=b_{21}$$

$$a_{22}=\frac{\rho-\beta}{l}x_{10}+\frac{\beta}{l}x_{20}=b_{22}$$

$$b_{11}=\lambda x_{10}-(\alpha_2+\lambda)x_{20}$$

$$b_{12}=\left(\frac{\rho-\beta}{l}-\alpha_2\right)x_{10}+\frac{\beta}{l}x_{20}$$

This solution defined on the equation (14) for the reactor kinetics, which reaches the target set within the allowable minimum time is then the optimal trajectory.

The equation(11) gives the switching condition when the sign of p_1 is changed at the moment of which p_1 meets zero but the initial vlaue of p_1 cannot be negative while power is increasing.

Therefore, only one switching point is recorded by the equation(14), which brings out the uniqueness of the optimal control as the value of ρ is on the upper bound of the control region so that Hamiltonian must keep maximum constant.

The trajectories depicted by equation(14) for the several different reactivities and target values are shown, x_1 versus x_2, in the figure 2 which is obtained by the analog computer.

The line S, whose slope is 45°, shows its target state with zero reactivity. The trajectory a-c-e on the phase plane in figure 2 and on R_3 space in figure 3 depicts the time optimal trajectory desired.

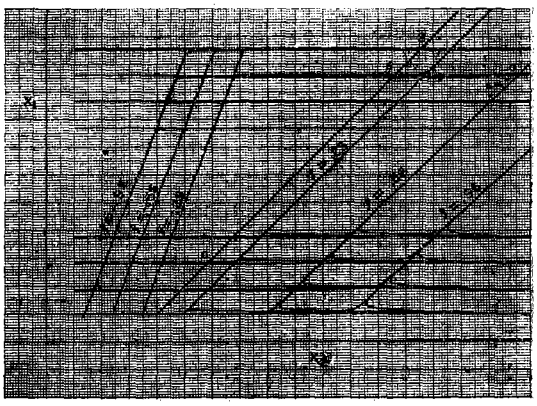

Figure 2. Time optimal trajectory, x_1 versus x_2, with the several different initial values and reactivities for U—235 (l=0.0001, λ=0.0784)

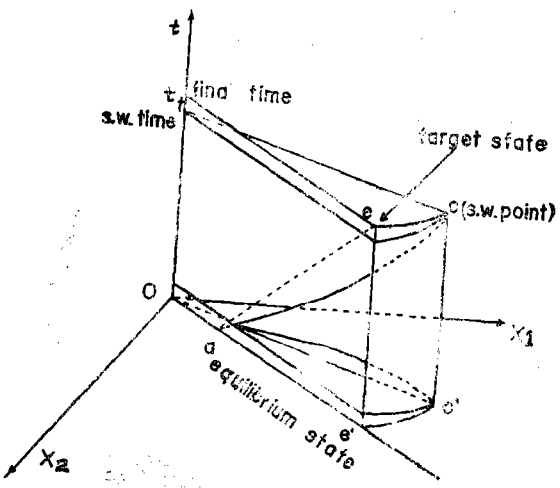

Figure 3. Time optimal trajectory in three dimension

The calculation of switching points according to the several reactor parameters shown in the table 1 and the several reactivities was found by the digital computer and their results are presented in the table 2.

Table 1. Fuel nuclide and reactor parameters

Fuel nuclide	β	λ	l(sec)
U^{235}	0.0065	0.00748	0.001
U^{238}	0.0026	0.0559	0.001
Pu^{239}	0.0021	0.686	0.001

Table 2. Switching points and switching time

Fuel nuclide	Reactivity dollars	Initial state(1.1), Target state(2.2) S.W.point X_1	X_2	S.W. time (sec)	Final time (sec)
U^{235}	0.2	2.4881	1.9979	36.162	0.1059
U^{235}	0.5	3.8939	1.9918	9.4937	0.1341
U^{235}	0.9	11.689	1.9528	2.3857	0.1936
U^{238}	0.2	2.4918	1.9986	21.562	0.0458
U^{238}	0.5	3.9253	1.9943	5.5809	0.0568
U^{238}	0.9	12.994	1.9623	1.2441	0.0846
Pu^{239}	0.2	2.4696	1.9949	42.867	0.03353
Pu^{239}	0.5	3.7424	1.9800	12.014	0.3936
Pu^{239}	0.9	8.4256	1.9220	4.1304	0.5058

These switching points for the time optimal control process are determined from equations (11) and (14) according to the predetermined target values of the output x_1. This means that $p_1(t)$ in equation(A−11) changes monotonically with the time and thus the switching of control occurs at most once at the moment of change of its sign.

The initial value p_1(to) of the adjoint system satisfies the non-zero value and its terminal condition, that is, transversality condition, is satisfied at the terminal time t_f by

$$\langle P(t_f),\ X-X(t_f)\rangle=0 \qquad (15)$$

That is, the vector of $P_1(t_f)$ is perpendicular to the tagent plane of $X(t_f)$ and the value of $p_1(t_f)$ must be zero in order to meet the transversality condition. The zero value of $p_1(t_f)$ gives also $\rho=0$ at the terminal moment in equilibrium state S in figure 2 as shown in figure 1.

4. Conclution

Applying the maximum principle to the process control of reactor kinetics, the exact optimal switching point, its switching time and its corresponding time optimal trajectory transfering the control system from the initial equilibrium state to the target equilibrium state in the shortest time are obtained by the digital computer.[8]

The time optimal trajectory a-c-e in figure 2 or 3 is decided only with the reactivity without concerning with the initial equilibrium state and the switching points are independent on the initial state, but dependent on the target state.

In power increase, the time optimal trajectory converges towards the switching point with the postitive reativity and switches to the power shutdown trajectory with the negative reactivity at the switching point.

The trajectory obtained from equation(14) is then the unique solution for the time optimal control, whose edge belongs to the boundary set of control region.

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Appendix

1. Solution of state equation

For a piecewise constant of reactivity for reactor kinetics, the set of equations(12) and(13) is replaced in the state matrix form;

$$\dot{X}=AX \tag{A—1}$$

Where $\dot{X}$ and X are column matrix given by;

$$\dot{X}=\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix}=\begin{bmatrix}\dot{n}\\ \dot{c}\end{bmatrix}$$

$$X=\begin{bmatrix}X_1\\ X_2\end{bmatrix}=\begin{bmatrix}n\\ c\end{bmatrix} \tag{A—2}$$

and A is a time unvariable constant square matrix given by;

$$A=\begin{bmatrix}\frac{\rho-\beta}{l} & \frac{\beta}{l}\\ \lambda & \lambda\end{bmatrix} \tag{A—3}$$

To find the eigen values α_1 and α_2 of A, the determinant of $A-\alpha I$ is set to zero

$$|A-\alpha I|=0 \tag{A—4}$$

which leads to

$$\alpha_1=\frac{\rho-\beta-l\lambda+\sqrt{(\rho-\beta-l\lambda)^2+4l\lambda\rho}}{2l} \tag{A—5}$$

$$\alpha_2=\frac{\rho-\beta-l\lambda-\sqrt{(\rho-\beta-l\lambda\rho+4l\lambda\rho}}{2l} \tag{A—6}$$

The transition matrix for this antomonous system (A—1) is obtained according to the Sylvester's theorem.

$$\phi(t-t_0)=e^{A(t-t_0)}$$
$$=\frac{e^{\alpha_1(t-t_0)}}{\alpha_1-\alpha_2}\begin{bmatrix}\frac{\rho-\beta-l\alpha_2}{l} & \frac{\beta}{l}\\ \lambda & -\lambda-\alpha_2\end{bmatrix}$$
$$+\frac{e^{\alpha_2(t-t_0)}}{\alpha_2-\alpha_1}\begin{bmatrix}\frac{\rho-\beta-l\alpha_1}{l} & \frac{\beta}{l}\\ \lambda & -\lambda-\alpha_1\end{bmatrix} \tag{A—7}$$

If the problem is to transfer the system from the initial state $X(t_0)=X_0$ at time t_0 to the target state $X(t_f)=X_f$ at time t_f, the solution of the equation(A—1) is;

$$X(t)=\phi(t-t_0)X(t_0) \tag{A—8}$$

That is,

$$\begin{bmatrix}X_1\\ X_2\end{bmatrix}=\frac{e^{\alpha_1(t-t_0)}}{\alpha_1-\alpha_2}\begin{bmatrix}\frac{\rho-\beta-l\alpha_2}{l} & \frac{\beta}{l}\\ \lambda & -\lambda-\alpha_2\end{bmatrix}\begin{bmatrix}X_{10}\\ X_{20}\end{bmatrix}$$
$$+\frac{e^{\alpha_2(t-t_0)}}{\alpha_2-\alpha_1}\begin{bmatrix}\frac{\rho-\beta-l\alpha_1}{l} & \frac{\beta}{l}\\ \lambda & -\lambda-\alpha_1\end{bmatrix}\begin{bmatrix}X_{10}\\ X_{20}\end{bmatrix} \tag{A—9}$$

Hence the sign of the reactivity ρ must be alternative in accordance with the movement of control rod so that it can give the suitable trajectory.

2. Solution of adjoint system equation

Since the adjoint system is, in general, adjoint to the original system, its solution is similar to the solution of the original system with time reversed and thus the solution of costate function to the adjoint system is;

$$P(t)=-\phi^T(t-t_0)P(t_0) \tag{A—10}$$

which is replaced as the form of

$$\begin{bmatrix}p_1\\ p_2\end{bmatrix}=\frac{e^{v_1(t-t_0)}}{v_1-v_2}\begin{bmatrix}\frac{\beta-\rho-lv_2}{l} & -\lambda\\ -\frac{\beta}{l} & \lambda-v_2\end{bmatrix}\begin{bmatrix}p_{10}\\ p_{20}\end{bmatrix}$$
$$+\frac{e^{v_2(t-t_0)}}{v_2-v_1}\begin{bmatrix}\frac{\beta-\rho-lv_1}{l} & -\lambda\\ -\frac{\beta}{l} & \lambda-v_2\end{bmatrix}\begin{bmatrix}p_{10}\\ p_{20}\end{bmatrix} \tag{A—11}$$

Where p_{10} and p_{20} are the initial conditions of the adjoint system and V_1 and V_2 are eigen values defined by;

$$v_1=\frac{-(\rho-\beta-l\lambda)+\sqrt{(\rho-\beta-l\lambda)^2+4l\lambda\rho}}{2l} \tag{A—12}$$

$$v_2=\frac{-(\rho-\beta-l\lambda)-\sqrt{(\rho-\beta-l\lambda)^2+4l\lambda\rho}}{2l} \tag{A—13}$$

Hence the sign of ρ is also alternative in accordance with the motion of control rod.