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A THEOREM ON MEIJER-LAPLACE TRANSFORM OF TWO VARIABLES

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1. Introduction.

The author has introduced the generalisation of the Laplace transform of two variables [3, p. 657]

$$(1.1)\qquad F(p,q)=pq\int_0^\infty\int_0^\infty e^{-px-qy}f(x,y)\,dxdy,\quad R(p,q)>0.$$

in the form [4]

$$(1.2)\quad F(p,q)=pq\int_0^\infty\int_0^\infty G^{m+1,0}_{m,m+1}\left(px\left|\begin{matrix}a_1+b_1,\cdots,a_m+b_m\\ a_1,\cdots\cdots,a_{m+1}\end{matrix}\right.\right)G^{n+1,0}_{n,n+1}\left(qy\left|\begin{matrix}c_1+d_1,\cdots,c_n+d_n\\ c_1,\cdots\cdots,c_{n+1}\end{matrix}\right.\right)$$

$$\times f(x,y)\,dxdy,\quad R(p,q)>0$$

and denote it symbolically by

$$F(p,q)=G\left[f(x,y):\begin{matrix}b_m,a_{m+1}\\ d_n,c_{n+1}\end{matrix}\right]$$

We shall call (1.2) as Meijer-Laplace transform of two variables.

Taking $b_i=0,\ i=1,2,\cdots,\ m-1;\ d_j=0,\ j=1,2,\cdots,\ n-1,$

(i) and $b_m=a_{m+1}=d_n=c_{n+1}=0$, (1.2) reduces to (1.1);

(ii) and $b_m=-m_1-k_1,\ a_m=m_1-k_1,\ a_{m+1}=-m_1-k_1,\ d_n=-m_2-k_2,\ c_n=m_2-k_2,$ $c_{n+1}=-m_2-k_2,$

(1.2) reduces to Meijer transform of two variables [6, p. 83]

$$(1.3)\qquad F(p,q)=pq\int_0^\infty\int_0^\infty (px)^{-k_1-\frac{1}{2}}(qy)^{-k_2-\frac{1}{2}}e^{-\frac{1}{2}px-\frac{1}{2}qy}W_{k_1+\frac{1}{2},m_1}$$

$$(px)W_{k_2+\frac{1}{2},m_2}(qy)f(x,y)\,dxdy,\quad R(p,q)>0;$$

(iii) and $b_m=\frac{1}{2}-m_1-k_1,\ a_m=2m_1,\ a_{m+1}=0;\ d_n=\frac{1}{2}-m_2-k_2,\ c_n=2m_2,\ c_{n+1}=0,$

we get (1.2) reduced to [8, p. 49]

$$(1.4)\qquad F(p,q)=pq\int_0^\infty\int_0^\infty (px)^{m_1-\frac{1}{2}}(qy)^{m_2-\frac{1}{2}}e^{-\frac{1}{2}px-\frac{1}{2}qy}W_{k_1,m_1}(px)W_{k_2,m_2}$$

$$\times(qy)f(x,y)\,dxdy,\quad R(p,q)>0$$

and shall call it as Varma's transform of two variables.

We denote (1.1), (1.3) and (1.4) symbolically as $F(p,q) \doteqdot f(x,y)$, $f(x,y) \xrightarrow[m_1,\ m_2]{k_1+\frac{1}{2},\ k_2+\frac{1}{2}} F(p,q)$ and $f(x,y) \xrightarrow[m_1,\ m_2]{k_1,\ k_2} F(p,q)$ respectively.

In this paper we have obtained a theorem for Meijer-Laplace transform of two variables, on parallel lines to that of Tricomi's theorem [9, p. 564] for Laplace transform of one variable. Several interesting particular cases have been obtained and the theorem is illustrated by an example.

In what follows, we have used the symbols (a_m), $\Delta(n,a)$, $\Delta((n,a_r))$ and $\Delta(n,\pm a)$ to denote respectively the set of parameters $a_1, a_2, \cdots\cdots, a_m$: $\frac{a}{n}, \frac{a+1}{n}, \cdots\cdots, \frac{a+n-1}{n}$; $\Delta(n,a_1), \cdots\cdots, \Delta(n,a_2)$, $\Delta(n,a_r)$ and $\Delta(n,a)$, $\Delta(n,-a)$ throughout this paper.

2. Theorem.

If

$$F(p,q)=G\left[f(x,y):\begin{matrix} b_m, & a_{m+1} \\ d_n, & c_{n+1}\end{matrix}\right],$$

then

$$(2.1)\qquad p^{\frac{n_1}{s_1}-r_1} q^{\frac{n_2}{s_2}-r_2} F\left(p^{-\frac{n_1}{s_1}},\ q^{-\frac{n_2}{s_2}}\right)=G\left[s^{r_1}t^{r_2}\int_0^\infty\int_0^\infty H(s,t,x,y)\ f(x,y)\ dxdy:\begin{matrix} f_u, & e_{u+1} \\ h_v, & g_{v+1}\end{matrix}\right]$$

where

$$(2.2)\quad H(s,t,x,y)=\frac{(2\pi)^{\frac{1}{2}(n_1+n_2-s_1-s_2)} s_1^{a_{m+1}-\sum_{i=1}^{m}b_i+\frac{1}{2}} s_2^{c_{n+1}-\sum_{j=1}^{n}d_j+\frac{1}{2}}}{n_1^{e_{u+1}-\sum_{i=1}^{u}f_i+r_1+\frac{1}{2}}\ n_2^{g_{v+1}-\sum_{j=1}^{v}h_j+r_2+\frac{1}{2}}}$$

$$\times G^{s_1m+s_1,\ n_1u}_{n_1u+s_1m,\ s_1m+s_1+n_1u+n_1}\left(\frac{x^{s_1}s^{n_1}}{s_1^{s_1}n_1^{n_1}}\middle|\begin{matrix}\Delta((n_1,\ -e_u-f_u-r_1)),\ \Delta((s_1,\ a_m+b_m)) \\ \Delta((s_1,\ a_{m+1})),\ \Delta((n_1,\ -e_{u+1}-r_1))\end{matrix}\right)$$

$$\times G^{s_2n+s_2,\ n_2v}_{n_2v+s_2n,\ s_2n+s_2+n_2v+n_2}\left(\frac{y^{s_2}t^{n_2}}{s_2^{s_2}\ n_2^{n_2}}\middle|\begin{matrix}\Delta((n_2,\ -g_v-h_v-r_2)),\ \Delta((s_2,\ c_n+d_n)) \\ \Delta((s_2,\ c_{n+1})),\ \Delta((n_2,\ -g_{v+1}-r_2))\end{matrix}\right)$$

provided n_r, s_r *are positive integers* $s_r>n_r$, $r=1,\ 2$:

$|\arg(x^{s_1}/p^{n_1})|<(s_1-n_1)\frac{\pi}{2}$, $\min\{R(e_i),\ R(e_{u+1})\}+n_1\min\left\{R(\frac{a_j}{s_1})\right\}>-1-R(r_1)>$

$-R(e_i+f_i+r_1)-2$ for $i=1,2,\ \cdots\cdots,\ u,\ j=1,2,\ \cdots\cdots,\ m+1$:

$|\arg(y^{s_2}/q^{n_2})|<(s_2-n_2)\frac{\pi}{2}$, $\min\{R(g_k),\ R(g_{v+1})\}+n_2\min\left\{R\left(\frac{c_l}{s_2}\right)\right\}>-1-R(r_2)>$

$-R(g_k+h_k+r_2)-2$ for $k=1,2,\ \cdots\cdots,\ v,\ l=1,2,\ \cdots\cdots,\ n+1$;

and the double integral in (2.1) *is absolutely convergenl and Meijer-Laplace transform of*

$$\left|s^{r_1}t^{r_2}\int_0^\infty\int_0^\infty H(s,t,x,y)\ dxdy\right| \text{ exists.}$$

PROOF. In (1.2), replacing p and q by $p^{-\frac{n_1}{s_1}}$ and $q^{-\frac{n_2}{s_2}}$ and multiplying by $p^{\frac{n_1}{s_1}-r_1}\ q^{\frac{n_2}{s_2}-r_2}$, we get

$$(2.3)\quad p^{\frac{n_1}{s_1}-r_1}q^{\frac{n_2}{s_2}-r_2}\ F\left(p^{-\frac{n_1}{s_1}},\ q^{-\frac{n_2}{s_2}}\right)=\int_0^\infty\int_0^\infty p^{-r_1}q^{-r_2}G^{m+1,\ 0}_{m,\ m+1}\left(p^{-\frac{n_1}{s_1}}x\left|\begin{matrix}(a_m+b_m)\\(a_{m+1})\end{matrix}\right.\right)$$

$$\times G^{n+1,\ 0}_{n,\ n+1}\left(q^{-\frac{n_2}{s_2}}y\left|\begin{matrix}(c_n+d_n)\\(c_{n+1})\end{matrix}\right.\right)f(x,y)\ dxdy$$

Further, it can be proved easily, using [1, p.3, (2.3)], that

$$(2.4)\quad p^{-r_1}q^{-r_2}\,G^{m+1,\ 0}_{m,\ m+1}\left(xp^{-\frac{n_1}{s_1}}\left|\begin{matrix}(a_m+b_m)\\(a_{m+1})\end{matrix}\right.\right)G^{n+1,\ 0}_{n,\ n+1}\left(yq^{-\frac{n_2}{s_2}}\left|\begin{matrix}(c_n+d_n)\\(c_{n+1})\end{matrix}\right.\right)$$

$$=pq\int_0^\infty\int_0^\infty G^{u+1,\ 0}_{u,\ u+1}\left(ps\left|\begin{matrix}(e_u+f_u)\\(e_{u+1})\end{matrix}\right.\right)G^{v+1,\ 0}_{v,\ v+1}\left(qt\left|\begin{matrix}(g_v+h_v)\\(g_{v+1})\end{matrix}\right.\right)s^{r_1}t^{r_2}\,H(s,t,x,y)\ dsdt$$

On using (2.4) in the right hand side of (2.3), and interchanging the order of integration, the result (2.1) follows.

To justify the change of order of integration we observe that

(i) s,t-integral in (2.4) is absolutely convergent [7, p. 84] as n_1, n_2, s_1 and s_2 are positive integers,

$s_1>n_1$, $|\arg(x^{s_1}/p^{n_1})|<(s_1-n_1)\pi/2$,

$\min\ \{R(e_i),\ R(e_{u+1})\}+n_1\min\ \{R(a_j/s_1)\}>-1-R(r_1)>-R(e_i+f_i+r_1)-2$

for $i=1,2,\ \cdots\cdots,\ u,\ j=1,2,\ \cdots\cdots m+1$;

$s_2>n_2$, $|\arg(y^{s_2}/q^{n_2})|<(s_2-n_2)\pi/2$,

$\min\{R(g_k),\ R(g_{v+1})\}+n_2,\ \min\{R(c_l/s_2)\}>-1-R(r_2)>-R(g_k+h_k+r_2)-2$

for $k=1,2,\ \cdots\cdots,\ v,\ l=1,2,\cdots\cdots,\ n+1$.

(ii) x,y-integral in (2.1) is absolutely convergent.

(iii) the repeated integral is absolutely convergent as Meijer-Laplace transform of

$$\left| s^{r_1} t^{r_2} \int_0^\infty \int_0^\infty H(s,t,x,y)\, dxdy \right| \text{ exists.}$$

3. Particular cases.

In what follows, we have assumed n_1, n_2, s_1 and s_2 as positive integers with

$s_1 > n_1$, $s_2 > n_2$:

$\left|\arg(x^{s_1}/p^{n_1})\right| < \frac{1}{2}(s_1 - n_1)\pi$, $\left|\arg(y^{s_2}/q^{n_2})\right| < \frac{1}{2}(s_2 - n_2)\pi$,

and integrals involved are absolutely convergent.

(i) Setting $b_j = 0$, $j = 1, 2, \cdots\cdots, m-1$: $b_m = a_{m+1} = -m_1 - k_1$, $a_m = m_1 - k_1$: $d_i = 0$, $i = 1, 2, \cdots\cdots, n-1$: $d_n = c_{n+1} = -m_2 - k_2$, $c_n = m_2 - k_2$, we obtain:

If

$$(3.1)\quad f(x,y) \xrightarrow[m_1,\ m_2]{k_1+\frac{1}{2},\ k_2+\frac{1}{2}} F(p,q),$$

then

$$(3.2)\quad p^{\frac{n_1}{s_1} - r_1} q^{\frac{n_2}{s_2} - r_2} F\left(p^{-\frac{n_1}{s_1}},\ q^{-\frac{n_2}{s_2}}\right) = \frac{(2\pi)^{(n_1+n_2-s_1-s_2)/2}(s_1 s_2)^{1/2}}{n_1^{e_{u+1} - \sum_{i=1}^{u} f_i + r_1 + \frac{1}{2}}\, n_2^{g_{v+1} - \sum_{j=1}^{v} h_j + r_2 + \frac{1}{2}}}$$

$$\times G\left[s^{r_1} t^{r_2} \int_0^\infty \int_0^\infty H(s,t,x,y)\ (fx,y)\ dxdy : \begin{matrix} f_u, & e_{u+1} \\ h_v, & g_{v+1} \end{matrix} \right],$$

where

$$(3.3)\quad H(s,t,x,y) = G^{2s_1,\ n_1 u}_{n_1 u + s_1,\ 2s_1 + n_1 u + n_1}\left(\frac{x^{s_1} s^{n_1}}{s_1^{s_1} n_1^{n_1}} \middle| \begin{matrix} \Delta(n_1,\ -e_u - f_u - r_1),\ \Delta(s_1,\ -2k_1) \\ \Delta(s_1, \pm m_1 - k_1),\ \Delta(n_1,\ -e_{u+1} - r_1) \end{matrix} \right)$$

$$\times G^{2s_2,\ n_2 v}_{n_2 v + s_2,\ 2s_2 + n_2 v + n_2}\left(\frac{y^{s_2} t^{n_2}}{s_2^{s_2} n_2^{n_2}} \middle| \begin{matrix} \Delta(n_2,\ -g_v - h_v - r_2),\ \Delta(s_2,\ -2k_2) \\ \Delta(s_2,\ \pm m_2 - k_2),\ \Delta(n_2,\ -g_{v+1} - r_2) \end{matrix} \right)$$

provided

$\min\{R(e_i), R(e_{u+1})\} + n_1 \min\{R(\pm m_1 - k_1)/s_1\} > -1 - R(r_1) > -R(e_i + f_i + r_i) - 2$, $i = 1, 2, \cdots\cdots, u$;

$\min\{R(g_j), R(g_{v+1})\} + n_2 \min\{R(\pm m_2 - k_2)/s_2\} > -1 - R(r_2) > -R(g_j + h_j + r_2) - 2$, $j = 1, 2, \cdots\cdots, v$.

(ia) Further with $k_1 = \pm m_1$, $k_2 = \pm m_2$ in (3.1) to (3.3) we get:

If $F(p,q) \doteqdot f(x,y)$, then

$$(3.4)\quad p^{\frac{n_1}{s_1}-r_1} q^{\frac{n_2}{s_2}-r_2} F\left(p^{-\frac{n_1}{s_1}},\ q^{-\frac{n_2}{s_2}}\right) = \frac{(2\pi)^{(n_1+n_2-s_1-s_2)/2}\sqrt{(s_1 s_2)}}{n_1^{e_{u+1}-\sum_{i=1}^{u} f_i + r_1 + \frac{1}{2}}\ n_2^{g_{v+1}-\sum_{j=1}^{v} h_j + r_2 + \frac{1}{2}}}$$

$$\times G\left[s^{r_1} t^{r_2} \int_0^\infty\int_0^\infty H(s,t,x,y)\ f(x,y)\,dxdy : \begin{matrix} f_u, & e_{u+1} \\ h_v, & g_{v+1} \end{matrix}\right]$$

where

$$(3.5)\quad H(s,t,x,y) = G^{s_1,\ n_1 u}_{n_1 u+s_1,\ s_1+n_1 u+n_1}\left(\frac{x^{s_1} s^{n_1}}{s_1^{s_1} n_1^{n_1}}\middle|\begin{matrix}\Delta(n_1,\ -e_u-f_u-r_1) \\ \Delta(s_1,0),\quad \Delta(n_1,\ -e_{u+1}-r_1)\end{matrix}\right)$$

$$\times G^{s_2,\ n_2 v}_{n_2 v+s_2,\ s_2+n_2 v+n_2}\left(\frac{y^{s_2} t^{n_2}}{s_2^{s_2} n_2^{n_2}}\middle|\begin{matrix}\Delta(n_2,\ -g_v-h_v-r_2) \\ \Delta(s_2,0),\quad \Delta(n_2,\ -g_{v+1}-r_2)\end{matrix}\right)$$

provided

$\min\{R(e_i),\ R(e_{u+1})\}+R(r_1)+1>0,\ R(e_i+f_i)>-1,\ i=1,2,\ \cdots\cdots,\ u;$

$\min\{R(g_j),\ R(g_{v+1})\}+R(r_2)+1>0,\ R(g_j+h_j)>-1,\ j=1,2,\ \cdots\cdots,\ v.$

(ib) Putting $k_1=\pm m_1,\ k_2=\pm m_2$ and $f_i=0,\ i=1,2,\ \cdots\cdots,\ u;\ e_{u+1}=0,\ h_j=0,$ $j=1,2,\cdots\cdots,v,\ g_{v+1}=0$ in (3.1) to (3.3) we have:

If $\qquad F(p,q) \doteqdot f(x,y),$

then

$$(3.6)\quad p^{\frac{n_1}{s_1}-r_1} q^{\frac{n_2}{s_2}-r_2} F\left(p^{-\frac{n_1}{s_1}},\ q^{-\frac{n_2}{s_2}}\right) \doteqdot \frac{(2\pi)^{(n_1+n_2-s_1-s_2)/2}\sqrt{(s_1 s_2)}\, s^{r_1} t^{r_2}}{n_1^{r_1+\frac{1}{2}}\ n_2^{r_2+\frac{1}{2}}}$$

$$\times \int_0^\infty\int_0^\infty H(s,t,x,y)\ f(x,y)\ dxdy,$$

where

$$(3.7)\quad H(s,t,x,y) = G^{s_1,\ 0}_{0,\ s_1+n_1}\left(\frac{x^{s_1} s^{n_1}}{s_1^{s_1} n_1^{n_1}}\middle|\begin{matrix}- \\ \Delta(s_1,0),\quad \Delta(n_1,r_1)\end{matrix}\right)$$

$$\times G^{s_2,0}_{0,\ s_2+n_2}\left(\frac{y^{s_2} t^{n_2}}{s_2^{s_2} n_2^{n_2}}\middle|\begin{matrix}- \\ \Delta(s_2,0),\quad \Delta(n_2,-r_2)\end{matrix}\right)$$

for $R(r_1,r_2)>-1$.

(ic) Setting $n_1=s_1=n_2=s_2=1,\ r_1=n_1,\ r_2=n_2$ in (3.6) and (3.7) we get a known result obtained earlier by the author [5].

(id) Further with $r_1=r_2=0$, we get a known result [2, p. 58, (2.58)]

(ii) Putting $b_i=0,\ i=1,2,\ \cdots\cdots,\ m-1,\ b_m=\frac{1}{2}-m_1-k_1,\ a_m=2m_1,\ a_{m+1}=0;$

$d_j=0,\ j=1,2,\ \cdots\cdots,\ n-1,\ d_n=\frac{1}{2}-m_2-k_2,\ c_n=2m_2,\ c_{n+1}=0,$ we get

If

$$f(x,y)\xrightarrow[m_1,\ m_2]{k_1,\ k_2}F(p,q),$$

then

$$(3.8)\quad p^{\frac{n_1}{s_1}-r_1}q^{\frac{n_2}{s_2}-r_2}F\left(p^{-\frac{n_1}{s_1}},\ q^{-\frac{n_2}{s_2}}\right)=\frac{(2\pi)^{(n_1+n_2-s_1-s_2)/2}s_1^{m_1+k_1}s_2^{m_2+k_2}}{n_1^{e_{u+1}-\sum_{i=1}^{u}f_i+r_1+\frac{1}{2}}n_2^{g_{v+1}-\sum_{j=1}^{v}h_j+r_2+\frac{1}{2}}}$$

$$\times G\left[s^{r_1}t^{r_2}\int_0^\infty\int_0^\infty H(s,t,x,y)\ f(x,y)dxdy;\ \begin{matrix}f_u,\ e_{u+1}\\ h_v,\ g_{v+1}\end{matrix}\right]$$

where

$$(3.9)\ H(s,t,x,y)=G^{2s_1,\ n_1u}_{n_1u+s_1,\ 2s_1+n_1u+n_1}\left(\frac{x^{s_1}s^{n_1}}{s_1^{s_1}n_1^{n_1}}\middle|\begin{matrix}\Delta((n_1,-e_u-f_u-r_1)),\quad \Delta(s_1,\frac{1}{2}+m_1-k_1)\\ \Delta(s_1,2m_1),\quad \Delta(s_1,0),\quad \Delta((n_1,-e_{u+1}-r_1))\end{matrix}\right)$$

$$\times G^{2s_2,\ n_2v}_{n_2v+s_2,\ 2s_2+n_2v+n_2}\left(\frac{y^{s_2}t^{n_2}}{s_2^{s_2}n_2^{n_2}}\middle|\begin{matrix}\Delta((n_2,-g_v-h_v-r_2)),\quad \Delta(s_2,\quad\frac{1}{2}+m_2-k_2)\\ \Delta(s_2,2m_2),\quad \Delta(s_2,0),\quad \Delta((n_2,-g_{v+1}-r_2))\end{matrix}\right)$$

$\min\{R(e_i),R(e_{u+1})\}+\frac{n_1}{s_1}\min\{R(2m_1)\}>-1-R(r_1)>-R(e_i+f_i+r_1)-2,\ i=1,2,\cdots,u;$

$\min\{R(g_k),R(g_{v+1})\}+\frac{n_2}{s_2}\min\{R(2m_2)\}>-1-R(r_2)>-R(g_k+h_k+r_2)-2,$

$k=1,2,\ \cdots\cdots,\ v.$

(iia) Further with $f_i=0,\ i=1,2,\ \cdots\cdots,\ u-1,\ f_u=\frac{1}{2}-u_1-v_1,\ e_u=2u_1,\ e_{u+1}=0$:

$h_j=0,\ j=1,2,\ \cdots\cdots,\ v-1,\ h_v=\frac{1}{2}-u_2-v_2,\ g_v=2u_2,\ g_{v+1}=0,$ we obtain:

If $\quad F(p,q)\xleftarrow[m_1,\ m_2]{k_1,\ k_2}f(x,y),$

then

$$(3.10)\quad p^{\frac{n_1}{s_1}-r_1}q^{\frac{n_2}{s_2}-r_2}F\left(p^{-\frac{n_1}{s_1}},\ q^{-\frac{n_2}{s_2}}\right)\xleftarrow[v_1,\ v_2]{u_1,\ u_2}\ s^{r_1}t^{r_2}\int_0^\infty\int_0^\infty H(s,t,x,y)\ f(x,y)\ dxdy$$

where

$$(3.11)\quad H(s,t,x,y)=\frac{(2\pi)^{(n_1+n_2-s_1-s_2)/2}s_1^{m_1+k_1}s_2^{m_2+k_2}}{n_1^{u_1+v_1}n_2^{u_2+v_2}}$$

$$\times G^{2s_1,\ n_1}_{n_1+s_1,\ 2s_1+2n_1}\left(\frac{x^{s_1}s^{n_1}}{s_1^{s_1}n_1^{n_1}}\middle|\begin{matrix}\Delta\left(n_1,-\frac{1}{2}-u_1-v_1\right),\quad \Delta\left(s_1,\ \frac{1}{2}+m_1-k_1\right)\\ \Delta(s_1,2m_1),\quad \Delta(s_1,0),\quad \Delta(n_1,-2u_1-r_1),\quad \Delta(n_1,r_1)\end{matrix}\right)$$

$$\times G^{2s_2,\ n_2}_{n_2+s_2,\ 2s_2+2n_2}\left(\frac{y^{s_2}t^{n_2}}{s_2^{s_2}n_2^{n_2}}\middle|\begin{matrix}\Delta\left(n_2,\ -\frac{1}{2}-u_2-v_2\right),\ \Delta\left(s_2,\ \frac{1}{2}+m_2-k_2\right)\\ \Delta(s_2,2m_2),\ \ \Delta(s_2,0),\ \ \Delta(n_2,-2u_2-r_2),\ \ \Delta(n_2,-r_2)\end{matrix}\right)$$

and

$$R(2u_1)+\frac{n_1}{s_1}R(2m_1)>-1-R(r_1)>-R(u_1-v_1+r_1)-\frac{5}{2},$$

$$R(2u_2)+\frac{n_2}{s_2}R(2m_2)>-1-R(r_2)>-R(u_2-v_2+r_2)-\frac{5}{2}.$$

4. Application.

Let

$$f(x,y)=I_\nu(2(xy)^{\frac{1}{2}}),$$

where $I_\nu(z)$ is Bessel's function for imaginary argument, then from (1.2)

$$(4.1)\quad F(p,q)=\frac{1}{(pq)^{\nu/2}}\cdot\frac{\prod_{j=1}^{m+1}\Gamma\left(a_j+\frac{\nu}{2}+1\right)\prod_{j=1}^{n+1}\Gamma\left(c_j+\frac{\nu}{2}+1\right)}{\Gamma(\nu+1)\prod_{j=1}^{m}\Gamma(a_j+b_j+\nu/2+1)\prod_{j=1}^{n}\Gamma(c_j+d_j+\nu/2+1)}$$

$$\times{}_{m+n+2}F_{m+n+1}\left[\begin{matrix}(a_{m+1}+\nu/2+1),\quad (c_{n+1}+\nu/2+1)\\ \nu+1,(a_m+b_m+\nu/2+1),\quad (c_n+d_n+\nu/2+1)\end{matrix};\frac{1}{pq}\right]$$

provided $R(a_j+v/2+1)>0,\ j=1,2,\ \cdots\cdots,\ m+1: R(c_i+\nu/2+1)>0,$
$i=1,2,\ \cdots\cdots,\ n+1: R(p,q)>0,\ |pq|>1,\ |\arg p|<\pi/2$ and $|\arg q|<\frac{\pi}{2}$.

Hence from (2.1), we get

$$(4.2)\quad p^{\frac{n_1}{s_1}-\frac{\nu}{2}-r_1}q^{\frac{n_2}{s_2}-\frac{\nu}{2}-r_2}\cdot\frac{\prod_{j=1}^{m+1}\Gamma(a_j+\nu/2+1)\prod_{j=1}^{n+1}\Gamma(c_j+\nu/2+1)}{\Gamma(\nu+1)\prod_{j=1}^{m}\Gamma(a_j+b_j+\nu/2+1)\prod_{j=1}^{n}\Gamma(c_j+d_j+\nu/2+1)}$$

$$\times{}_{m+n+2}F_{m+n+1}\left[\begin{matrix}(a_{m+1}+\nu/2+1),\ \left(c_{n+1}+\frac{\nu}{2}+1\right)\\ \nu+1,\ \left(a_m+b_m+\frac{\nu}{2}+1\right),\ (c_n+d_n+\nu/2+1)\end{matrix};p^{\frac{n_1}{s_1}}q^{\frac{n_2}{s_2}}\right]$$

$$=G\left[s^{r_1}t^{r_2}\int_0^\infty\int_0^\infty H(s,t,x,y)\ I_\nu(2(xy)^{1/2})dxdy:\begin{matrix}f_u,\ e_{u+1}\\ h_v,\ g_{v+1}\end{matrix}\right]$$

where $H(s,t,x,y)$ is given by (2.2), provided condition given in (2.2) and (4.1) are satisfied.

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REFERENCES

[1] Bhise V.M., *Certain properties of Meijer Laplace transform*, Compositio Mathematica, Vol. 18 Fase 1, 2(1967) pp. 1—6

[2] Ditkin V.A. and Prudnikov A.P., *Operational Calculus of two variables and its applications*, Pergamon Press 1962.

[3] Humbert P., *Le Calcul symbolique a deux variables*, Comptes Rendus Acad. Sci. 199 (1934) pp. 657—660

[4] Jain N.C., *Some inversion formulae for Meijer-Laplace transform of two variables*, communicated for publication.

[5] Jain N.C., *A theorem on Laplace transform of two variables*, communicated for publication.

[6] Mehra A.N., *On Meijer transform of two variables*, Bull. Cal. Math. Soc. Vol. 48, No. 2 (1956) pp. 83—94.

[7] Meijer C.S., *Neue Integraldarstellungen fur whiltakersche Funktionen*, Proc. Ned. Acad. V. Wetenseh, 44 (1941) pp. 82—92.

[8] Mukherjee S.N., *Some inversion formulae for the generalised Laplace transform of two variables*,, Vijnana Anusandhan Patrika, 5(1962), pp. 49—55.

[9] Tricomi F., *Sulla transformazione il teorema reciprocita di Hankel*, Rendi. Atti. Acc. Lincei (6), 22(1935) pp. 564—571.