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A NOTE ON A MINIMAL HAUSDORFF SPACE

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It is well known that the property of being minimal Hausdorff is not closed-hereditary. In [1], if a subspace of a minimal Hausdorff space is closed and open, then it is minimal Hausdorff.

The purpose of this note is to investigate a hereditary subset of a minimal Hausdorff space.

R. F. Dickman, Jr. and A. Zame introduced a functionally compact space and an r-closed subset and they have shown that the r-closed is a hereditary property in a functionally compact space.

All of the definitions used but not given in this paper may be found in [2] and [3].

DEFINITION 1. A topological space $(X, \mathscr{T})$ is said to be *minimal Hausdorff* if $\mathscr{T}$ is Hausdorff and there exists no Hausdorff topology strictly weaker than $\mathscr{T}$.

DEFINITION 2. A closed subset C of a space X is said to be *r-closed* if whenever B is closed in C, $x \notin B$ there exist disjoint open sets in X containing x and B respectively.

THEOREM. *An r-closed subset C of a minimal Hausdorff space X is minimal Hausdorff.*

Proof. A Hausdorff space X is minimal if and only if for every point $x \in X$ and every open filter-base $\mathscr{U}$ on X such that $\{x\}$ is the intersection of the closures of the elements of $\mathscr{U}$, then $\mathscr{U}$ is a base for the neighborhoods of x. [3].

Let $\mathscr{U}$ be an open filter-base on C such that

$$\bigcap \{\mathrm{cl}_C U : U \in \mathscr{U}\} = \{x\},$$

and $\mathscr{V}$ be the open filter-base on X consisting of all open sets V of X such that $V \cap C \in \mathscr{U}$.

Clearly

$$\bigcap \{cl_X V : V \in \mathcal{V}\} \ni x.$$

For any element y distinct from x, there exists $U \in \mathcal{U}$ such that $y \notin cl_C U$. Since the set C is an r-closed set, there exist disjoint open sets O_1, O_2 in X containing y and $cl_C U$ respectively.

Let $V \cap C = U$ and $V_1 = O_2 \cap V$, then $V_1 \in \mathcal{V}$ since $V_1 \cap C = U$. On the other hand $C_1 \cap V_1 = \phi$. Therefore

$$y \notin \bigcap \{cl_X V : V \in \mathcal{V}\}.$$

This shows that

$$\{x\} = \bigcap \{cl_X V : V \in \mathcal{V}\}.$$

Hence $\mathcal{V}$ is a base for the neighborhoods of $\{x\}$. Of course this implies that $\mathcal{U}$ is a base for the neighborhoods $\{x\}$ relative to C. This completes the proof.

We now investigate some propositions derived from the above theorem. R.F. Dickman, Jr. and A. Zame have shown that a functionally compact space is minimal Hausdorff, but the converse is not true.

A space X is *Urysohn* if any two distinct points in X can be seperated by closed neighborhoods.

COROLLARY. *An r-closed subset of a functionally compact space is functionally compact.*

Proof. Every Urysohn minimal Hausdorff space is compact [2]. An r-closed subset is a Urysohn subspace. Therefore by the above theorem an r-closed subset of a functionally compact space is compact. The fact that compactness implies functionally compactness is found in [3], [4].

References

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