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# ≪Original≫ Calculated Critical Masses for Spherical Plutonium Core with Various Spherical Reflectors

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**Abstract**

Requirements necessary for the construction of a fast critical assembly in this country are briefly reviewed.

The critial masses of a spherical plutonium core with various spherical reflectors and its thicknesses are calculated using lDX, which is a computer code written for fast reactor analysis. The compilation of this code was made on CDC 3300/Master and UNIVAC 1106/EXEC 8. The calculated results are completely depicted, and compared with other results in this report. The Russian formatted 29 group cross section set generated by ETOX-2 using ENDF/B Version II file at Hanford is used in the calculations.

**요 약**

국내에서 고속임계장치를 건조하기 위한 장치의 필요 요건에 대하여 간단히 고찰하였다. 구형 프루토늄의 임계질량을 여러가지 반사물질과 그 두께에 따라 계산되었다. 계산에 사용된 코드는 주로 고속원자로의 분석에 이용되도록 프로그램된 lDX를 사용하였으며 이는 CDC 3300/MASTER와 UNIVAC 1106/EXEC 8에 Compile되었다.

이 보고서에 계산된 결과는 모두 도표로 완성하고 고찰되었다. 사용된 핵자료는 Hanford 연구소에서 ENDF/B VERSION II의 자료를 이용 ETOX-2에 의하여 계산된 Russian Format의 29그룹 단면적 자료이다.

## 1. Introduction

Rapid growth of nuclear power generation in the world certainly forces the development of new types of power reactor such as advanced thermal reactor and fast breeder reactor for more efficient use of uranium resources as compared with conventional light watet nuclear power plant. A simple calculation shows that a reactor should have a breeding ratio larger than unity for the porpose of sustaining fuel cycle. The fact breeder reactor is most attractive with $^{239}Pu$-$^{238}U$ cycle, and tremendous efforts have been given to the development of the Liquid Metal-cooled Fast Breeder Reactor by a number of leading institutions in the world. Needless to say, the interest in the fast reactor concept began as early as 1950's, and various experiments have been conducted with fast critical assemblies and experimental reactors.

The goal of the development program on the fast breeder technology is being pursued to build the first commercial power plant by the early part of 1980's. It would be quite natural for us to take sincere considerations on the extensive study of fast breeder system at this stage. However, the development of the fast breeder reactor involves quite a regorous task, especially in the view of our difficult situation in many respects. For the purpose of establishing the analytical methods and design techniques for fast reactor system, we have been carefully expedited the feasibility to construct a fact critical assembly at the Atomic Energy Research Institute. Due to difficulties in financial and technological capabilities, we considered that the critical assembly should satisfy following conditions:

1. Low power,
2. Air cooling, or no cooling,
3. Simple control mechanism,
4. Minimum fuel requirement, and
5. Minimum requirement of hardware technology.

The fast pulsed reactor(or fast burst reactor) such as GODIVA of the U. S. A., IBR of the U. S. S. R., is a very useful device that can have a wide range of applications in nuclear investigations[1]. The idea of fast pulsed reactor came from the DRAGON experiment[2] in 1945 and a number of such reactors have been built since then[3-8]. While the GODIVA type reactor produces individual pulses at intervals of an hour or so, IBR[3,9] of the U. S. S. R. produces repetitive pulses, and Euratom has been working on the development of SORA[10], also a repetitively pulsed system.

The fast pulsed reactor considered here is a type similar to the IBR, except the pulsing method. The pulse will be produced by the movement of reflector as in the SORA instead of insertion of uranium in the core. The detailed core configuration is not given here at this preliminary stage of the design. The purpose of this report is to give all the computation results of critical masses of the simple spherical core with Pu fuel and variations of thickness and material of the reflector. The theoretical masses for pure fuel and air and stainless steel dilute systems are given. The computations were carried out with two forms of fuel, $PuO_2$ and Pu-metal.

The results of the calculated critical masses for Pu-metal with iron and beryllium reflectors are compared with the experimental results and other computational results from the transport calculation, which are obtained by other investigators.

## 2. Computation Model

In connection with the considerations for setting up a fast critical assembly in this institute, the critical masses of a small, high density, plutonium fuelled spherical reactor have been computed as a very first approach to the assembly design. In this computations two kind of fuels, plutonium oxide and plutonium metal(with 9 atom % of aluminum for stability in the $\delta$-phase), are used with five different materials and thicknesses of reflector. Table 1 contains the nuclides, densities, and nuclear densities that were used in the computations.

In order to consider the effects of cladding material and cooling channel void to the variation of critical masses, both diluted system with S. S. 304 and air were taken into consideration with various degrees of dilution.

**Table 1. Nuclide, density, and atom density for all materials.**

Reflector

| Nuclide | Density | Atom density ($\times 10^{-24}$) |
|---|---|---|
| Be | 1.84 | 0.123011 |
| Cu | 8.96 | 0.084930 |
| Ni | 8.90 | 0.091300 |
| Mo | 10.20 | 0.064030 |
| Fe | 7.87 | 0.084900 |

Clad(S. S. 304) Density…7.92

| Nuclide | Atom density($\times 10^{-24}$) |
|---|---|
| Fe | 0.0637015 |
| Ni | 0.0068959 |
| Cr | 0.0155159 |
| C | 0.0008610 |

Fuel

| Fuel form | Density | Nuclide | Atom density($\times 10^{-24}$) |
|---|---|---|---|
| Pu(pure) | 19.7 | Pu | 0.04966 |
| Pu-Al | 16.0 | Pu | 0.039928 |
| | | Al | 0.003472 |
| $PuO_2$ | 11.46 | Pu | 0.02547 |
| | | O | 0.05094 |

All the modes of computation are summarized as follows:

1. Plutonium metal with various reflector and its thickness,
2. Plutonium oxide with various reflector and its thickness,
3. Plutonium metal diluted by S. S. 304 with various reflectors, and
4. Plutonium metal diluted by air with various reflectors.

All the computations were performed using a computer code, 1DX, which was compiled on CDC 3300/Master and UNIVAC 1106/EXEC 8 from its original listing with minimum changes due to the system dependences.

1DX[11] is a multipurpose, one-dimensional (plane, sphere, cylinder) diffusion theory code for use in fast reactor analysis, and solves the multigroup diffusion equation which can generally be written in the form

$$D_i \nabla^2 \phi_i - \Sigma_r^i \phi_i + S_i = 0 \text{ for } i = 1, 2, \cdots, IGM \quad (1)$$

where

$$S_i = \frac{x_i}{k_{eff}} \sum_{j=1}^{IGM} (\nu\Sigma_f)^i \phi_i + \sum_{j=1}^{i-1} \Sigma(j \to i)\phi_j \quad (2)$$

and $IGM$ : number of energy groups,

$i$ : energy group index,

$\phi_i$ : neutron flux in group $i$,

$S_i$ : source in group $i$,

$D_i$ : diffusion constant for group $i$ $(1/3\Sigma_{tr}^i)$,

$(\nu\Sigma_f)^i$ : fission source cross section for group $i$,

$\Sigma(j \to i)$ : downscattering cross section from group $j$ to $i$

$\Sigma_r^i$ : removal cross section for group $i$,

$$\left[ = \Sigma_a^i + \sum_{j=i+1}^{IGM} \Sigma(i \to j) \right]$$

$x_i$ : fission source fraction in group $i$,

$k_{eff}$ : effective multiplication constant.

The spatial difference equations in 1DX are set up such that the mesh point is placed in the center of the homogeneous mesh interval. Equation (1) and (2) are then integrated over the volume associated with each mesh point. For mesh point k the following expression is given after the integration.

$$-\alpha_k \phi_{k-1} + \beta_k \phi_k - \alpha_{k+1} \phi_{k+1} = S_k V_k, \ k = 1, 2, \cdots, IM \quad (3)$$

where

$$\alpha_k = (\overline{D}_{k,k-1} A_{k,k-1}) / (r_k - r_{k-1})$$

$$\beta_k = \alpha_k + \alpha_{k+1} + \Sigma_r^k V_k$$

$A_{k,k-1}$ : area of boundary between mesh point $k$ and mesh point $k-1$

$\overline{D}_{k,k-1}$ : effective diffusion constant between mesh point $k$ and mesh point $k-1$

$$\left[ \overline{D}_{k,k-1} = \frac{D_k D_{k-1} (\delta r_k + \delta r_{k-1})}{D_k \delta r_{k-1} + D_{k-1} \delta r_k} \right]$$

$IM$ : number of mesh intervals

$\phi_k$ : flux associated with mesh point $k$

$r_k$ : radial position of mesh point $k$

$V_k$ : volume associated with mesh point $k$

$\Sigma_r^k$ : removal cross section associated with mesh point $k$

In above difference equation the group indices are omitted. With one of three boundary conditions, reflective, vacuum, and periodic, 1DX computes the eigenvalue and spatial flux profiles using standard source-iteration techniques. In addition to the above computations, it performs criticality searches on time absorption, reactor composition, reactor dimensions, and buckling, and also computes and punches resonance shielded cross sections using data in the Russian format and collapsed group cross sections with given energy boundaries and indicated weighting fluxes. However, only $k_{eff}$ calculation option was used with the cross section data in the Russian format[12].

The cross section set used in the computations is the 29 group constant, which was generated by ETOX-2 based on the latest revisions of ENDF/B Version II for fast reactor analysis at Hanford[12]. The Russian formatted cross section data consist of infinite dilute cross sections, inelastic scattering matrices, and self-shielding factors.

*Infinite Dilution Cross Section*

If an element is present in a medium in low concentrations, the element's particular resonance structure does not affect the neutron spectrum. In this case, the neutron flux, $\phi_0$, used as a weighting function in the average process is given by

$$\phi_0 = 1/E \qquad \text{for } 0 \leq E \leq 2.23\text{Mev}$$
$$= \sqrt{E}e^{(-E/1.41)} \qquad \text{for } E > 2.23\text{Mev}.$$

Using the weighting function above the infinitely dilute cross section for group, $i$, for isotope, $j$, and for reaction type, $x$, is then given by:

$$<\sigma_{x_j}>^i = \int^i \sigma_{x_j}(E, T)\phi_0(E)dE \Big/ \int^i \phi_0(E)dE \quad (4)$$

The infinitely dilute cross section consists of total, capture, elastic, fission cross section, neutron per fission ($\nu$), scattering cosine ($\mu$), and the average logarithmic energy decrement($\xi$).

*Inelastic Scattering Constants*

The second data block in Russian format contains the inelastic scattering matrix, $\sigma(i \to j)$. This matrix contains NXCM+1 terms(where NXCM is the number of downscattering terms) for each energy group: one for in-group scattering and NXCM for scattering to the first NXCM lower groups. Since most multigroup codes do not handle the $(n, 2n)$ reaction explicitly, it is combined with the $(n, n)$ reaction to form the "inelastic constants" in the following manner:

$$<\sigma_{in_j}>^i = <\sigma_n, n'_j>^i + <\sigma_n, 2n_j>^i \quad (5)$$
$$<\sigma_{in_j}(i \to j)>^i = <\sigma_n, n'_j(i \to j)>^i + 2 \times <\sigma_n, 2n_j(i \to j)>^i \quad (6)$$

where $<\sigma_{x_j}(i \to j)>^i$ is the infinitely dilute $x$ transfer cross section from group $i$ to $j$.

*Self-shielding Constants*

When an element's concentration is not negligible, its resonance structure does affect the neutron spectrum. For the purpose of cross section averaging, the flux is assumed to have following form:

$$\phi(E) = \phi_0(E)/\Sigma_t(E), \quad (7)$$

where $\Sigma_t(E)$ is the total macroscopic cross section of the medium. This definition allows for flux depression at a resonance. The element's effective group cross sections are then given by:

$$\bar{\sigma}^i_{x_j} = \int^i \sigma_{x_j}(E, T)\ \phi(E)dE \Big/ \int^i \phi(E)dE \quad (8)$$

But this equation implies a very time consuming method of calculating the effective group average cross section for every different temperature or composition. The introduction of correction factors that will adjust the infinitely dilute cross sections has proven to be more convenient:

$$\bar{\sigma}^i_{x_j} = f^i_{x_j} <\sigma_{x_j}>^i \text{ for } x = f, c, e, t \quad (9)$$

In order to reduce composition dependence to a single variable, $\sigma_{0_j}(E, T)$ is introduced which accounts for all other isotopes. It is defined by:

$$\Sigma_t(E, T) = N_j\sigma_{t_j}(E, T) + N_j\sigma_{0_j}(E, T) \quad (10)$$

Using Equation (4), (7), (8), (9) and (10) the self-shielding factor for fission, capture and elastic cross section can be calculated by following equation:

$$f^i_{x_j}(\sigma^i_{0_j}, T) = \frac{\int^i \frac{\sigma_{x_j}(E,T)\phi_0(E)}{\sigma_{t_j}(E,T)+\sigma^i_{0_j}} dE}{\int^i \frac{{}_0(E)}{\sigma_{t_j}(E,T)+\sigma^i_{0_j}} dE} \cdot \frac{\int^i \phi_0(E)dE}{\int^i \sigma_{x_j}(E,T)\phi_0(E)dE} \qquad (11)$$

and for total cross section by:

$$f^i_{t_j}(\sigma^i_{0_j}, T) = \left\{\left[\int^i \frac{\phi_0(E)dE}{[\sigma_{t_j})E, T)+\sigma^i_{0_j}]} \Big/ \int \frac{{}^i\phi_0(E)dE}{[\sigma_{t_j}(E, T)+\sigma^i_{0_j}]^2}\right] - \sigma^i_{0_j}\right\}$$

$$\times \left\{\int^i \phi_0(E)dE \Big/ \int^i \sigma^i_{t_j}(E, T)\phi_0(E)dE\right. \qquad (12)$$

*Elastic Downscattering Cross Section*

When the energy loss in elastic scattering is small compared to the group width then elastic downscattering, $\sigma_d$, can be approximated in the following manner:

$$\sigma^i_d = \frac{\langle\xi\rangle^i \langle\sigma_s\rangle^i \phi_0(E^i)E^i}{\int^i \phi_0(E)dE}$$

where $\xi$ is the average logarithmic energy decrement and $E^i$ is the lower energy group $i$.

## 3. Results and Discussion

Critical masses for spherical plutonium core with various reflector have been computed using spherical geometry with suitable boundary condition of 1DX. All computation results are depicted in Fig. 1, 2, 3, and 4.

As show n in Fig. 1 and 2, the critical masses do not increase greatly as the reflector is getting thicker over the thickness about 30cm. However, the critical mass of plutonium core with iron reflector requires more than that with other reflector such as nickel, copper, molybdenum, or beryllium oxide. The minimum critical mass can be achieved with beryllium oxide reflector having enough thickness. Plutonium oxide fuelled system

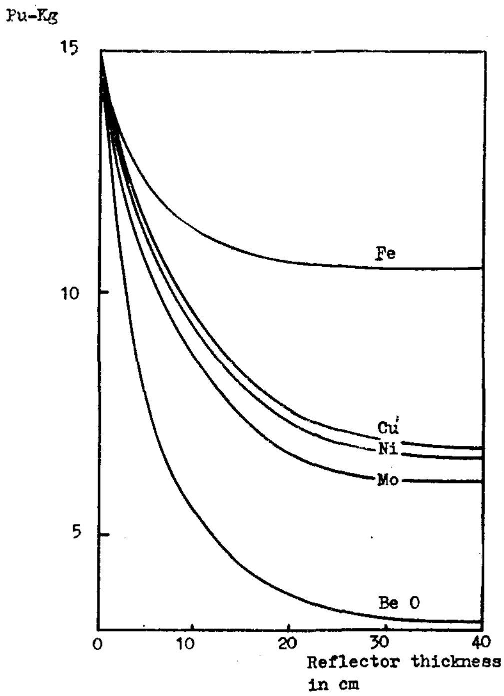


**Fig. 1. Critical Masses of Pu-metal with various reflector material**

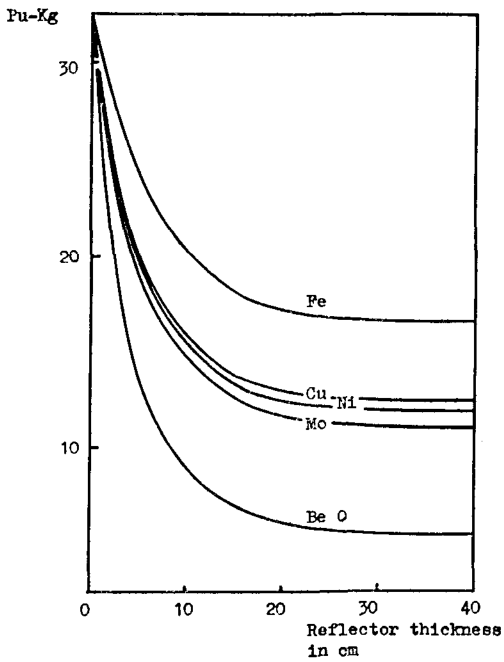


**Fig. 2. Critical Masses of Pu Oxide with various reflector materials**

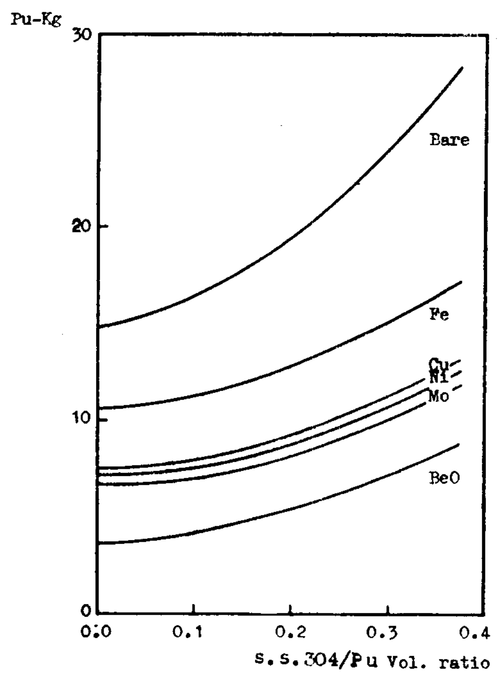


**Fig. 3. Critical Masses for Pu-S.S. 304 dilute system with various reflector**

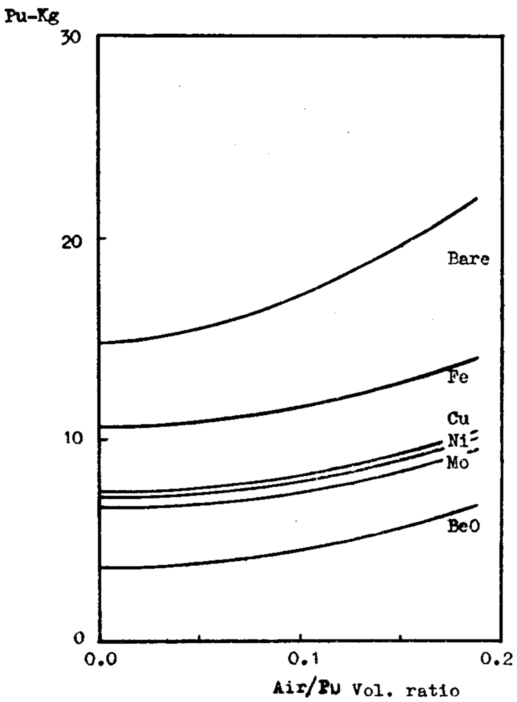


**Fig. 4. Critical Masses for Pu-Air dilute system with various reflector**

needs much more critical mass compared with plutonium metal system because of lower atom density as well as its absorption. It would be possible to consider two layers of reflector consisted of two different material in order to save critical mass and to make the spectrum hard.

On the diluted system. 0.1, 0.2, and 0.3 of the volumetric ratio of S.S. 304/Plutonium are equivalent to the thickness of cladding of 0.03, 0.05, and 0.07cm, respectively. In the air dilute system the volumetric ratio of 0.05, 0.10, and 0.15 are equivalent to the spacings of 0.03, 0.05, and 0.10 cm, respectively, in case of 1 cm diameter of fuel rods. However, the results shows that the critical mass of plutonium fuelled core, which consists of hexagonal, pitch 0.03, arrangement of fuel rods, which diameter assumes 1 cm with 0.05cm S.S. 304 cladding, is expected about 10 Kgs. For all the dilute systems the thickness of the reflectors were taken to be a constant value of 20cm.

The results obtained for iron and beryllium reflectors were compared with the critical masses calculated by Ferguson *et al*[13] in Fig. 5. The experimental values for a beryllium reflector reported in Ref. 14 are also plotted in the figure. Ferguson computed the critical masses of the spherical plutonium core using the DTF code, in which Carlson's $S_N$ method was employed in approximating a solution to Boltzmann transport equation, with the Hanson-Roach's 16 group neutron cross sections and five angular directions.

As shown in Fig. 5, 1DX calculation gave higher values than DTF code for both cases of iron and beryllium reflectors. It should be pointed out that the cross sections of iron used in the calculation is much different from each other. The results for beryllium reflector using 1DX and Russian formatted data gave

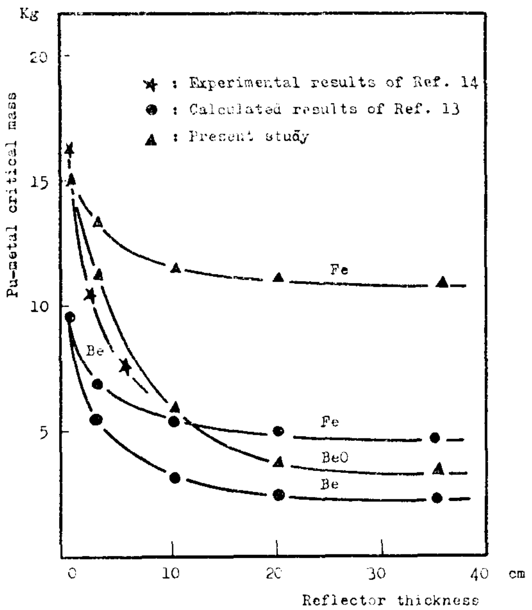


**Fig. 5. Pu-metal critical masses for Fe and Be reflectors as function of reflector thickness**

better agreemeents with the experimental data.

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