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常磁性 푸쉬플메이서의 反轉比

On Inversion Ratios for Push-Pull Paramagnetic Masers

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Abstract

Electron paramagnetic resonance is one of the concrete forms of magnetic resonance and the superposition of an external magnetic field causes an orientation of the magnetic moment of an atom.

On the assumption that both pumped levels be saturated and all relaxation times equal the inversion ratio for a push-pull paramagnetic maser is obtained and compared with those of three-level paramagnetic masers and the magnetic field intensities for 9 and 10 Gc push-pull ruby maser oriented with an angle of 54°44′ between the caxis and the magnetic field are, also, obtained.

1. INTRODUCTION

Electron paramagnetic resonance is one of the concrete forms of magnetic resonance and the superposition of an external magnetic field causes an orientation of the magnetic moment of an atom. If a paramagnetic system has more than two energy levels, there appear new possibilities for application of such systems in amplification and generation of microwaves[1].

The first operating maser was due to Gordon, Zeiger, and Townes. Bloembergen proposed the three level solid state maser on theoretical grounds and the first successful operation was done by Scovil, Feher, and Seidel, who used a lanthanum ethylsulfate crystal containing 0.5 per cent gadolinium. A three level maser is generally a lower noise device than a two level solid-state maser[2].

When a paramagnetic crystal is placed in a magnetic field, the magnetic atoms assume only certain allowed energy states, and to obtain maser action, the level population inversion must take place in the crystal. A number of different techniques exist for creating a population inversion. However, threre exist relaxation processes which act to destroy the population inversion. In solid-state masers, interactions between the paramagnetic atoms and the vibrations of the surrounding crystal lattice lead to the spin-lattice relaxation process, and this is the dominant relaxation process in paramagnetic solids, and the dominant spin-lattice interaction of paramagnetic ions in crystals is by the phonon modulation of the crystalline electric field[3]-[6].

A push-pull paramagnetic maser operates under the method of double maser pumping and it is of interest, here, to discuss its inversion ratio to compare with those of three level masers under the specific conditions, and also, to find some characteristic data of a push-pull paramagnetic maser.

2. THEORETICAL CONSIDERATIONS

When a paramagnetic spin is placed in an external magnetic field $\vec{H}$ and subjected to a perturbation by an alternating magnetic field with resonant frequency f_o such that the quantum hf_o is exactly the same as the separation between two energy levels, and the direction of the alternating field is perpendicular to the direction of the static magnetic field, then there are induced transitions between two levels and the condition for magnetic resonance is

$$hf_o = g\beta H, \qquad (1)$$

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where g and β are the g factor and Bohr magneton, respectively.

If the assembly of magnetic moments of all paramagnetic spins can interact with lattices and transmit the energy received form the r_of_o field to to the lattice, then the resonant magnetic absorption can be steady state. The condition for thermal equilibrium between the spin system and the lattice is that the number of transitions in both directions beween the two states must be equal, i. e.,

$$N_j w_{ji} = N_i w_{ij}, \tag{2}$$

where w_{ij} and N_i are the relaxation transition probability caused by thermal lattice vibrations per spin per unit time and the thermal-equilibrium population of level i, respectively. To deal with the problem of interaction between the spin system and the lattice, it is assumed initially that the rf field is absent and that the constant magnetic field is vanishingly small. In this case the amount of level splitting in the magnetic field is vanishingly small and at the initial moment the spin system and the lattice are not in equilibrium. At this moment, the rate equation of change of the excess n of particles per volume per unit time in the lower state is

$$\frac{dn}{dt} = 2(w_{ji}N_j - w_{ij}N_i) \tag{3}$$

Substituting the appropreate value of w_i and integrating equation (3) one gets

$$(n_o - n) = (n_o - n_o)e^{-2wt} \quad , \tag{4}$$

where

$$n_o = N\beta H/kT$$

$$w = \frac{1}{2}(w_{ij} + w_{ji})$$

and n_a is the initial value of n.

when an oscillating magnetic field is present, the initial equation for n becomes

$$\frac{dn}{dt} = \frac{n_o - n}{T_1} - 2nW, \tag{5}$$

where $2nW$ represents the difference between transitions induced by the rf field, W the stimulated transition probability and T_1 the spin-lattice relaxation time.

As a result of the above discussion, rate equations for multilevel systems can be written as

$$\frac{dn_i}{dt} = \sum_j [-w_{ij}n_i + w_{ji}n_j + W_{ij}(n_j - n_i)], \tag{6}$$

where n_i is the instantaneous population of level i and W is the stimulated transition probability per spin per unit time from level i to level j. Furthermore, thermalequilibrium considerations of the systems demand the following relationship

$$N_i/N_j = w_{ji}/w_{ij} = e^{-\Delta ij} \quad , \tag{7}$$

where

$$\Delta_{ij} = -\Delta_{ji} = \frac{E_i - E_j}{kT} \tag{8}$$

with E_i for the energy of level i, and a pseudo spin-lattice relaxation time is defined by

$$T_1^{(ij)} = (w_{ij} + W_{ji})^{-1} \quad , \tag{6}$$

equation (6) can, therefore, be rewritten as

$$\frac{dn_i}{dt} = \sum_j \left[\frac{\Delta n_{ij} - [(n_i + n_j)/(N_i + N_j)]\Delta N_{ij}}{2T_1^{(ij)}} + W_{ij}\Delta n_{ij} \right] \tag{10}$$

with

$$\left. \begin{aligned} \Delta n_{ij} &= -\Delta n_{ji} = n_i - n_j \\ \Delta N_{ij} &= -\Delta N_{ji} = N_i - N_j \end{aligned} \right\} . \tag{11}$$

The power output of a multilevel system is the induced radiated power which is equal to the product of the difference in populations of the levels used for amplification, the quantum of energy, and the induced transition probability, and which is expressed by

$$P = (n_j - n_i)hf_{ji}W_{ji} \quad . \tag{12}$$

3. DISCUSSION

For the case of a push-pull paramagnetic maser the rate equation (10) can now be written, assuming that at steady state the number of transitions to each level from the other levels is equal to the

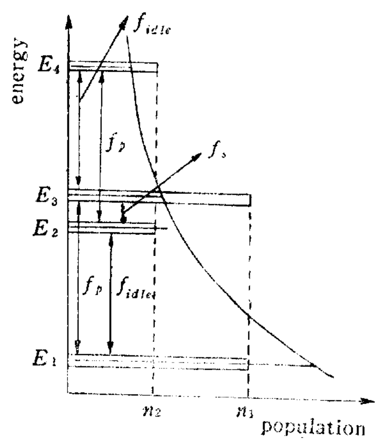

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Fig. E-n relations of a push-pull paramagnetic maser number of transitions from the given level to the other levels, as

$$\left.\begin{aligned}
\frac{dn_1}{dt} &= -\frac{\Delta n_{12}-\Delta N_{12}}{2T_1^{(12)}}-\frac{\Delta n_{13}-\Delta N_{13}}{2T_1^{(13)}}-\frac{\Delta n_{14}-\Delta N_{14}}{2T_1^{(14)}} \\
&\quad -W_{12}\Delta n_{12}-W_{13}\Delta n_{13}-W_{14}\Delta n_{14} \\
\frac{dn_2}{dt} &= -\frac{\Delta n_{21}-\Delta N_{21}}{2T_1^{(21)}}-\frac{\Delta n_{23}-\Delta N_{23}}{2T_1^{(23)}}-\frac{\Delta n_{24}-\Delta N_{24}}{2T_1^{(24)}} \\
&\quad -W_{21}\Delta n_{21}-W_{23}\Delta n_{23}-W_{24}\Delta n_{24} \\
\frac{dn_3}{dt} &= -\frac{\Delta n_{31}-\Delta N_{31}}{2T_1^{(31)}}-\frac{\Delta n_{32}-\Delta N_{32}}{2T_1^{(32)}}-\frac{\Delta n_{34}-\Delta N_{34}}{2T_1^{(34)}} \\
&\quad -W_{31}\Delta n_{31}-W_{32}\Delta n_{32}-W_{34}\Delta n_{34} \\
\frac{dn_4}{dt} &= -\frac{\Delta n_{41}-\Delta N_{41}}{2T_1^{(41)}}-\frac{\Delta n_{42}-\Delta N_{42}}{2T_1^{(42)}}-\frac{\Delta n_{43}-\Delta N_{43}}{2T_1^{(43)}} \\
&\quad -W_{41}\Delta n_{41}-W_{42}\Delta n_{42}-W_{43}\Delta n_{43}
\end{aligned}\right\} \quad (13)$$

Deriving equations (13) from equation (10) it is assumed that $\Delta N_{ij} \ll N_i$ and N_j (which means essentially that $\Delta_{ij} \ll 1$) and, also, $\Delta n_{ij} \ll n_i$ and n_j and under these conditions the real push-pull maser operates.

Now, the complete set of four equations are set equal to zero, because we are interested in the steady-state equilibrium condition with both pump and signal applied, i. e.,

$$\frac{dn_i}{dt}=0$$

and we neglect W_{14}, i. e., $W_{14}=W_{12}=0$, because we assume that there is no stimulated transition probability between level 1 and 4(no power relation). And the pump transition rate W_p is large enough to completely saturate the pump transition, while the signal transition rate W_s will be small enough so that it can be neglected, i. e.,

$$\Delta n_{31}=n_3-n_1=0$$

$$\Delta n_{42}=n_4-n_2=0$$

$$0 \fallingdotseq W_s \ll W_p$$

and from Figure

$$\Delta n_{12}=\Delta n_{14}=\Delta n_{32}=-n_{21}=-\Delta n_{41}=-\Delta n_{23}=-\Delta n_{43} \quad .$$

Putting these relations into the second equation of equation (13) we get

$$\frac{\Delta n_{32}+\Delta N_{21}}{2T_1^{(21)}}+\frac{\Delta n_{32}+\Delta N_{23}}{2T_1^{(23)}}+\frac{\Delta N_{24}}{2T_1^{(24)}}=0 \quad .$$

Then, Δn_{32} is expressed as

$$\Delta n_{32}=\frac{\Delta N_{12}/T_1^{(12)}-\Delta N_{23}/T_1^{(23)}-\Delta N_{24}/T_1^{(24)}}{1/T_1^{(12)}+1/T_1^{(23)}} \quad (14)$$

The approximation $\Delta^{ij} \ll 1$ also makes it possible to write

$$\Delta N_{ij}=N_i-N_j \simeq (hf_{ji}/nkT)N \ ,$$

where n is the total number of levels. Hence ,

$$\Delta N_{23}=\frac{hf_{32}}{4kT}N \quad (15)$$

With the above relationship equation (14) becomes as

$$\Delta n_{32}=\frac{\Delta N_{23}}{f_{32}}\;\frac{f_{21}/T_1^{(12)}-f_{32}/T_1^{(23)}-f_{42}/T_1^{(24)}}{1+T_1^{(12)}/T_1^{(23)}}$$

$$\therefore \frac{\Delta n_{32}}{\Delta N_{23}}=\frac{1}{f_{32}}\;\frac{f_{21}-f_{32}T_1^{(12)}/T_1^{(23)}-f_{42}T_1^{(12)}/T_1^{(24)}}{1+T_1^{(12)}/T_1^{(23)}}$$

$$=\frac{f_i/f_s-T_1^{idle}/T_1^{sig}+f_p/f_s\cdot T_1^{idle}/T_1^{pump}}{1+T_1^{idle}/T_1^{sig}} \quad (16)$$

The condition for population inversion, valid regardless of which transition becomes the inverted or singnal transition, is

$$f_p/f_s=1+f_i/f_s>1+T_1^{idle}/T_1^{signal}$$

Then equation (16) becomes

$$I=\frac{\Delta n_{32}}{\Delta N_{23}}$$

$$=\frac{f_p/f_s-1-T_1^{idle}/T_1^{sig}+f_p/f_s\cdot T_1^{idle}/T_1^{pump}}{1+T_1^{idle}/T_1^{sig}}$$

and I is the inversion ratio of the push-pull paramagnetic maser.

For the case where all the T_1's are equal I becomes

$$I=\frac{f_p}{f_s}-1$$

and, for comparison with the result, in the case of the threelevel paramagnetic maser the ratio becomes

$$I_3=\frac{f_p}{2f_s}-1 \quad .$$

The magnetic field intensity for a push-pull ruby maser can obtained from the relation of

$$W=\sqrt{D^2+\frac{5}{4}(g\beta B)^2 \pm g\beta B\sqrt{3D^2+(g\beta B)^2}} \ ,$$

where W is the energy level and D the constant determined by the symmetry and energy consideration of ruby crystals, and above expression is obtained by considering the spin Hamiltonian for rubies. The magnetic field intensities for 9 and 10Gc push-pull ruby maser oriented with an angle of 54° 44′ between the c axis and the magnetic field are, using the above expression,

$$B_{9Gc}=900 \text{ and } 3200 \text{ gauss}$$

and

$$B_{10Gc}=380 \text{ and } 4020 \text{ gauss}$$

The power output of the maser is, using the appropriate data, of the order of erg per second in magnitude.

4. CONCLUSIONS

The inversion ratios for a push-pull paramagnetic maser, where the signal transition rate is small and the pumping transition rate is very large and also all the $T's$ are equal, and for a three-level paramagnetic maser are compared with each other and the magnetic field intensities for 9 and 10 Gc push-pull ruby maser, oriented with an angle of 54°44′ between the c axis and the magnetic field, are approximately 900 and 3200 gausses for 9 Gc and 380 and 4020 gausses for 10 Gc.

5. REFERENCES

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