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Application of Optimum Control to 600 MWe Pressurized Water Reactor

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Abstract

This paper presents an approach to control that is a result of modern control theory, and is based on the control philosophy of feeding back all the state variable through constant gain frequency independent elements. The values of these elements or feedback coefficients are determined by equating like coefficients of the desired system transfer function to the transfer function of the system containing the unspecified coefficients.

This application of modern control law is a simple design method depending on feedingback all the system variables for reactor control and it is particuraly amenable to the control of Pressurized Water Reactor.

요 약

각 상태변수의 출력에서 일정한 이득의 신호를 다시 궤환시키는 최신의 제어방법을 실제에 적용시켜 보았다. 이러한 궤환 신호의 값은 계 전체의 전달함수를 구하고 우리가 원하는 계의 전달함수와 비교하여 s의 같은 차수끼리 서로 같도록 놓음으로서 결정하였다.

이 방법은 복잡한 원자로의 궤환 계수를 매우 간단하게 결정할 수 있으며 특히 PWR의 제어에 간편 방법으로 사용하여 보았다.

1. Introduction

A large dynamical system defined by

$$\dot{X}=AX+Bm(t)$$

is characterized by a matrix of high order A. For such a system the analysis is laborious and time consuming. Especially, it is very difficult to determine feedback cofficients or weighting factors.

The optimization procedure described here is closely associated with the Ellert's procedure.

The design specifications are given in terms of a desired overall system transfer function, which is realized exactly through the feedback constants. These feedback coefficients are determined by equating like coefficients of the desired system transfer function to the transfer function of the system containing the unspecified coefficients.

2. State Variable Feedback Design

It is assumed that control elements are described by

$$\dot{X}=A(t)X+B(t)m \cdots\cdots (1)$$

where X: State Vector

m: Control Vector

A, B: $n \times n$, $n \times r$ Matrix

and the performance index is given by substituting

$$f_o(X,m,t)=\frac{1}{2}[\langle (X^d-X), Q(X^d-X)\rangle + \langle m, Zm\rangle] \cdots\cdots (2)$$

into

$$I=\int_{t_o}^{t_f} f_o(X,m,t)dt \cdots\cdots (3)$$

X^d is the desired state behavior, Q and Z are symmetric matrices which possibly time-varying. The dimensions of Q are less than $(n \times n)$ unless all components of (X^d-X) are included in f_o. We can use the minimum principle to obtain the necessary conditions for the optimal control and so derive the extremal controls. The Hamiltonian H for the system (1) and (2) is

$$H^o(X,m,p,t)=\frac{1}{2}[\langle (X^d-X^o), Q(X^d-X^o)\rangle + \langle m^o, Zm^o\rangle] + \langle p^o, AX^o+Bm^o\rangle \cdots\cdots (4)$$

In terms of the optimum H, the first necessary condition for an optimum for the case of unspecified terminal conditions on the state variables can be written as

$$grad_{x^o}H^o=-\dot{P}^o$$
$$grad_{m^o}H^o=O \cdots\cdots (5)$$
$$grad_{p^o}H^o=\dot{X}^o$$

subject to the boundary conditions $X^o(t_o)=X(t_o)$ and $p^o(t_f)=0$. For specified terminal conditions an X^o, the latter boundary condition is replaced by $X^o(t_f)=X(t_f)$.

From Eq. (5) we deduce that

$$m^o(t)=-Z^{-1}B^TP^o \cdots\cdots (6)$$

The assumption that Z is positive definite for all gurantees and the existence of Z^{-1} is for all $t \in [t_o, t_f]$.

The control law requires the optimum control signal m^o in terms of X^o.

Define the matrix R, by setting

$$R=BZ^{-1}B^T \cdots\cdots (7)$$

Using the matrix R, we can combine the canonical equations in the form

$$\begin{pmatrix} \dot{X}(t) \\ \dot{P}(t) \end{pmatrix} = \begin{pmatrix} A & -R \\ -Q & -A^T \end{pmatrix} \begin{pmatrix} X^o \\ P^o \end{pmatrix} + Q \begin{pmatrix} O \\ X^d \end{pmatrix} \cdots (8)$$

Equation (8) consists of a set of interrelated linear differential equations for X^o and P^o. Thus X^o and P^o must be related by a linear transformation. This transformation may be expressed by

$$P^o=KX^o-V^o \cdots\cdots (9)$$

where K is a square matrix of time-varying gains and V^o is a time-varying vector. The control law for the optimum system is given by substituting Eq. (9) into (6) to obtain

$$m^o=-Z^{-1}B^T(KX^o-V^o) \cdots\cdots (10)$$

Thus, for this case, the control law is linear, and the controller feedback gains K are independent of the state of the controlled elements.

Once m^o is determined, the response of the optimum system can be obtained from

$$\dot{X}^o=(A-RK)X^o+RV^o \cdots\cdots (11)$$

which results from subtituting Eq. (10) into Eq. (1).

It is known that as $T\rightarrow\infty$, the gain matrix K of the Eq. (11) tends to the constant matrix K.

From Eq. (11), the system transfer function can be obtained by taking the Laplace transform with the usual assumption in transfer function determination that all initial conditions are zero. Transforming Eq. (11) and assuming that V and y are the single input and output,

$$sX^o(s)=(A-RK)X^o(s)+RV^o(s)$$
$$y(s)=CX^o(s)$$
$$\frac{y(s)}{V^o(s)}=RC(s\mathbf{1}-A+RK)^{-1} \cdots\cdots (12)$$

where C: $1 \times n$ matrix

$\mathbf{1}$: unit matrix

The design philosophy is to specify the sys-

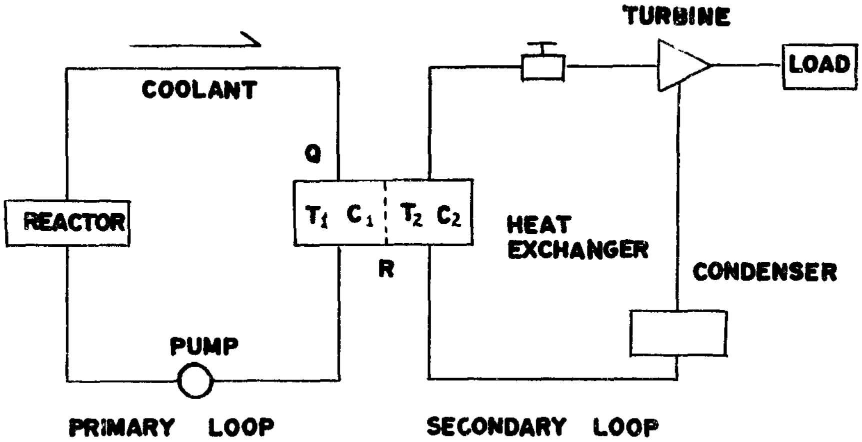

Fig. 1. **Primary and secondary loops of nuclear reactor plant**

tem dynamics in terms of the desired system transfer function, and to realize these specifications through the feedback coefficient K.

Whiteley's standard forms for the characteristic equations are used.

3. Application to 600 MWe Pressurized Water Reactor

A. Characteristic Plant Equation

The general block diagram of linear reactor system to be considered is shown in Fig. 1.

Typical equations of state for a pressurized water reactor plant are given in the following. These can be made more simple or complex as the situation demands. The one group neutron kinetics equations are

$$\frac{dn}{d\tau}=\frac{\rho-\beta}{l}n+\lambda C \cdots\cdots\cdots(13)$$

$$\frac{dC}{d\tau}=\frac{\beta}{l}n-\lambda C \cdots\cdots\cdots(14)$$

where n is the neutron power as a fraction of rated value, ρ is the reactivity, β is the fraction of neutrons that are delayed, l is the prompt neutron lifetime, C is the concentration of delayed neutron emitters, and λ is the decay constant. The symbol τ represents time.

The transfer of heat from the primary loop to the secondary loop is represented as

$$\frac{dT_1}{d\tau}=\frac{1}{C_1}Q-\frac{1}{C_1R_1}(T_1-T_2)$$

$$=\frac{1}{C_1}n-\frac{1}{C_1R_1}(T_1-T_2) \qquad (15)$$

$$\frac{dT_2}{d\tau}=\frac{1}{C_2R_1}(T_1-T_2) -\left(\frac{Ka}{C_2R_1}\psi\frac{\partial P_2}{\partial T_2}\right)T_2 \qquad (16)$$

where T_1 is the coolant temperature of the reactor, T_2 is the steam temperature, R_1 is the heat resistance, Q is inlet thermal power, C_1 and C_2 are the heat capacity of primary loop and secondary loop, A and Ka are constants, $\frac{\partial P_2}{\partial T_2}$ is the slope relation between the secondary system pressure and temperature in the temperature range of interest. The symbol ψ is the load demand as a fraction of the rated steam flow.

The set of equations (13)-(16) is nonlinear by virtue of (13) which contains the product of reactivity ρ and neutron power n. Total neutron power n is considered to be composed of steady part n_0, and a fluctuating part δn. Since the reactor is critical, there is no steady source. Then Eq. (13)-(16) are linearized to

$$\frac{d\,\delta n/n_0}{d\tau}=\frac{1}{l}\rho-\frac{\beta}{l}\frac{\delta n}{n_0}+\frac{\lambda}{n_0}\delta C \cdots\cdots(17)$$

$$\frac{d\delta C}{d\tau}=\frac{n_0\beta}{l}\frac{\delta n}{n_0}-\lambda\delta C \cdots\cdots(18)$$

$$\frac{d\delta T_1}{d\tau}=\frac{An_0}{C_1}\frac{\delta n}{n_0}-\frac{\delta T_1}{C_1R_1}+\frac{\delta T_2}{C_1R_1} \cdots\cdots(19)$$

$$\frac{d\delta T_2}{d\tau}=\frac{\delta T_1}{C_2R_1}-\frac{1}{C_2R_1}\left(1+Ka\psi\frac{\partial P_2}{\partial T_2}\right)\delta T_2 \cdots(20)$$

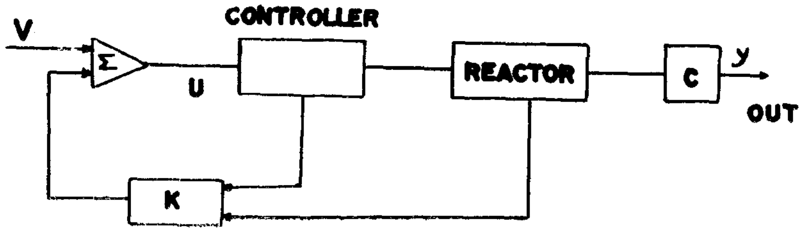

Fig. 2. Block diagram of state variable feedback control

where $\lambda C_0-\beta n_0/l=0$ ··························(21)

$An_0/C_1-T_{10}/C_1R_1+T_{20}/C_1R_1=0$ ·········(22)

$T_{10}/C_2R_1-\left(1+Ka\phi\frac{\partial P_2}{\partial T_2}\right)T_{20}/C_2R_1=0$ (23)

The block diagram of linear reactor system to be considered is shown in Fig. 2

The optimum control system was employed with 600 MWe PWR using a β value of 75×10^{-4}, a decay constant λ of 0.1 second^{-1}, and a prompt neutron life time l of 10^{-4} seconds. The heat capacity of primary loop and secondary loop C_1, C_2 are 4×10^{-5} Btu/°F and 3.2×10^{-5}Btu/°F. The heat resistance R_1 is 43.56×10^{-6} sec °F/Btu.

The differential equations defining the system are

$\dot{X}_1=-75X_1+0.2667\times10^{-10}X_2+10^4X_5$

$\dot{X}_2=2.25\times10^{11}X_1-0.08X_2$

$\dot{X}_3=1.5\times10^4X_1-8.7392\times10^{-2}X_3$

$+8.7392\times10^{-2}X_4$ ··························(24)

$\dot{X}_4=7.17401\times10^{-2}X_3-0.928166X_4$

$\dot{X}_5=-0.1X_5+u.$

where X_1: n the neutron power as a fraction of rated value.

X_1: C the concentration of delayed neutron emitters.

X_3: T_1 the coolant temperature of the reactor

X_4: T_2 the steam temperature

X_5: the reactivity input from controller

u: input control signal

Referring to Eqs. (24), the terms in Eqs. (1), (2) are given by

$$A=\begin{pmatrix} -75 & 0.2667\times10^{-10} & 0 & 0 & 10^4 \\ 2.25\times10^{11} & -0.08 & 0 & 0 & 0 \\ 1.5\times10^4 & 0 & -8.7392\times10^{-2} & 8.7392\times10^{-2} & 0 \\ 0 & 0 & 7.17401\times10^{-2} & -0.928166 & 0 \\ 0 & 0 & 0 & 0 & -0.1 \end{pmatrix}$$

$$B=\begin{pmatrix} 0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&1 \end{pmatrix} \qquad C=\begin{pmatrix} 1&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0 \end{pmatrix}$$

$$Z=\begin{pmatrix} \xi_{11}&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0 \end{pmatrix}$$

From Eq. (6), m° is unchanged if Z is replaced by

$$Z=\begin{pmatrix} \xi_{11}&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1 \end{pmatrix}$$

Therfore
$$Z^{-1}=\begin{pmatrix} \xi_{11} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad K=\begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} \end{pmatrix}$$

Using the method of Eq. (12), the closed-loop transfer function of optimized reactor system can be determined to be

$$\frac{X_1(S)}{V(S)}=\frac{10^4\{s^3+1.095558s^2+0.156089s+0.005988}{s^5+as^4+bs^3+cs^2+ds+e} \quad\cdots\cdots(25)$$

where

$a=76.195558+k_{55}$

$b=167.790145+10^4k_{51}+76.095558k_{55}$

$c=13.251656+1.095558\times10^4k_{51}+2.25\times10^{15}k_{52}$
$\quad+1.5\times10^8k_{53}+76.24809k_{55}$

$d=0.561974+0.15609k_{51}+2.285006\times10^{15}k_{52}$
$\quad+1.512249\times10^8k_{53}+0.10761\times10^8k_{54}$
$\quad+5.61974k_{55}$

$e=0.005988\times10^4k_{51}+0.168401\times10^{15}k_{52}$
$\quad+0.11138\times10^8k_{53}+0.008609\times10^8k_{54}$

Suppose that the desired dynamics of the system is given by the second order transfer function

$$T_d=\frac{10^4}{s^2+200s+10^4} \quad\cdots\cdots(26)$$

which has well-behaved transient characteristics with a damping ratio of 10% and a desirable frequency response. To realize these desired system characteristics, Eq. (26) must equal

$$T=\frac{10^4(s^3+1.095558s^2+0.156089s+0.005988)}{(s^2+200s+10^4)(s^3+1.095558s^2+0.156089s+0.005988)} \quad\cdots\cdots(27)$$

or

$$T=\frac{10^4(s^3+1.095558s^2+0.156089s+0.005988)}{s^5+201.095558s^4+10219.267689s^3+10988.0004s^2+1562.0876s+59.88} \quad\cdots\cdots(28)$$

Equating the coefficients of like powers of s in the denominator of Eq. (28) to Eq. (26) and solving the resultant linear algebraic simultaneous equations for K yields

$$\begin{aligned} k_{51}&=0.6310\times10^{-1}\\ k_{52}&=0.5356\times10^{-13}\\ k_{53}&=0.3000\times10^{-5} \quad\cdots\cdots(29)\\ k_{54}&=0.30882\times10^{-6}\\ k_{55}&=0.1249\times10^{3} \end{aligned}$$

From the response curve of $X_1(n)$ in Fig. 3, it can be found that the transient response behavior corresponds exactly to that expected from the desired ststem transfer function.

The power transient shown in this case runs between rated power$\left(\psi=1,\ \frac{\delta n}{n_0}=1\right)$

4. Conclusions

The above example has demonstrated a new design technique for reactor control based on feeding back all the system variables or state variables through constant gain elements. The reactor dynamical system is characterized by a matrix of high order A. This design method is a very simple mode for reactor control. The design specifications are given in terms of a desired overall system transfer function, which is realized exactly through the feedback constants. These feedback coefficients are determined by equating like coefficients of the desired system transfer function to the transfer function of the system containing the unspecified coefficients. The initial design procedure is to assume that all state variables are available, and then determine the feedback gains. This design technique is adaptable to digital computation.

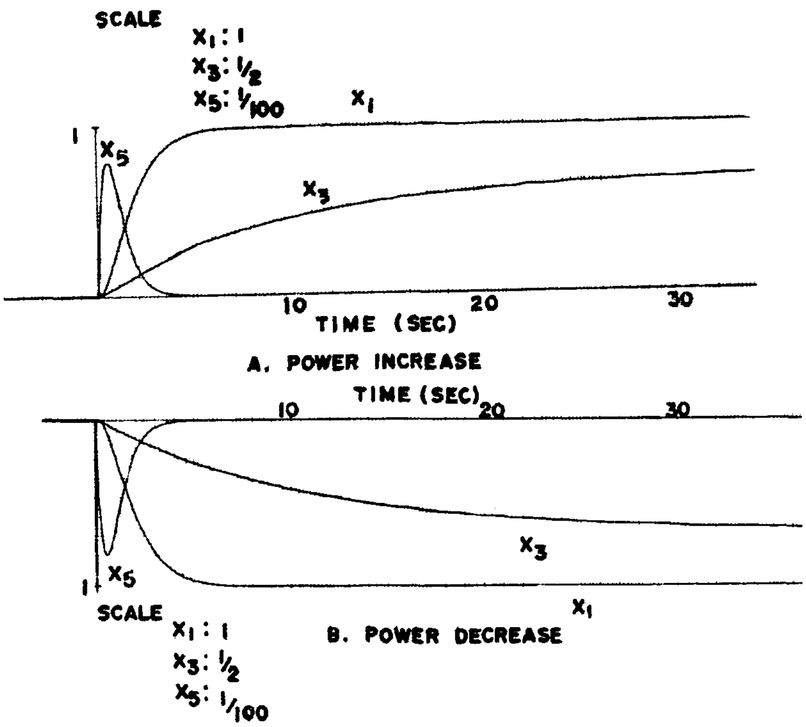

Fig. 3. State variable response of the various system parameters to a step changes in power.

In order to specify Ω, Eqs. (8) are solved for ω. Substituting the result of K into Eqs. (8), we can obtain the value of ω_{ij}.

$$\begin{aligned} \omega_{11} &= 0.192183 \\ \omega_{22} &= 0.367293\times10^{-25} \\ \omega_{33} &= 0.193649\times10^{-10} \\ \omega_{44} &= 0.502496\times10^{-9} \\ \omega_{55} &= 0.143630\times10^{5} \end{aligned} \qquad (30)$$

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