SOME RESULTS INVOLVING GENERALISED FUNCTION OF TWO VARIABLES

By S. L. Bora and S. L. Kalla

1. Introduction.

Recently Munot and Kalla [6] have defined and represented the generalised function of two variables as follows:

$$(1.1)\qquad H\left[\begin{matrix}\left[\begin{smallmatrix}m_1, & 0\\ p_1-m_1, & q_1\end{smallmatrix}\right]\\ \left(\begin{smallmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{smallmatrix}\right)\\ \left(\begin{smallmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{smallmatrix}\right)\end{matrix}\left|\begin{matrix}(a_{p_1}, A_{p_1}) : (b_{q_1}, B_{q_1})\\ (c_{p_2}, C_{p_2}) : (d_{q_2}, D_{q_2})\\ (e_{p_3}, E_{p_3}) : (f_{q_3}, F_{q_3})\end{matrix}\right| X, Y\right]$$

$$=\frac{1}{(2\pi i)^2}\int_{L_1}\int_{L_2}F(\xi+\eta)\phi(\xi, \eta)d\xi d\eta,$$

where on the left the symbol (a_p, A_p) stands for the set of p ordered pairs $(a_1, A_1), \dots, (a_p, A_p)$, on the right L_1 and L_2 are two suitable contours and,

$$F(\xi+\eta)=\frac{\prod_{j=1}^{m_1}\Gamma(a_j+A_j\xi+A_j\eta)}{\prod_{j=m_1+1}^{p_1}\Gamma(1-a_j-A_j\xi-A_j\eta)\prod_{j=1}^{q_1}\Gamma(b_j+B_j\xi+B_j\eta)}$$

$$\Phi(\xi, \eta)=\frac{\prod_{j=1}^{m_2}\Gamma(1-c_j+C_j\xi)\prod_{j=1}^{n_2}\Gamma(d_j-D_j\xi)\prod_{j=1}^{m_3}\Gamma(1-e_j+E_j\eta)\prod_{j=1}^{n_3}\Gamma(f_j-F_j\eta)\, x^{\xi}\, y^{\eta}}{\prod_{j=m_2+1}^{p_2}\Gamma(c_j-C_j\xi)\prod_{j=n_2+1}^{q_2}\Gamma(1-d_j+D_j\xi)\prod_{j=m_3+1}^{p_3}\Gamma(e_j-E\eta)\prod_{j=n_3+1}^{q_3}\Gamma(1-f_j+F_j\eta)}$$

with $p_1\geqslant m_1\geqslant 0, p_2\geqslant m_2\geqslant 0, p_3\geqslant m_3\geqslant 0, q_1\geqslant 0, q_2\geqslant n_2\geqslant 0, q_3\geqslant n_3\geqslant 0, q_1+q_2\geqslant p_1+p_2,$

$$q_1+q_3\geqslant p_1+p_3$$

and each p's, n's and m's is a non negative integer.

The object of this paper is to evaluate some finite and infinite integrals involving generalised function of two variables. The results established here are of general character and include as particular cases some known results. It is interesting to observe that some integrals involving product of two Fox's H-function [4] follow as special cases of our main results.

The following results [2, p. 331 eq(26), (28), p. 371(51), p. 398(2) and p. 399(3)], will be required in the sequel.

$$(1.2)\qquad \int_0^\infty x^{\lambda-1} K_\nu(ax)dx = a^{-\lambda} 2^{\lambda-2} \Gamma\left(\frac{1}{2}\lambda \pm \frac{1}{2}\nu\right)$$

valid for $R(a)>0,\ R(\lambda)>|R(\nu)|$.

$$(1.3)\qquad \int_0^\infty x^{\lambda-1} e^{-ax} K_\nu(ax)dx = \frac{\sqrt{\pi}\,\Gamma(\lambda\pm\nu)}{2^\lambda a^\lambda \Gamma\left(\lambda+\frac{1}{2}\right)},$$

valid for $R(a)>0,\ R(\lambda)>R(\nu)|$.

$$(1.4)\qquad \int_0^\infty x^{\rho-1} K_\mu(\alpha x) K_\nu(\alpha x) dx$$

$$= 2^{\rho-3}\alpha^{-\rho}\{\Gamma(\rho)\}^{-1}\Gamma\left\{\frac{1}{2}(\rho+\mu+\nu)\right\}\Gamma\left\{\frac{1}{2}(\rho+\mu-\nu)\right\}$$

$$\times\Gamma\left\{\frac{1}{2}(\rho-\mu+\nu)\right\}\Gamma\left\{\frac{1}{2}(\rho-\mu-\nu)\right\},$$

provided $|\mathrm{Re}(\alpha)|>0,\ \mathrm{Re}(\rho)>|\mathrm{Re}(\mu)|+|\mathrm{Re}(\nu)|$.

$$(1.5)\qquad \int_0^1 x^{\rho-1}(1-x)^{\beta-\gamma-n}\,{}_2F_1(-n,\ \beta;\gamma;x)dx$$

$$= \frac{\Gamma(\gamma)\Gamma(\beta-\gamma+1)\Gamma(\rho)\Gamma(\gamma+n-\rho)}{\Gamma(\gamma+n)\Gamma(\gamma-\rho)\Gamma(\beta-\gamma+\rho+1)}$$

where $n=0,\ 1,\ 2,\cdots\cdots,\ R(\rho)>0,\ R(\beta-\gamma)>n-1$ and

$$(1.6)\qquad \int_0^1 x^{\rho-1}(1-x)^{\beta-\rho-1}\,{}_2F_1(\alpha,\ \beta;\gamma;x)dx = \frac{\Gamma(\gamma)\Gamma(\rho)\Gamma(\beta-\rho)\Gamma(\gamma-\alpha-\rho)}{\Gamma(\beta)\Gamma(\gamma-\alpha)\Gamma(\gamma-\rho)}$$

valid for $R(\rho)>0,\ R(\beta-\rho)>0,\ R(\gamma-\alpha-\rho)>0$.

2. Main results to be established here are:

$$(2.1)\qquad \int_0^\infty x^{\lambda-1} K_\nu(\alpha x) H\left[\begin{array}{c|c|c} \left[\begin{smallmatrix} 0, & 0 \\ p_1, & q_1 \end{smallmatrix}\right] & (a_{p_1}, A_{p_1}) : (b_{q_1}, B_{q_1}) & \\ \left(\begin{smallmatrix} m_2, & n_2 \\ p_2-m_2, & q_2-n_2 \end{smallmatrix}\right) & (c_{p_2}, C_{p_2}) : (d_{q_2}, D_{q_2}) & \beta x^{2h},\ \delta x^{2h} \\ \left(\begin{smallmatrix} m_3, & n_3 \\ p_3-m_3, & q_3-n_3 \end{smallmatrix}\right) & (e_{p_3}, E_{p_3}) : (f_{q_3}, F_{q_3}) & \end{array}\right] dx$$

$$= 2^{\lambda-2}\alpha^{-\lambda} H\left[\begin{array}{c|c|c} \left[\begin{smallmatrix} 2, & 0 \\ p_1, & q_1 \end{smallmatrix}\right] & \left(\frac{\lambda+\nu}{2}, h\right), \left(\frac{\lambda-\nu}{2}, h\right), (a_{p_1}, A_{p_1}) : (b_{q_1}, B_{q_1}) & \\ \left(\begin{smallmatrix} m_2, & n_2 \\ p_2-m_2, & q_2-n_2 \end{smallmatrix}\right) & (c_{p_2}, C_{p_2}) \quad : (d_{q_2}, D_{q_2}) & \frac{2^{2h}\beta}{\alpha^{2h}}, \frac{2^{2h}\delta}{\alpha^{2h}} \\ \left(\begin{smallmatrix} m_3, & n_3 \\ p_3-m_3, & q_3-n_3 \end{smallmatrix}\right) & (e_{p_3}, E_{p_3}) \quad : (f_{q_3}, F_{q_3}) & \end{array}\right]$$

where $|\arg\beta| < \left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_2}C_j-\sum_{j=m_2+1}^{p_2}C_j+\sum_{j=1}^{n_2}D_j-\sum_{j=n_2+1}^{q_2}D_j\right)\frac{\pi}{2}$,

$|\arg\delta| < \left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_3}E_j-\sum_{j=m_3+1}^{p_3}E_j+\sum_{j=1}^{n_3}F_j-\sum_{j=n_3+1}^{q_3}F_j\right)\frac{\pi}{2}$,

$\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_2}C_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_2}D_j<0$; $\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0$,

$\mathrm{Re}(\alpha)>0$, $R\left(\lambda\pm\nu+2h\frac{d_j}{D_j}+2h\frac{f_k}{F_k}\right)>0$; $j=1,2,\cdots,n_2$; $k=1,2,\cdots,n_3$.

$p_1\geqslant 0$, $p_2\geqslant m_2\geqslant 0$, $p_3\geqslant m_3\geqslant 0$, $q_1\geqslant 0$, $q_2\geqslant n_2\geqslant 0$, $q_3\geqslant n_3\geqslant 0$,

$q_1+q_2\geqslant p_1+p_2$ and $q_1+q_3\geqslant p_1+p_3$.

$$(2.2)\quad \int_0^\infty x^{\lambda-1}e^{-\alpha x}K_\nu(\alpha x)H\left[\begin{array}{c|c|c}\begin{bmatrix}0, & 0\\ p_1, & q_1\end{bmatrix} & (a_{p_1},A_{p_1}):(b_{q_1},B_{q_1}) & \\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix} & (c_{p_2},C_{p_2}):(d_{q_2},D_{q_2}) & \beta x^h,\ \delta x^h\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix} & (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3}) & \end{array}\right]dx$$

$$=\frac{\sqrt{\pi}}{2^\lambda\alpha^\lambda}H\left[\begin{array}{c|c|c}\begin{bmatrix}2, & 0\\ p_1, & q_1+1\end{bmatrix} & (\lambda+\nu,h),(\lambda-\nu,h),(a_{p_1},A_{p_1}):\left(\lambda+\frac{1}{2},h\right),(b_{q_1},B_{q_1}) & \\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix} & (c_{p_2},C_{p_2}):(d_{q_2},D_{q_2}) & \frac{\beta}{(2\alpha)^h},\frac{\delta}{(2\alpha)^h}\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & p_3-n_3\end{pmatrix} & (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3}) & \end{array}\right]$$

where $|\arg\beta| < \left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_2}c_j-\sum_{j=m_2+1}^{p_2}C_j+\sum_{j=1}^{n_2}D_j-\sum_{j=n_2+1}^{q_2}D_j\right)\frac{\pi}{2}$,

$|\arg\delta| < \left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_3}E_j-\sum_{j=m_3+1}^{p_3}E_j+\sum_{j=1}^{n_3}F_j-\sum_{j=n_3+1}^{q_3}F_j\right)\frac{\pi}{2}$,

$\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_2}C_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_2}D_j<0$; $\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0$,

$\mathrm{Re}(\alpha)>0$, $R\left(\lambda\pm\nu+h\frac{d_j}{D_j}+h\frac{f_k}{F_k}\right)<0$, $j=1,2,\cdots,n_2$; $k=1,2,\cdots,n_3$,

$p_1\geqslant 0$, $p_2\geqslant m_2\geqslant 0$, $p_3\geqslant m_3\geqslant 0$, $q_1\geqslant 0$, $q_2\geqslant n_2\geqslant 0$, $q_3\geqslant n_3\geqslant 0$, $q_1+q_2\geqslant p_1+p_2$

and $q_1+q_3\geqslant p_1+p_3$.

$$(2.3)\quad \int_0^\infty x^{\rho-1}K_\mu(\alpha x)K_\nu(\alpha x)H\left[\begin{array}{c|c|c}\begin{bmatrix}0, & 0\\ p_1, & q_1\end{bmatrix} & (a_{p_1},A_{p_1}):(b_{q_1},B_{q_1}) & \\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix} & (c_{p_2},C_{p_2}):(d_{q_2},D_{q_2}) & \beta x^{2h},\delta x^{2h}\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix} & (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3}) & \end{array}\right]dx$$

$$=2^{\rho-3}\alpha^{-\rho}H\left[\begin{matrix}\begin{bmatrix}4,\ 0\\ p_1,\ q_1+1\end{bmatrix}\\ \\ \begin{pmatrix}m_2,\ n_2\\ p_2-m_2,\ q_2-n_2\end{pmatrix}\\ \begin{pmatrix}m_3,\ n_3\\ p_3-m_3,\ q_2-n_3\end{pmatrix}\end{matrix}\middle|\begin{matrix}\left(\frac{\rho+\mu+\nu}{2},h\right),\ \left(\frac{\rho-\mu+\nu}{2},h\right),\ \left(\frac{\rho+\mu-\nu}{2},h\right),\ \left(\frac{\rho-\mu-\nu}{2},h\right),\\ (a_{p_1},A_{p_1}):(b_{q_1},B_{q_1}),\ (\rho,\ 2h)\\ (c_{p_2},C_{p_2}):(d_{q_2},D_{q_2})\\ (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3})\end{matrix}\middle|\ \left(\frac{2}{\alpha}\right)^{2h}\beta,\ \left(\frac{2}{\alpha}\right)^{2h}\delta\right],$$

provided that $|\arg\beta|<\left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_2}C_j-\sum_{j=m_2+1}^{p_2}C_j+\sum_{j=1}^{n_2}D_j-\sum_{j=n_2+1}^{q_2}D_j\right)\frac{\pi}{2}$,

$|\arg\delta|<\left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_3}E_j-\sum_{j=m_3+1}^{p_3}E_j+\sum_{j=1}^{n_3}F_j-\sum_{j=n_3+1}^{q_3}F_j\right)\frac{\pi}{2}$,

$\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_2}C_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_2}D_j<0$; $\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0$,

$\mathrm{Re}(\alpha)>0$, $R\left(\rho\pm\mu\pm\nu+2h\frac{d_j}{D_j}+2h\frac{f_k}{F_k}\right)>0$, $(j=1,2,\cdots,n_2;k=1,2,\cdots,n_3)$,

$p_1\geqslant 0$, $p_2\geqslant m_2\geqslant 0$, $p_3\geqslant m_3\geqslant 0$, $q_1\geqslant 0$, $q_2\geqslant n_2\geqslant 0$, $q_3\geqslant n_3\geqslant 0$, $q_1+q_2\geqslant p_1+p_2$ and $q_1+q_3\geqslant p_1+p_3$.

$$(2.4)\quad \int_0^1 x^{\rho-1}(1-x)^{\beta-\gamma-n}{}_2F_1(-n,\beta;\gamma;x)H\left[\begin{matrix}\begin{bmatrix}0,\ 0\\ p_1,\ q_1\end{bmatrix}\\ \begin{pmatrix}m_2,\ n_2\\ p_2-m_2,\ q_2-n_2\end{pmatrix}\\ \begin{pmatrix}m_3,\ n_3\\ p_3-m_3,\ q_3-n_3\end{pmatrix}\end{matrix}\middle|\begin{matrix}(a_{p_1},A_{p_1}):(b_{q_1},B_{q_1})\\ (c_{p_2},C_{p_2}):(d_{q_2},D_{q_2})\\ (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3})\end{matrix}\middle|\ \delta x^h,\ \sigma\right]dx$$

$$=\frac{\Gamma(\gamma)\Gamma(\beta-\gamma+1)}{\Gamma(\gamma+n)}H\left[\begin{matrix}\begin{bmatrix}0,\ 0\\ p_1,\ q_1\end{bmatrix}\\ \begin{pmatrix}m_2+1,\ n_2+2\\ p_2-m_2+1,\ q_2-n_2\end{pmatrix}\\ \begin{pmatrix}m_3,\ n_3\\ p_3-m_3,\ q_3-n_3\end{pmatrix}\end{matrix}\middle|\begin{matrix}(a_{p_1},A_{p_1}):(b_{q_1},B_{q_1})\\ (1-\rho,h),(c_{p_2},C_{p_2}),(\gamma-\rho,h):\\ (\gamma-\rho+n,h),(\gamma-\beta-\rho,h),(d_{q_2},D_{q_2})\\ (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3})\end{matrix}\middle|\ \delta,\ \sigma\right]$$

where $|\arg\delta|<\left(-\sum_{q=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_2}C_j-\sum_{j=m_2+1}^{p_2}C_j+\sum_{j=1}^{n_2}D_j-\sum_{j=n_2+1}^{q_2}D_j\right)\frac{\pi}{2}$,

$|\arg\sigma|<\left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_3}E_j-\sum_{j=m_3+1}^{p_3}E_j+\sum_{j=1}^{n_3}F_j-\sum_{j=n_3+1}^{q_3}F_j\right)\frac{\pi}{2}$,

$\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_2}C_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_2}D_j<0$; $\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0$,

$R(\beta-\gamma)>n-1,\ n=0,1,2,\cdots;\ R\left(\rho+h\frac{d_j}{D_j}\right)>0,\ j=1,2,\cdots,n_2,$

$\gamma\neq 0,\ -1,\ -2,\cdots;\ p_1\geqslant 0,\ p_2\geqslant m_2\geqslant 0,\ p_3\geqslant m_3\geqslant 0,\ q_1\geqslant 0,\ q_2\geqslant n_2\geqslant 0,$
$q_3\geqslant n_3\geqslant 0,\ q_1+q_2\geqslant p_1+p_2$ and $q_1+q_3\geqslant p_1+p_3$.

$$(2.5)\quad \int_0^1 x^{\rho-1}(1-x)^{\beta-\rho-1}{}_2F_1(\alpha,\ \beta;\gamma;x)$$

$$\times H\left[\begin{array}{c|c|c} \begin{bmatrix}0,\ 0\\ p_1,\ q_1\end{bmatrix} & (a_{p_1},A_{p_1}):(b_{q_1},B_{q_1}) & \\ \begin{pmatrix}m_2,\ n_2\\ p_2-m_2,\ q_2-n_2\end{pmatrix} & (c_{p_2},C_{p_2}):(d_{q_2},D_{q_2}) & \dfrac{\delta x^h}{(1-x)^h},\ \sigma \\ \begin{pmatrix}m_3,\ n_3\\ p_3-m_3,\ q_3-n_3\end{pmatrix} & (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3}) & \end{array}\right]dx$$

$$=\frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\alpha)}H\left[\begin{array}{c|c|c} \begin{bmatrix}0,\ 0\\ p_1,\ q_1\end{bmatrix} & (a_{p_1},A_{p_1}):(b_{q_1},B_{q_1}) & \\ \begin{pmatrix}m_2+1,\ n_2+2\\ p_2-m_2+1,\ q_2-n_2\end{pmatrix} & \begin{array}{c}(1-\rho,h),\ (c_{p_2},C_{p_2})(\gamma-\rho,h):\\ (\beta-\rho,h),\ (\gamma-\alpha-\rho,h),\ (d_{q_2},D_{q_2})\end{array} & \delta,\ \sigma \\ \begin{pmatrix}m_3,\ n_3\\ p_3-m_3,\ q_3-n_3\end{pmatrix} & (e_{p_3},E_{p_3}):(f_{q_3},F_{q_3}) & \end{array}\right],$$

where $|\arg\delta|<\left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_2}C_j-\sum_{j=m_2+1}^{p_2}C_j+\sum_{j=1}^{n_2}D_j-\sum_{j=n_2+1}^{q_2}D_j\right)\frac{\pi}{2},$

$|\arg\sigma|<\left(-\sum_{j=1}^{p_1}A_j-\sum_{j=1}^{q_1}B_j+\sum_{j=1}^{m_3}E_j-\sum_{j=m_3+1}^{p_3}E_j+\sum_{j=1}^{n_3}F_j-\sum_{j=n_3+1}^{q_3}F_j\right)\frac{\pi}{2},$

$\sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_2}C_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_2}D_j<0;\ \sum_{j=1}^{p_1}A_j+\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_1}B_j-\sum_{j=1}^{q_3}F_j<0,$

$\mathrm{Re}\left(\rho+h\frac{d_j}{D_j}\right)>0;\ (j=1,2,\cdots,n_2),\ \mathrm{Re}(\gamma-\alpha-\beta)>0;\ \gamma\neq 0,-1,-2,\cdots,$

$R\left(\rho-\beta+h\frac{c_j-1}{c_j}+2\right)<0,\ (j=1,2,\cdots,\ m_2);\ p_1\geqslant 0,\ p_2\geqslant m_2\geqslant 0,\ p_3\geqslant m_3\geqslant 0,$

$q_1\geqslant 0,\ q_2\geqslant n_2\geqslant 0,\ q_3\geqslant n_3\geqslant 0,\ q_1+q_2\geqslant p_1+p_2$ and $q_1+q_3\geqslant p_1+p_3$.

PROOF. In order to prove the results (2.1), (2.2), (2.3), (2.4) and (2.5), we first express the generalised function of two variables from (1.1) in the integrals, change the order of integrations and interpreting the results on evaluating the inner integrals by virtue of the results (1.2), (1.3), (1.4), (1.5), and (1.6) respectively, we obtain the main results.

The change of order of integrations are justified [1] due to the absolute convergence of the integrals involved in the process.

3. Particular cases:

On setting $p_1=q_1=0$, we obtain a number of particular cases by using the property [6].

$$(3.1)\quad H\left[\begin{matrix}\begin{bmatrix}0, & 0\\ 0, & 0\end{bmatrix}\\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix}\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix}\end{matrix}\left|\begin{matrix}:\\ (a_{p_2}, A_{p_2}):(b_{q_2}, B_{q_2})\\ (c_{p_3}, C_{p_3}):(d_{q_3}, D_{q_3})\end{matrix}\right| x, \quad y\right]$$

$$=H^{n_2, m_2}_{p_2, q_2}\left[x\left|\begin{matrix}(a_{p_2}, A_{p_2})\\ (b_{q_2}, B_{q_2})\end{matrix}\right.\right]\times H^{n_3, m_3}_{p_3, q_3}\left[y\left|\begin{matrix}(c_{p_3}, C_{p_3})\\ (d_{q_3}, D_{q_3})\end{matrix}\right.\right],$$

where $H^{m, n}_{p, q}\left[x\left|\begin{matrix}(a_p, A_p)\\ (b_q, B_q)\end{matrix}\right.\right]$ is Fox's [4] generalisation of Meijer's G-function [5].

Thus the results (2.1), (2.2) and (2.3) reduces as follows:

$$(3.2)\quad \int_0^\infty x^{\lambda-1}K_\nu(\alpha x)H^{n_2, m_2}_{p_2, q_2}\left[\beta x^{2h}\left|\begin{matrix}(c_{p_2}, C_{p_2})\\ (d_{q_2}, D_{q_2})\end{matrix}\right.\right]H^{n_3, m_3}_{p_3, q_3}\left[\delta x^{2h}\left|\begin{matrix}(e_{p_3}, E_{p_3})\\ (f_{q_3}, F_{q_3})\end{matrix}\right.\right]dx$$

$$=2^{\lambda-2}\alpha^{-\lambda}H\left[\begin{matrix}\begin{bmatrix}2, & 0\\ 0, & 0\end{bmatrix}\\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix}\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix}\end{matrix}\left|\begin{matrix}\left(\frac{\lambda+\nu}{2}, h\right), \left(\frac{\lambda-\nu}{2}, h\right):\\ (c_{p_2}, C_{p_2}):(d_{q_2}, D_{q_2})\\ (e_{p_3}, E_{p_3}):(f_{q_3}, F_{q_3})\end{matrix}\right| \left(\frac{2}{\alpha}\right)^{2h}\beta, \left(\frac{2}{\alpha}\right)^{2h}\delta\right]$$

where $|\arg\beta|<\left(\sum_{j=1}^{m_2}C_j-\sum_{j=m_2+1}^{p_2}C_j+\sum_{j=1}^{n_2}D_j-\sum_{j=n_2+1}^{q_2}D_j\right)\frac{\pi}{2}$,

$|\arg\delta|<\left(\sum_{j=1}^{m_3}E_j-\sum_{j=m_3+1}^{p_3}E_j+\sum_{j=1}^{n_3}F_j-\sum_{j=n_3+1}^{q_3}F_j\right)\frac{\pi}{2}$,

$\sum_{j=1}^{p_1}C_j-\sum_{j=1}^{q_2}D_j<0$; $\sum_{j=1}^{p_3}E_j-\sum_{j=1}^{q_3}F_j<0$, $\mathrm{Re}(\alpha)>0$,

$\mathrm{Re}\left(\lambda\pm\nu+2h\frac{d_j}{D_j}+2h\frac{f_k}{F_k}\right)>0$, $(j=1, 2, \cdots, n_2;\ k=1, 2, \cdots, n_3)$,

$p_2\geqslant m_2\geqslant 0$, $p_3\geqslant m_3\geqslant 0$, $q_2\geqslant n_2\geqslant 0$, $q_3\geqslant n_3\geqslant 0$, $q_2\geqslant p_2$, and $q_3\geqslant p_3$.

$$(3.3)\quad \int_0^\infty x^{\lambda-1}e^{-\alpha x}K_\nu(\alpha x)H^{n_2, m_2}_{p_2, q_2}\left[\beta x^{h}\left|\begin{matrix}(c_{p_2}, C_{p_2})\\ (d_{q_2}, D_{q_2})\end{matrix}\right.\right]H^{n_3, m_3}_{p_3, q_3}\left[\delta x^{h}\left|\begin{matrix}(e_{p_3}, E_{p_3})\\ (f_{q_3}, F_{q_3})\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2^\lambda\alpha^\lambda}H\left[\begin{matrix}\begin{bmatrix}2, & 0\\ 0, & 1\end{bmatrix}\\ \begin{pmatrix}m_2, & n_2\\ p_2-m_2, & q_2-n_2\end{pmatrix}\\ \begin{pmatrix}m_3, & n_3\\ p_3-m_3, & q_3-n_3\end{pmatrix}\end{matrix}\left|\begin{matrix}(\lambda+\nu, h), (\lambda-\nu, h):\left(\lambda+\frac{1}{2}, h\right)\\ (e_{p_2}, c_{p_2}):(d_{q_2}, D_{q_2})\\ (e_{p_3}, E_{p_3}):(f_{q_3}, F_{q_3})\end{matrix}\right| \frac{\beta}{(2\alpha)^h}, \frac{\delta}{(2\alpha)^h}\right]$$

where $|\arg\beta| < \left(\sum_{j=1}^{m_2} C_j - \sum_{j=m_2+1}^{p_2} C_j + \sum_{j=1}^{n_2} D_j - \sum_{j=n_2+1}^{q_2} D_j\right)\frac{\pi}{2}$,

$|\arg\delta| < \left(\sum_{j=1}^{m_3} E_j - \sum_{j=m_3+1}^{p_3} E_j + \sum_{j=1}^{n_3} F_j - \sum_{j=n_3+1}^{q_3} F_j\right)\frac{\pi}{2}$, $\sum_{j=1}^{p_2} C_j - \sum_{j=1}^{q_2} D_j < 0$,

$\sum_{j=1}^{p_3} E_j - \sum_{j=1}^{q_3} F_j < 0$, $\mathrm{Re}(\alpha) > 0$, $R\left(\lambda \pm \nu + h\frac{d_j}{D_j} + h\frac{f_k}{F_k}\right) > 0$;

$(j=1, 2, \cdots, n_2;\ k=1, 2, \cdots, n_3)$, $p_2 \geqslant m_2 \geqslant 0$, $p_3 \geqslant m_3 \geqslant 0$, $q_2 \geqslant n_2 \geqslant 0$,

$q_3 \geqslant n_3 \geqslant 0$, $q_2 \geqslant p_2$ and $q_3 \geqslant p_3$.

$$(3.4)\quad \int_0^\infty x^{\rho-1} K_\mu(\alpha x) K_\nu(\alpha x) H_{p_2, q_2}^{n_2, m_2}\left[\beta x^{2h} \left| \begin{matrix} (c_{p_2}, C_{p_2}) \\ (d_{q_2}, D_{q_2}) \end{matrix} \right.\right] H_{p_3, q_3}^{n_3, m_3}\left[\delta x^{2h} \left| \begin{matrix} (e_{p_3}, E_{p_3}) \\ (f_{q_3}, F_{q_3}) \end{matrix} \right.\right] dx$$

$$= 2^{\rho-3}\alpha^{-\rho} H\left[\begin{matrix} \begin{bmatrix} 4, & 0 \\ 0, & 1 \end{bmatrix} \\ \begin{pmatrix} m_2, & n_2 \\ p_2-m_2, & q_2-n_2 \end{pmatrix} \\ \begin{pmatrix} m_3, & n_3 \\ p_3-m_3, & q_3-n_3 \end{pmatrix} \end{matrix} \left| \begin{matrix} \left(\frac{\rho+\mu+\nu}{2}, h\right), \left(\frac{\rho-\mu+\nu}{2}, h\right), \left(\frac{\rho+\mu-\nu}{2}, h\right), \\ \left(\frac{\rho-\mu-\nu}{2}, h\right) : (\rho, 2h) \\ (c_{p_2}, C_{p_2}) : (d_{q_2}, D_{q_2}) \\ (e_{p_3}, E_{p_3}) : (f_{q_3}, F_{q_3}) \end{matrix} \right| \left(\frac{2}{\alpha}\right)^{2h}\beta, \left(\frac{2}{\alpha}\right)^{2h}\delta \right],$$

provided that $|\arg\beta| < \left(\sum_{j=1}^{m_2} C_j - \sum_{j=m_2+1}^{p_2} C_j + \sum_{j=1}^{n_2} D_j - \sum_{j=n_2+1}^{q_2} D_j\right)\frac{\pi}{2}$,

$|\arg\delta| < \left(\sum_{j=1}^{m_3} E_j - \sum_{j=m_3+1}^{p_3} E_j + \sum_{j=1}^{n_3} F_j - \sum_{j=n_3+1}^{q_3} F_j\right)\frac{\pi}{2}$,

$\sum_{j=1}^{p_2} C_j - \sum_{j=1}^{q_2} D_j < 0$, $\sum_{j=1}^{p_3} E_j - \sum_{j=1}^{q_3} F_j < 0$; $p_2 \geqslant m_2 \geqslant 0$, $p_3 \geqslant m_3 \geqslant 0$, $q_2 \geqslant n_2 \geqslant 0$,

$q_3 \geqslant n_3 \geqslant 0$, $q_2 \geqslant p_2$, $q_3 \geqslant p_3$, $R(\alpha) > 0$, $R\left(\rho \pm \mu \pm \nu + 2h\frac{d_j}{D_j} + 2h\frac{f_k}{F_k}\right) > 0$

$(j=1, 2, \cdots, n_2;\ k=1, 2, \cdots, n_3)$.

The interesting results of Rathie [7] which themselves are the generalisation of Sharma's [10] results, follow as particular cases of our results (2.4) and (2.5) when $p_1 = q_1 = 0$.

In (2.2), if we put $h=1$, $A_{p_1} = B_{q_1} = C_{p_2} = D_{q_2} = E_{p_3} = F_{q_3} = 1$, we obtain a known result due to Sharma [9].

Again if $A_{p_1} = B_{q_1} = C_{p_2} = D_{q_2} = E_{p_3} = F_{q_3} = 1$, then the results (2.1) and (2.2) reduces to Sharma's [9] results by virtue of [3], where

$$S\left[\begin{matrix} \begin{bmatrix} m_1, & 0 \\ p_1-m_1, & q_1 \end{bmatrix} \\ \begin{pmatrix} m_2, & n_2 \\ p_2-m_2, & q_2-n_2 \end{pmatrix} \\ \begin{pmatrix} m_3, & n_3 \\ p_3-m_3, & q_3-n_3 \end{pmatrix} \end{matrix} \left| \begin{matrix} a_1, \cdots, a_{p_1} : b_1, \cdots, b_{q_1} \\ c_1, \cdots, c_{p_2} : d_1, \cdots, d_{q_2} \\ e_1, \cdots, e_{p_3} : f_1, \cdots, f_{q_3} \end{matrix} \right| x, y \right]$$

is the generalised function of two variables introduced by Sharma [8].

University of Jodhpur,
India

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