A NOTE ON COUNTABLE-DIMENSIONAL SPACES

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1. Introduction

In this paper we consider countable dimensional in a metric space. Up to the present we do not know so much about *infinite-dimensional* spaces as about *finite-dimensional* spaces. The purpose of this note is to extend some results of the theory of finite-dimensional spaces to countable-dimensional spaces. We shall give a brief study of countable-dimensional spaces.

2. Preliminary concepts.

Let be a collection in a topological space R and P a point of R. Then we mean by the order of **U** at P the number of members of **U** which contain P, and we denote it by ord $_P\mathbf{U}$.

If there exist infinitely many such members, then ord $_P\mathbf{U}=+\infty$. We denote B(U) as the boundary of U. If every closed set of a normal space R is a $G\delta$-set, then R is called perfectly normal.

2.1 Let $\mathbf{U}=\{U_\alpha|\alpha\in A\}$ be a locally finite open covering of a normal space R. Then there exists an open covering $\mathbf{B}=\{V_\alpha|\alpha\in A\}$ such that

$$V_\alpha\subset U_\alpha \text{ for every } \alpha\in A.$$

2.2 A space R is countable dimensional if and only if for every open collection $\{U_\alpha|\alpha<\tau\}$ and closed collection $\{F_\alpha|\alpha<\tau\}$ such that

$$F_\alpha\subset U_\alpha$$

and such that $\{U_\beta|\beta<\alpha\}$ is locally finite for every $\alpha<\tau$, there exists an open collection $\{V_\alpha|\alpha\vee\tau\}$ such that

$$F_\alpha\subset V_\alpha\subset U_\alpha$$

$$\text{ord }_P\{B(V_\alpha)|\alpha<\tau\}+\infty \text{ at every point } P\in R.$$

2.3 Let $\{F_i|i=1,2,\cdots\}$ be a closed covering of a space R such that

$$\text{Ind } F_i\leqq n,\quad i=1,2,\cdots$$

Then $\text{Ind } R\leqq n$

General notations and definitions can be found in [1], [2].

3. Countable-dimensional space.

A metric spase R is called countable-dimensional if $R=\bigcup_{i=1}^{\infty}R_i$ for some subspaces R_i of dimension$\leqq 0$.

Let A_n, $n=1,2,\cdots\cdots$ be a sequence of 0-dimensional sets of a space R.

Let $U=\{U_\alpha|\alpha<\tau\}$ be a locally finite open covering. Then we note also that there exists a closed covering $\mathbf{F}=\{F_\alpha|\alpha<\tau\}$ such that

$$F_\alpha\subset U_\alpha$$

$$\text{ord }_P\mathbf{F}\leqq n \text{ for every } P\in A_n$$

so we can obtain a proposition as the following.

THEOREM Let R be metric space. If R is countable-dimensional, then for every locally finite open convering: $\{U_\alpha | \alpha < \tau\}$ of R there exists a closed convering $\mathbf{F} = \{F_\alpha | \alpha < \tau\}$ of R such that

$$F_\alpha \subset U_\alpha \text{ for every } \alpha < \tau$$

$$\text{ord }_P \mathbf{F} < +\infty \text{ at every point } P \in R.$$

PROOF. Let R be countable-dimensional; Then R is decomposable into countably many O-dimensional spaces A_i, $i=1, 2, 3, \cdots\cdots$ For every locally finite open covering $\{U_\alpha | \alpha < \tau\}$ or R, by the normality of R, there exists a closed covering $\{H_\alpha | \alpha < \tau\}$ such that

$$H_\alpha \subset U_\alpha \text{ for every } \alpha < \tau,$$

by virtue of 2. 2 there exists an open covering $\mathbf{B} = \{V_\alpha | \alpha < \tau\}$ such that

$$H_\alpha \subset V_\alpha \subset U_\alpha$$

$$\text{ord }_P \{B(V_\alpha) | \alpha < \tau\} < +\infty \text{ at every point. } P \in R.$$

Let $F_0 = V_0$

$$F_\alpha = V_\alpha - \cup \{V_\beta | \beta < \alpha\} \text{ for } \alpha < \tau$$

then $\{F_\alpha | \alpha < \tau\}$ is the desired closed covering. Now we consider on strong inductive dimension of a metric space. If for any disjoint closed sets F and G of a topological space R there exists an open set U such that

$$F \subset U \subset R - G, \text{ Ind } B(U) \leqq n-1,$$

we say that R has strong inductive dimension$\leqq n$, Ind R$\leqq n$.

PROPOSITION. Let R be a metric space. If $\mathbf{U} = \{U_i\ i=1, 2, \cdots\cdots\}$ is an open covering of R such that Ind $U_i \leqq n$, then

$$\text{Ind } R \leqq n.$$

PROOF. Since R is perfectly normal, we decompose each U_i as $U_i = \bigcup_{j=1}^{\infty} F_i^j$ where F_j^j is closed sets. Then $\{F_i^j | i, j=1, 2, \cdots\cdots\}$ is a closed covering of R such that

$$\text{Ind } F_i^j \leqq n$$

by virtue of 2. 3, Ind R$\leqq n$.

The following is hold on a topological space in general. Here let A be a subset of a space R. We denote **C** A as the complement of A and IntA as the interior of A.

PROPOSITION Let R be a topological space

If D is a dense subset in R, then dim Int (CD) $= -1$.

PROOF Since non-empty basic open set in R contains an element of D, the complement of D has empty interior. Therefore dim Int (CD) $= -1$.

References

〔1〕 J. Nagata, Modern Dimension Theory, Interscience, New York, 1965.

〔2〕 W. Hurewicz and H. Wallman, Dimension Theory, Princeton, 1941.

요 약

제목 : A note on countable-dimensional spaces.

이 논문에서는 거리공간 R가 가산적인 차원이 될 필요한 조건을 얻었으며 다음에는 거리공간 R가 가산개의 개집합에 의하여 덮어지고 각 개집합이 강한 귀납적 차원이 n 이하면 R의 강한 귀납적 차원도 n 이하이다.