Linearized Ship Boundary Value Problems

by

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船舶流體力學의 線型理論에 關한 問題點들

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要 約

船舶流體力學의 境界條件의 線型化를 위한 퍼터베이션(Perturbation)方法을 따라 배를 가늘고 길게 보느냐, 엷은 水平板으로 보느냐 등등에 따라 여러가지 다른 種類의 數學的 定義에 따르는 理論을 추궁함으로서, 1) Neumann Ship, 2) Thin Ship, 3) Michell Ship, 4) Slender Ship 등을 理論的인 面에서와 數値計算 例로서 比較檢討하였다. 同時에 이에 따르는 "直接問題" "間接問題" 等에 關하여도 言及하고 그 差異點을 지적하였다. 이에 依하면 定速前進時에 L/B가 10인 경우에는 최대광폭의 差가 5~8%정도 되고 또 船首水線半角에 상당한 差가 있음으로 같은 理論的인 計算値에 對하여 抵抗은 적어도 10~15%정도 差가 생길 것이 아닌가 보게끔 되었다.

Introduction

In dealing with theoretical ship hydrodynamics, we assume mathematically more tractable potential flow with usual Laplace equation and boundary conditions given for the particular problem. For a completely submerged body, the theoretical as well as a practical problem of representing a hull by an equivalent potential, or an equivalent singulasity distribution as the potential can be so represented by a distribution of mathematical singularities, is an essentially solved one [1]. But when the free surface is involved, the problem is not only non-linear but also the boundary conditions have to be applied on the unknown surfaces. The problem must thereupon be solved by a perturbation technique, the first meaningful fallout of this being a linearized problem. The perturbation technique as we now have command, however, introduces approximations on the hull boundary condition different in nature associated with the particular problem at hand and conceivably with the method of attack subject to the particular methematical tract being used.

T. Inui was the first to realize that there is a significant differences between the Michell approximation and what he termed "the exact hull boundary condition," which in the present case being Neumann ship, or one of the different ships derivable [2, 3].

Also, the difference in choice of perturbation parameter has led J.J. Stoker to discuss three first order ship types such as "flat ship," "thin ship," and "yacht type ship" [4]. The significance of such distinction was also discussed at the International Seminar on Theoretical Wave-Resistance in Ann Arbor, 1963 [5].

The purpose of this paper is to point out more formally the differences arising from these approximations on the hull boundary conditions, examplified by a numerical example of a completely submerged singularity

Received: Feb. 15, 1970

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system moving at a constant forward speed.

In the present paper, only the steady case is discussed in some detail, but the result can be in principle extendable to motion problems as well.

Formulation of the Problem

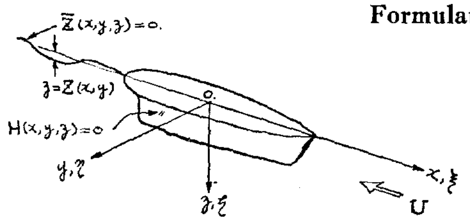

Fig. 1. Coordinate System

We fix the coordinate system in the ship to coincide with the free surface at rest as shown in Fig. 1, where x-axis is toward the bow; z-axis down; and y-axis to the starboard. Ship's constant forward speed is expressed by U in the direction as shown.

Denote the equation of the free surface by

$$Z(x,y,z)=0 \text{ or } z=Z(x,y) \tag{1}$$

and the portion of ship's hull surface in contact with water by

$$H(x,y,z)=0 \tag{2}$$

We assume Z and H are regular and has continuous first and second derivatives.

Further, we assume the flow to be inviscid and irrotational, and for the present case the fluid region extends to infinite half-space bounded by $z=0$ plane at rest. Then for the fluid we take a potential whose gradient expresses the velocity vector in the fluid:

$$\mathbf{u}=\nabla\cdot\phi(x,\ y,\ z) \tag{3}$$
$$=(u,\ v,\ w)$$

Hence the equation of motion is the well known Laplace equation

$$\nabla^2\phi=0 \tag{4}$$

The boundary conditions that must be satisfied are:

(i) Free surface condition

(ii) Ship hull surface condition

(iii) Radiation condition

(iv) Sea bed condition

The radiation condition will be taken so that there will be no waves ahead of the ship at infinity and the sea bed condition is the vanishing of normal velocity component at the sea floor, which in this case is at $z\to\infty$. Free surface and ship hull surface conditions are discussed in the following.

Free Surface Boundary Condition

The important characteristics of the free surface is that the pressure is prescribed but the form of the surface is not known *a priori*. The boundary conditions that must be satisfied are the kinematic and the dynamic conditions.

a. Kinematic condition

$$\frac{DZ}{Dt}=0 \qquad \text{at } Z=0 \tag{5}$$

where

$$\frac{D}{Dt}=\Phi_x\cdot\frac{\partial}{\partial x}+\Phi_y\cdot\frac{\partial}{\partial y}+\Phi_z\cdot\frac{\partial}{\partial z}$$

Therefore,

$$(\phi_x-U)Z_x+\Phi_y Z_y-\phi_z=0 \quad \text{at } z=Z \tag{6}$$

Where $\Phi=\phi-Ux$. Φ is the potential referred to a stationary coordinate system fixed in space and ϕ is that referred to the moving coordinate system fixed on the ship.

b. Dynamic condition

This is the Bernoulli's equation with the static pressure on the surface set equal to zero.

$$\frac{1}{2}(\Phi_x^2+\Phi_y^2+\Phi_z^2)+gZ=\frac{1}{2}U^2 \tag{7}$$

or

$$-U\phi_x+\frac{1}{2}q^2+gZ=0 \quad \text{on } z=Z \tag{7}'$$

where

$$q^2=u^2+v^2+w^2$$
$$=\phi_x^2+\phi_y^2+\phi_z^2$$

The above exact conditions, (6) and (7), are difficult to apply not only because the equation of the free surface is unknown but also the dynamic condition is non-linear and hence a superposition of soulution is not permitted. One of the favored techniques for attacking such a problem is by a formal development of hull equation, free, surface equation, potential, etc., in series with respect to "a small parameter," assuming the series converge.(2) This procedure for linearizing a non-linear problem is mathematically consistent and it gives means for determining any desired higher order corrections provided the "right parameter"(2) is chosen. This scheme, known as *perturbation procedure* or *perturbation technique*, will be used in the following for the two boundary conditions to correctly place the meaning of a study of the ship boundary value problems involving the free surface.

c. Perturbation

Assume henceforth that the velocity potential and the free surface equation possess the power series expansion is terms of a small parameter β as follows:

$$\phi(x, y, z)=\beta\phi^{(1)}(x, y, z)+\beta^2\phi^{(2)}(x, y, z)+\cdots\cdots \tag{8}$$

$$z=Z(x, y)=Z^{(0)}(x, y)+\beta Z^{(1)}(x, y)+\beta^2 Z^{(2)}(x, y)+\cdots\cdots \tag{9}$$

Also

$$\nabla^2\phi^{(n)}=0 \quad (n=1, 2, 3, \cdots\cdots) \tag{10}$$

The dynamic condition Eq. (7)′ becomes:

$$-U(\beta\phi_x^{(1)}+\beta^2\phi_x^{(2)}+\frac{1}{2}[(\beta\phi_x^{(1)}+\beta\phi_x^{(2)}+\cdots)^2+(\beta\phi_y^{(1)}+\beta^2\phi_y^{(2)}+\cdots)^2$$
$$+(\beta\phi_z^{(1)}+\beta^2\phi_z^{(2)}+\cdots)^2]+g(Z^{(0)}+\beta Z^{(1)}+\beta^2 Z^{(2)}+\cdots)=0$$

at

$$z=Z^{(0)}+\beta Z^{(1)}+\beta^2 Z^{(2)}+\cdots$$

Collecting terms of the same order with respect to β,

$$\text{For } \beta^0: \; gZ^{(0)}=0 \text{ at } z=Z^{(0)}, \text{ but } Z^{(0)}=0 \tag{11}$$

$$\text{For } \beta^1: \; -U\phi_x^{(1)}(x, y, \beta Z^{(1)})+gZ^{(1)}=0 \tag{12}$$
$$\text{at } z=\beta Z^{(1)}$$

For β^2: …………

……………

Note that the equation (12) collected in terms of order of β is to be satisfied at $z=Z^{(0)}$, since $Z^{(1)}=0[\phi^{(1)}]$ and the dimensional homogenuity must be observed.

Hence

$$-U\phi_x^{(1)}(x, y, 0)+gZ^{(1)}=0 \text{ at } z=0 \tag{12}'$$

The kinematic condition Eq. (6) becomes:

(1) J.J. Stoker, *Water Waves* (New York: Interscience, 1957), p. 19 and p. 269.

(2) Mathematically, the parameter needs only to be small compared to another number of unity. Its inter pretation in terms of a physical parameter is arbitrary. Professor Stoker interprets the term as "thick-ness-length ratio or draft-length ratio." In general it is considered to be the ratio of the wave height to the wave length.

$$(\beta\phi_x^{(1)}+\beta^2\phi_x^{(2)}+\cdots\cdots)(Z_x^{(0)}+\beta Z_x^{(1)}+\beta^2 Z_x^{(2)}+\cdots\cdots)$$
$$-U(Z_x^{(0)}+\beta Z_x^{(1)}+\beta^2 Z_x^{(2)}+\cdots)+(\beta\phi_y^{(1)}+\beta^2\phi_y^{(2)}+\cdots)(Z_y^{(0)}+\beta Z_y^{(1)}+\beta^2 Z_y^{(2)}+\cdots)$$
$$-(\beta\phi_z^{(1)}+\beta^2\phi_z^{(2)}+\cdots\cdots)=0 \quad \text{at} \quad z=Z^{(0)}+\beta^1 Z^{(1)}+\beta^2 Z^{(2)}+\cdots \tag{13}$$

Collecting terms of the same order with respect to β in the same manner as before,

$$\beta^0: \quad -UZ_x^{(0)}=0 \text{ at } z=Z_0=0 \tag{14}$$
$$\text{or} \quad Z_x^{(0)}=0 \tag{14)'$$
$$\beta^1: \quad \phi_x^{(1)}Z_x^{(0)}-UZ_x^{(1)}+\phi_y^{(1)}Z_y^{(0)}-\phi_z^{(1)}(x,\ y,\ 0)=0 \tag{15}$$
$$\text{at} \quad z=Z_0=0$$
$$\beta^2: \quad \cdots\cdots\cdots\cdots$$
$$\cdots\cdots\cdots\cdots\cdots$$

Eliminating the unknown surface Z_1 between Eqs. (12)′ and (15), where higher order terms like $\phi_x^{(1)}\ Z_x^{(0)}$ and $\phi_y^{(1)}\ Z_y^{(0)}$ are ignored we get a steady state free surface boundary condition:

$$-U^2\phi_{xx}^{(1)}-g\phi_z^{(1)}=0 \text{ at } \quad z=0 \tag{16}$$
$$\text{or} \quad \phi_{xx}+K_0\phi_z=0 \quad \text{at} \quad z=0 \tag{17}$$
$$K_0\equiv g/U^2$$

The equation (17) is the linearized boundary conditions on the free surface. Equations (11) and (14)′ signifies a rigid wall at the free surface and a double model. In terms of the perturbation procedure, the above shows that the linearized problem is of order one in β, whereas the Neumann problem is of the lower order $[\beta^\circ]$.

A physical interpretation of the sequential application of the perturbation procedure can be understood as a formal application of step-by-step corrections to the approximate initial solution, and therefore, it suggests reasonableness in assuming that the first term involving β° is greater than the second involving β^1, the second greater than the third and so forth.

The free surface boundary condition in (17) is linearized under the assumption that, according to the interpretation of Levi-Civita(3) and Struik(4), the amplitudes of the surface waves to be small compared to their wave lengths. This is interpreted to require at once that the ship must have a shape of thin disk or needle so that for a translational problem it can have a finite forward speed and still create a small disturbance or the forward speed itself must be approaching zero so that the free surface becomes a rigid wall.

Hull Boundary Condition

The hull surface boundary (2) is expressed as

$$H(x,y,z)\equiv y-h(x,z)=0 \tag{18}$$

The kinematic condition is

$$\frac{DH}{Dt}=0 \quad \text{on} \quad H=0 \tag{19}$$

$$\text{or} \quad -\Phi_x\cdot h_x+\Phi_y-\Phi_z\cdot h_z \quad \text{at } y=h(x,\ z) \tag{20}$$
$$\text{or} \quad -\phi_x\cdot h_x+\phi_y-\phi_z\cdot h_z=-Uh_x \quad \text{at } y=h \tag{21}$$

This is exact linear condition, and we shall call the ship satisfied by (21) a "Neumann Ship."

If now β is taken to be the beam-length ratio (B/L), where B is the total beam and L the half-length of the ship, the disturbance disappears as $\beta\rightarrow 0$, and it satisfies the assumptions of the finite forward speed and the small wave amplitudes.

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(3), (4) J.J. Stoker, *op. cit.*, p. 17 (secondary source)

Let $H(x, y, z) \equiv y - h(x, z) = y - \beta h^{(1)} - \beta^2 h^{(2)} - \cdots\cdots = 0$ (22)

where $y = \beta h(x, z) + \beta^2 h^{(2)} + \cdots\cdots$ is half-breadth of the ship, and β is small.

Since $\phi = \beta\phi^{(1)} + \beta^2\phi^{(2)} + \cdots\cdots$,

β^1 order term by a similar method becomes:

$$\phi_y^{(1)} = -Uh_x^{(1)} \quad \text{at } y = h^{(1)} \tag{23}$$

This is the "Thin Ship" approximation. Here $h_x^{(1)}$ and $h_z^{(1)}$ are assumed to be of the same order as $h^{(1)}(x,z)$, but this assumption is artificial and will have to be violated when applied to ordinary ship forms.

For a distribution of singularities on the longitudinal centerplane of a ship expressed as $\sigma(x,z)$, we have the following:[5]

$$\phi = \frac{1}{4\pi}\int_{-t}^{t}\int_{-l}^{l}\frac{\sigma(\xi,\zeta)d\xi d\zeta}{[(x-\xi)^2+y^2+(z-\zeta)^2]^{\frac{1}{2}}}$$
$$+\frac{1}{4\pi}\int_{-t}^{t}\int_{-l}^{l}\sigma(\xi,\zeta)\left\{[\text{P.V.}]\int_0^\infty\int_0^{2\pi}\frac{k+K_0\sec^2\theta}{k-K_0\sec^2\theta}\exp[-k(z+\zeta)-ik[(x-\zeta)\cos\theta+y\sin\theta]d\theta dk\right.$$
$$\left.+4K_0\int_0^{\frac{\pi}{2}}\exp[-K_0\sec^2\theta(z+\zeta)]\cos(K_0\sec^2\theta\, y\sin\theta)\sin[K_0\sec\theta\cdot(x-\xi)]d\theta\right\}d\xi d\zeta \tag{24}$$

where $-L \leq -l \leq \xi \leq l \leq L$

$-H \leq -t \leq \zeta \leq t \leq H$

$2L$: length of the ship

H: draft of the ship

$2l$: length of the singularity distribution plane; $2l \fallingdotseq 2L$.

t: depth of the distribution plane from the free surface

z: positive down:

$$\phi_y \equiv v = -\frac{1}{4\pi}y\int_{-t}^{t}\int_{-l}^{l}\frac{\sigma(\xi,\zeta)d\xi d\zeta}{[(x-\xi)^2+y^2+(z-\zeta)^2]^{3/2}}$$
$$+\frac{1}{4\pi}\int_{-t}^{t}\int_{-l}^{l}\sigma(\xi,\zeta)\left\{[\text{P.V.}]\int_0^\infty\int_0^{2\pi}\frac{k+K_0\sec^2\theta}{k-K_0\sec^2\theta}(-ik\sin\theta)\right.$$
$$\exp[-k(z+\zeta)-ik[(x-\xi)\cos\theta+y\sin\theta]]d\theta dk$$
$$\left.+4K_0^2\int_0^{\frac{\pi}{2}}-\exp[-K_0(z+\zeta)\sec^2\theta]\sec^2\theta\sin\theta\sin(K_0\sec^2\theta\, y\sin\theta)\sin[K_0\sec\theta(x-\xi)]d\theta\right\}d\xi d\zeta \tag{25}$$

Substituting the above into Eq. (23),

$$-Uh_x^{(1)} = [\text{Eq.}(25)] \tag{26}$$

Equation (26) is a Fredholm integral equation of the first kind in $\sigma(x, z)$ for a given thin ship (to the order β^1). The kernel of the equation is very complex but well behaved.

In the absence of the free surface terms, we have:

$$\int_{-t}^{t}\int_{-l}^{l}\frac{\sigma(\xi,\zeta)d\xi d\zeta}{[(x-\xi)^2+h^2(x,z)+(z-\zeta)^2]^{3/2}} = \frac{4\pi U}{h(x,z)}h_x(x,z) \tag{27}$$

It has often been said that a linearized theory to the order of β^1 gives a Michell ship, where β^1 is the beam-length ratio, but it can be shown that there is distinction. To show such a distinction, we take a singularity distribution $\sigma(x, y, z)$ on S surface and evaluate the y-component of velocity at a point (x, y, z) on S. In general we have then

$$v(=v_n) = \frac{1}{4\pi}\left[2\pi\sigma(x,y,z) - \frac{\partial}{\partial y}\int_S\frac{\sigma(\xi,\eta,\zeta)d\xi d\eta d\zeta}{[(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2]^{1/2}}\right. \tag{28}$$

Particularly at $y=0$ and for the centerplane distribution,

[5] For example, see J.K. Lunde, "On the Linearized Theory of Wave Resistance for Displacement Ships in Steady and Accelerated Motion," *Transactions of SNAME*, Vol. 59 (1951), p. 29, Eq. (3.20). To this the last term will be added to make the potential unique.

$$v \equiv \phi_y = \frac{1}{4\pi}\left[2\pi\sigma(x,z) - \frac{\partial}{\partial y}\int_S \frac{\sigma(\xi,\zeta)\,d\xi d\eta}{[(x-\xi)^2+y^2+(z-\zeta)^2]^{1/2}}\right. \qquad (29)$$

evaluated at $y=0$

Since the last term vanishes at $y=0$,

$$v = \frac{1}{2}\sigma(x,z)^{(6)} \qquad (30)$$

Using this, we have from Eq. (23),

$$\frac{1}{2}\sigma(x,z) = -Uh_x \quad \text{at} \quad y=0 \qquad (31)$$

or

$$\sigma(x,z) = -2Uh_x(x,z) \quad \text{at} \quad y=0 \qquad (32)$$

Equation (32) is the Michell approximation to the hull boundary condition, which even in the absence of the free surface effect, is clearly different from that of the first order thin ship, Eq. (27). As in the first order thin ship, when $\beta\to 0$, the "Michell Ship" disappears altogether, and it is of order β^1. The important fact is that the Michell approximation is derived only from the principal value right at the source and at the center-plane. It is a useful approximation to the solution of the integral equation, Eq. (27).

In the slender ship theory, the Laplace equation is assumed to take the following form right near the hull.[7]

$$\nabla^2\phi \fallingdotseq \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad \text{where} \quad \frac{\partial^2\phi}{\partial x^2} \approx 0 \qquad (33)$$

This means that the flow is two-dimensional on y-z plane and $u=\phi_x$ is independent function of x near the hull. This can be met if h_x is very small, and thus we can derive the "Slender Ship" theory as

$$\phi_y - \phi_z h_z = -Uh_x \qquad \text{at } y=h \qquad (34)$$

For a given form of singularity distribution, this becomes the slender ship approximation in a form of the integral equation of the first kind.

The thin ship and the slender ship approximations as originally conceptualized include the effect of free surface, but because the expressions are difficult to calculate, the whole expressions are seldom used. It is customary to neglect the free surface terms as we have done in the above, that is, Eq. (27) was used where Eq. (26) should have been, and the effect of free surface is added after the singularity distribution is found. The last process is really an indirect method. It can be used for all four cases, that is, Neumann, slender ship, thin ship and the Michell ship. The disadvantage of this lies in the fact that the hull form given by the singularity distribution so found depends on the speed, and hence there are infinitely many corresponding geometrical models. In order to avoid this, a complete expression such as Eq. (26) must be used, and consequently we now have only one model, but an infinite number of equivalent singularity distributions corresponding to it. It is undoubtedly the method which gives a single geometrical model that is consistent in theoretical foundations but more difficult to deal that we have to choose eventually.

The original Michell approximation does not contain the effect of the free surface. But, the free surface

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(6) Also from the Gauss flux theorem, the normal component of velocity at the source (x,y,z) due to a source distribution $\sigma'(x,y,z)$ in the absence of free surface is given by

$$4\pi\sigma'(x,y,z)\ ds = 2v ds$$

or $\quad v = 2\pi\sigma'(x,y,z)$.

The original Michell approximation is to be satisfied at $y=0$,

or $\quad v = 2\pi\sigma'(x,z)$, but $\sigma'(x,z) = \frac{1}{4\pi}\sigma(x,z)$ here.

(7) Hajime Maruo, "Calculation of the Wave Resistance of Ships, the Draught of Which is As Small As the Beam," *Transactions of Society of Naval Architects of Japan*, Vol. 112 (1962), pp. 21-37. More elegant treatment of this can be found in T. F. Ogilvie and E. O. Tuck, ***A Rational Strip Theory of Ship Motions***, Part 1. The Univetsity of Michigan, Department of Naval Architecture, Pub. No. 013 (March, 1969).

effect can be added to generate a different singularity distribution as in the following. For a three-dimensional case, using the linearized total potential for a source distribution $\sigma(x,z)$, and using Eq. (23) at $y=0$,

$$\frac{1}{2}\sigma(x,z)+\frac{1}{4\pi}\int_{-l}^{l}\int_{-l}^{l}\sigma(\xi,\xi)\,[\text{P.V.}]\int_{0}^{\infty}\int_{0}^{2\pi}\frac{k+K_o\sec^2\theta}{k-K_o\sec^2\theta}(-ik\sin\theta)$$
$$e^{-k(z+\zeta)-ik(x-\xi)\cos\theta}d\theta dkd\zeta d\xi=-Uh_x(x,z)\,|_{y=0} \qquad (35)$$

This is an integral equation of the second kind, which has a unique solution. Similarly, the formulae for the Neumann problem and the slender ship may be written down including the effect of the free surface.

Comparative Studies

Regarding all of the cases above, we can make up the following table (Table 1), where the expressions of the potentials for a source distribution of $\sigma(x,0,z)$ type

are: $$\phi=\frac{1}{4\pi}\int_{-l}^{l}\int_{-l}^{l}\frac{\sigma(\xi,\zeta)d\xi d\zeta}{[(x-\xi)^2+y^2+(z-\xi)^2]^{1/2}}$$

$\phi'=[\text{Eq}\cdot(24)]$

$\phi''=[\text{Eq}\cdot(24)$ with $\sigma_0(x,z)$ (known) instead of $\sigma(x,z)]$.

For a given hull, how well each of these cases will compare with one another is not known at this time. Certainly they present an interesting item to study. We might add that eight of these cases can be studied by the simpler indirect method, and that a comparison between the the Michell ship and the ship given by the

Table 1. 12 Cases of Comparative Studies

	Neumann Ship	Slender Ship	Thin Ship	Michell Ship
Without Free Surface β^0; $\sigma_0(x,z)$	$-\phi_x h_x+\phi_y$ $-\phi_z h_z=-Uh_x$	ϕ_y $-\phi_z h_z=-Uh_x$	$\phi_y=-Uh_x$	$-\frac{1}{2}2\sigma_0(x,z)=-Uh_x$ @ $y=0$
With Free Surface (Direct) β^1; $\sigma_1(x,z)$	$-\phi'_x h_x+\phi'_y$ $-\phi'_z h_z=-Uh_x$	ϕ'_y $-\phi'_z h_z=-Uh_x$	$\phi'_y=-Uh_x$	$\phi'_y=-Uh_x$ @ $y=0$
With Free Surface[a] (Indirect) β^1;$\sigma_0(x,y)+\cdots$	$-\phi''_x h_x+\phi''_y$ $-\phi''_z h_z=-Uh_x$	ϕ''_y $-\phi''_z h_z=-Uh_x$	$\phi''_y=-Uh_x$	$\phi'_y=-Uh_x$ @ $y=0$
Corresponding Streamline Equation	$\frac{dx}{u-U}=\frac{dy}{v}=\frac{dz}{w}$ $\mathbf{u}=\nabla\cdot\phi$	$\frac{dx}{-U}=\frac{dy}{v}=\frac{dz}{w}$	$\frac{dx}{-U}=\frac{dy}{v}$	$\frac{dx}{-U}=\frac{dy}{-\frac{1}{2}\sigma_0(x,z)}$

a Expressions are similar to those of direct method except the free surface effect is added after the corresponding singularity distribution in the absence of free surface is found.

Neumann problem for a simple given source distribution was made by T. Inui.[8] Indirect solutions are relatively easy to obtain and one such comparison is given in the following.

In order to show the distinction among the boundary conditions, we employ in the following an indirect method using a very simple source distribution in two-dimensions.

Take a simple source distribution corresponding to T. Inui's S-series,[9] i.e.

$$\sigma(\xi)=a\xi,\quad -l\leq\xi\leq l;\ l\approx L \qquad (36)$$

in a uniform flow.

We have then:

1) For the Michell ship (two dimensional) in the absence of free surface terms:

[8] T.Inui "Study on Wave-Making Resistance of Ships," 60*th Anniversary Volumes, Society of Naval Architects of Japan*, vol. 2, 1957, p. 173.

[9] Inui's S-series is a three-dimensional case. Eq. (36) is a two-dmiensional case.

$$\sigma(x)=ax=-2U\frac{dh(x)}{dx} \tag{37}$$

$$h(x)=-\frac{a}{4U}x^2+\text{constant}. \tag{37}$$

with $h(x)=0$ at $x=+L$ and $-L$,

$$h(x)=\frac{a}{4U}(L^2-x^2) \tag{39}$$

2) For the thin ship (two-dimensional) in the absence of free surface terms:

$$-\frac{v}{U}=\frac{dy}{dx}$$

$$\text{where} \quad y=h(x), \quad v=\frac{y}{4\pi}\int_{-l}^{l}\frac{a\xi d\xi}{[(x-\xi)^2+y^2]^{3/2}} \tag{40}$$

or

$$\int_{-l}^{l-L}\frac{a\xi d\xi}{[(x-\xi)^2+h^2(x)]^{3/2}}=-\frac{4\pi U}{h(x)}\frac{dh(x)}{dx} \tag{41}$$

where now $h(x)$ is the unknown to be found.

The integration with respect to ξ can be carried out explicitly [6].

3) For the Neumann ship (two-dimensional) in the absence of free surface terms:

$$\frac{v}{u-U}=\frac{dy}{dx} \tag{42}$$

$$\text{where} \quad v=\frac{y}{4\pi}\int_{-l}^{l}\frac{a\xi d\xi}{[(x-\xi)^2+y^2]^{3/2}}$$

$$u=\frac{1}{4\pi}\int_{-l}^{l}\frac{a(x-\xi)\xi d\xi}{[(x-\xi)^2+y^2]^{3/2}}$$

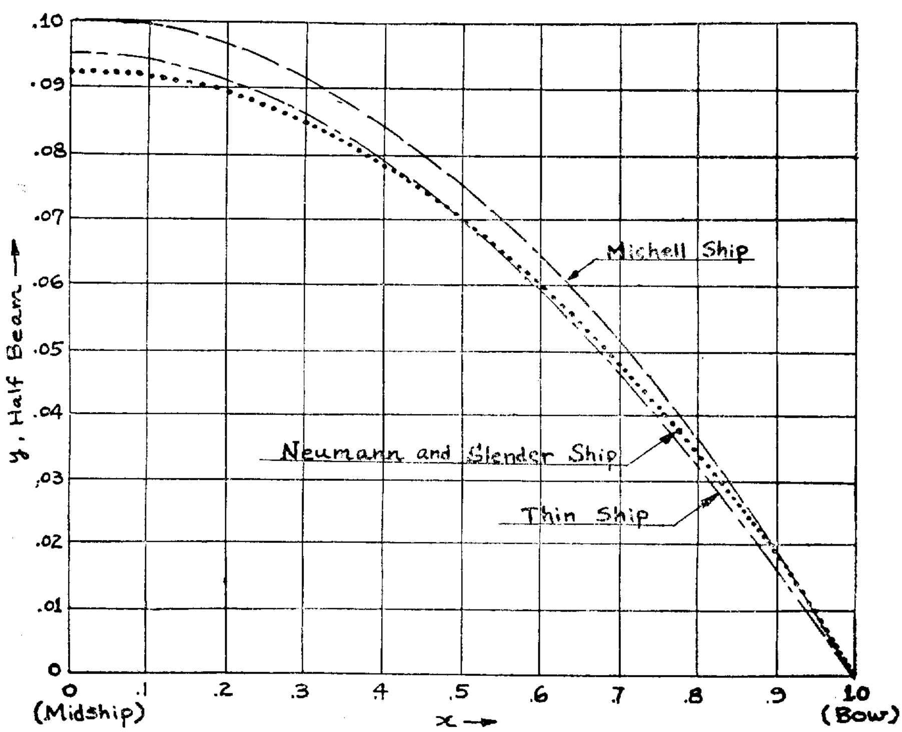

Fig. 2. Half Breadths for Various "Ships" from a 2-Dimensional Singularity Distribution $\sigma(\xi)=0.4\xi$

Also the integration can be carried out explicitly [6]. Result of numerical intergrations for Eqs. (41) and (42) are shown in Fig. 2, and Table 2 together with that of the Michell ship.

The figure is self-explanatory. The maximum difference in beam is approximately 5—8% depending on the "Ships" when $L/B \approx 10.0$. There is some diffence in half angle of entrance. Since the wave resistance is proportional to B^2, the difference in resistance must be more pronounced, say 10—15%.

We must, moreover, note that at a finite forward speed the Michell ship and the thin ship are assumed to have small amplitude waves according to the "linearization" based on the perturbation procedure, but that no such statement can be made for the Neumann ship. As can be seen in the figure, the Neumann ship fall in between the Michell ship and the thin ship with respect to the beam in the absence of free surface terms. Since it is not expected that the relationship of Fig. 2 will be greatly altered by the introduction of the free surface terms, we are led to believe that the waves created by the Neumann ship must also be small to the order of β^1.

In a three-dimensional case, the ship given by the Neumann problem may actually have a smaller beam than any other approximations. For a singularity distribution of S-101 type ($a=0.4$; depth of singularity distribution: $t=0.1$), the maximum beams of the thin ship, slender ship and the Neumann ship will not only be greatly reduced in general but also the difference among them will become smaller. The beam of the Michell ship, of course, does not vary from a two-dimensional case to a three-dimensional case.

Table 2. Comparison Among Michell Ship, Thin Ship and a Neumann Ship Using Simple Distribution.

x	MICHELL	NEUMANN	THIN[a]
1.00	0	.0002	.0002
.90	.0190	.0164	.0184
.80	.0360	.0324	.0343
.70	.0510	.0466	.0482
.60	.0640	.0592	.0601
.50	.0750	.0699	.0701
.40	.0841	.0788	.0782
.30	.0910	.0858	.0846
.20	.0960	.0910	.0892
.10	.0990	.0942	.0921
0	.1000	.0955	.0932

[a] The same computer program was used throughout.

Conclusion

A formal treatment of linearized ship boundary value problems yields "Neumann Ship," " Slender Ship," "Thin Ship," and "Michell Ship," as defined in the text. The distinction presents an interesting comparison. A numerical study of hull forms given by a two dimensional simple source distribution indicates characterestically different hull forms. The miximum difference in maximum beam amounts to approximately 8%. A comparative study for a three dimensional case is in order.

References

1. Hess, J.L. and A.M.O. Smith. *Calculation of Non-Lifting Potential Flow about Arbitrary Three-Dimensional Bodies*, Douglas Aircraft Div., Long Beach, Calif, 1962.

2. 乾崇夫 "正しい船型條件による 造造波抵抗理論の 新展開, 第一報" 日本造船協會論文集 85號 (1948).
3. Inui, T. "A New Theory of Wave-Making Resistance, Based on the Exact Condition of the Surface of Ships, Part II" *Journal of Zosen Kyokai*, vol. 93 (1953).
4. Stoker, J.J. *Water Waves*, New York: Interscience, 1957, pp. 245—269.
5. *Proceedings of International Seminar on Theoretical Wave-Resistance*, The University of Michigan, Ann Arbor, August, 1963.
6. Kim, Hun Chol. *Determination of the Non-Lifting Potential for An Arbitrary Ship Hull Form* (unpublished thesis), The University of Michigan, 1964