論 文

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六点給電안테나의 電流分布

(Current Distribution on a Six Points Fed Linear Antenna)

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要 約

本論文에서는 길이에 비해서 그 반지름을 無視할 수 있는 線形多波長 안테나에서 그 中央点에 關하여 對稱인 3雙의 給電点을 取하고 各雙마다에 同一한 起電力을 給電하는 6点給電안테나系의 電流分布를 Hallén의 逐次近似法에 따라서 理論的으로 解析하였으며, 이 結果 얻어진 電流分布式은 1次近似式으로서 일반전으로 6点給電안테나上 各部의 電流分布를 가늠하는 데 使用될 수 있을 것임을 例示하였다.

ABSTRACT

In this paper, the current distribution on a 6 points fed linear antenna is theoretically introduced. The antenna that we call the 6 points fed linear antenna is an antenna which are fed by emf E_1 at the first two points, emf E_2 at the second two points, and emf E_3 at the third two points. respectively symmetrical with respect to the center.

In this analysis, Hallėns theory has been extended with the approximation in the same order.

1. 序 論

그림 1 (a)와 같이 線形안테나의 中央点에 對해서 對稱인 세 雙의 点에 各雙마다 同一한 起電力 E_1, E_2 및 E_3을 給電한 6点給電 안테나의 電流分布와 給電임피이던스를 窮明할 수 있으면 그 結果를 그림 2와 같이 導體柱에 포울딛드 다이폴 안테나 素子들을 中央에 對해서 對稱으로 平行하게 固定한 안테나 系의 解析에 利用할 수가 있다.

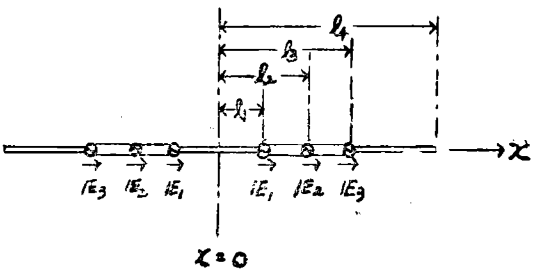

그림1. 六點給電線形안테나

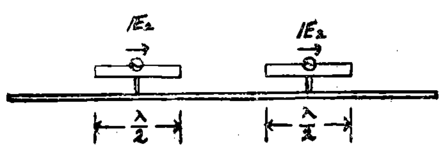

그림2. 포울딛드 다이폴 2段 取付 안테나

이러한 目的을 위하여 Hallèn과 同一한 手法으로 逐次近似式을 세워서 線形6点給電안테나의 電流分布式을 誘導 하였다.

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2. 理論解析

그림 1 과 같이 안테나의 軸方向을 x 軸으로 取하고 各 部分의 電流를 그림 3 (原안테나)의 記號와 같이 나타내기도 한다.

$\Phi_1(x)$를 導體表面의 $-l_1<x<l_1$ 部分의 Hertz 벡터, $\Phi_2(x)$를 $-l_2<x<-l_1$, $l_1<x<l_2$ 部分의, 또 $\Phi_3(x)$를 $-l_3<x<-l_2$, $l_2<x<l_3$ 部分의, $\Phi_4(x)$를 남어지部分 ($-l_4<x<-l_3$, $l_3<x<l_4$ 部分)의 Hertz 벡터라 한다면[1)]

$$\left.\begin{aligned}
\Phi_1(x)=&\frac{1}{j4\pi\omega\varepsilon}\left[\int_{-\ell_1}^{\ell_2}\frac{I_1(\xi)e^{-j\beta r_1}}{r_1}d\xi+\int_{l_2,\,l_1}^{-l_1,\,l_2}\frac{I_2(\xi)e^{-j\beta r_1}}{r_1}d\xi\right.\\
&\left.+\int_{-\ell_3,\,\ell_2}^{-\ell_2,\,\ell_3}\frac{I_3(\xi)e^{-j\beta r_1}}{r_1}d\xi+\int_{-\ell_4,\,\ell_3}^{-\ell_3,\,\ell_4}\frac{I_4(\xi)e^{-j\beta r_1}}{r_1}d\xi\right]\\
\Phi_2(x)=&\frac{1}{j4\pi\omega\varepsilon}\left[\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r_2}}{r_2}d\xi+\int_{-\ell_2,\,\ell_1}^{-\ell_1,\,\ell_2}\frac{I_2(\xi)e^{-j\beta r_2}}{r_2}d\xi\right.\\
&\left.+\int_{-\ell_3,\,\ell_2}^{-\ell_2,\,\ell_3}\frac{I_3(\xi)e^{-j\beta r_2}}{r_2}d\xi+\int_{-\ell_4,\,\ell_3}^{-\ell_3,\,\ell_4}\frac{I_4(\xi)e^{-j\beta r_2}}{r_2}\right]d\xi\\
\Phi_3(x)=&\frac{1}{j4\pi\omega\varepsilon}\left[\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r_3}}{r_3}d\xi+\int_{-\ell_2,\,\ell_1}^{-\ell_1,\,\ell_2}\frac{I_2(\xi)e^{-j\beta r_3}}{r_3}d\xi\right.\\
&+\int_{-\ell_3,\,\ell_2}^{-\ell_2,\,\ell_3}\frac{I_3(\xi)e^{-j\beta r_3}}{r_3}d\xi+\int_{-\ell_4,\,\ell_3}^{-\ell_3,\,\ell_4}\frac{I_4(\xi)e^{-j\beta r_3}}{r_3}d\xi\\
\Phi_4(x)=&\frac{1}{j4\pi\Phi\varepsilon}\left[\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r_4}}{r_4}d\xi+\int_{-\ell_2,\,\ell_1}^{-\ell_1,\,\ell_2}\frac{I_2(\xi)e^{-j\beta r_4}}{r_4}d\xi\right.\\
&\left.+\int_{-\ell_3,\,\ell_2}^{-\ell_2,\,\ell_3}\frac{I_3(\xi)e^{-j\beta r_4}}{r_4}d\xi+{}_{-\ell_4,\,\ell_3}^{-\ell_3,\,\ell_4}\frac{I_4(\xi)c^{-j\beta r_4}}{r_4}d\xi\right]
\end{aligned}\right\}\cdots\cdots(1)$$

但, β는 位相定數이며 減衰는 없다고 한다.
또

$$r_1=\sqrt{\rho_1^2+(x-\xi)^2},\quad r_2=\sqrt{\rho_2^2+(x-\xi)^2},$$
$$r_3=\sqrt{\rho_3^2+(x-\xi)^2},\quad r_4=\sqrt{\rho_4^2+(x-\xi)^2}\quad\cdots\cdots(2)$$

윗式에서는 x 軸上 任意点의 座標이며 ρ_1, ρ_2, ρ_3 ρ_4 는 안테나 各部分의 半지름이다.

이제 안테나가 完全導體라고 假定한다면 안테나 導體表面上의 電界中 表面에 對한 接線方向의 成分$E_x(x)$는 零이 된다. 따라서

$$\left.\begin{aligned}
E_x(x)&=\left(\frac{\partial^2}{\partial x^2}+\beta^2\right)\Phi_1(x)=o, && (o<|x|<\ell_1)\\
E_x(x)&=\left(\frac{\partial^2}{\partial x^2}+\beta^2\right)\Phi_2(x)=o, && (\ell_1<|x|<\ell_2)\\
E_x(x)&=\left(\frac{\partial^2}{\partial x^2}+\beta^2\right)\Phi_3(x)=o, && (\ell_2<|x|<\ell_3)\\
E_x(x)&=\left(\frac{\partial^2}{\partial x^2}+\beta^2\right)\Phi_4(x)=o, && (\ell_3<|x|<\ell_4)
\end{aligned}\right\}\cdots\cdots(3)$$

式(3)을 만족하는 解는

$$\left.\begin{aligned}
\Phi_1(x)&=A_1\cos\beta x+B_1\sin\beta|x|,\\
\Phi_2(x)&=A_2\cos\beta x+B_2\sin\beta|x|,\\
\Phi_3(x)&=A_3\cos\beta x+B_3\sin\beta|x|,\\
\Phi_4(x)&=A_4\cos\beta x+B_4\sin\beta|x|
\end{aligned}\right\}\cdots\cdots(4)$$

한편 理想的인 給電点의 경우를 생각하면(1)

$$\left.\begin{aligned}
o&=-2\frac{\partial\Phi_1(+o)}{\partial x}\\
E_1&=-\frac{\partial\Phi_2(\ell_1+o)}{\partial x}+\frac{\partial\Phi_1(\ell_1-o)}{\partial x}\\
E_2&=-\frac{\partial\Phi_3(\ell_{32}+o)}{\partial x}+\frac{\partial\Phi_2(\ell_2-o)}{\partial x}\\
E_3&=-\frac{\partial\Phi_4(\ell_3+o)}{\partial x}+\frac{\partial\Phi_3(\ell_3-o)}{\partial x}
\end{aligned}\right\}\cdots(5)$$

(4)식과 (5)식에서

$$\left.\begin{aligned}
o&=2\beta B_1\\
E_1&=\beta\{-(A_1-A_2)\sin\beta\ell_1+(B_1-B_2)\cos\beta\ell_1\}\\
E_2&=\beta\{-(A_2-A_3)\sin\beta\ell_2+(B_2-B_3)\cos\beta\ell_2\}\\
E_3&=\beta\{-(A_3-A_4)\sin\beta\ell_3+(B_3-B_4)\cos\beta\ell_3\}
\end{aligned}\right\}\cdots\cdots(6)$$

$A_1 \sim A_4$ 및 $B_1 \sim B_4$는 積分常數이며 (6)식과 안테나電流의 境界條件에 의하여 결정된다. 안테나의 電流分布를 결정하게 되는 (1)식의 究分方程式들을 풀기위하여서는 Hallèn의 逐次近似法[2]에 의하여 (1)식의 右邊을 다음과 같이 變形한다.

$$j4\pi\omega\varepsilon\Phi_1(x)=I_1(x)\int_{\ell_4}^{\ell_4}\frac{d\xi}{r_1}$$
$$+\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi$$
$$+\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I_2(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi$$
$$+\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I_3(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi$$
$$+\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi \cdots(7)$$

Hallèn과 마찬가지로 윗式의 右邊의 第1項 $|x|=\ell_3$ 올부근을 除外하고 다음과 같이 나타낸다.

$$\int_{-\ell_4}^{\ell_4}\frac{d\xi}{r_1}\doteqdot\Omega_1+ln\frac{\ell^2{}_4-x^2}{\ell_4{}^2},$$
$$\text{단 }\Omega_1=2ln\frac{2\ell_4}{\rho_1}\quad\cdots\cdots\cdots\cdots(8)$$

여기서 r_1 대신 $|x-\xi|$를 대입하여도 안테나의 길이에 대한 굵기의 비율 정도의 誤差밖에는 나지 않는다.

(4)식과 (8)식을 (7)식에 代入하므로서

$$I_1(x)=\frac{j4\pi\omega\varepsilon}{\Omega_1}\{A_1cos\beta x+B_1sin\beta|x|\}$$
$$-\frac{I_1(x)}{\Omega_1}ln\frac{\ell_4{}^2-x^2}{\ell_4{}^2}$$
$$-\frac{1}{\Omega_1}\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi$$
$$-\frac{1}{{}_1\Omega}\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I_2(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi$$
$$-\frac{1}{\Omega_1}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I_3(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi$$
$$-\frac{1}{\Omega_1}\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_1}-I_1(x)}{r_1}d\xi$$
$$\cdots\cdots\cdots\cdots(9)$$

같은 方法으로

$$I_2(x)=\frac{j4\pi\omega\varepsilon}{\Omega_2}\{A_2cos\beta x+B_2sin\beta|x|\}$$
$$-\frac{I_2(x)}{\Omega_2}ln\frac{\ell_4{}^2-x^2}{\ell_4{}^2}$$
$$-\frac{1}{\Omega_2}\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r_2}-I_2(x)}{r_2}d\xi$$
$$-\frac{1}{\Omega_2}\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I_2(\xi)e^{-j\beta r_2}-I^2(x)}{r_2}d\xi$$
$$-\frac{1}{\Omega_2}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I_3(\xi)e^{-j\beta r_2}-I_2(x)}{r_2}d\xi$$
$$-\frac{1}{\Omega_2}\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_2}-I_2(x)}{r_2}d\xi$$
$$\cdots\cdots\cdots\cdots(10)$$

단, $\Omega_2=2\ell n\frac{2\ell_4}{\rho_2}$

$$I_3(x)=\frac{j4\pi\omega\varepsilon}{\Omega_3}\{A_3cos\beta x+B_3sin\beta|x|\}$$
$$-\frac{I_3(x)}{\Omega_3}\ell n\frac{\ell_4{}^2-x^2}{\ell_4{}^2}$$
$$-\frac{1}{\Omega_3}\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r}-I_3(x)}{r_3}d\xi$$
$$-\frac{1}{\Omega_3}\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I_2(\xi)e^{-j\beta r3}-I_3(x)}{r_3}d\xi$$
$$-\frac{1}{\Omega_3}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I_3(\xi)e^{-j\beta r3}-I_3(x)}{r_3}d\xi$$
$$-\frac{1}{\Omega_3}\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r3}-I_3(x)}{r_3}d\xi$$
$$\cdots\cdots\cdots\cdots(11)$$

단, $\Omega_3=2ln\frac{2\ell_4}{\rho_3}$

$$I_4(x)=\frac{j4\pi\omega\varepsilon}{\Omega_4}\{A_4cos\beta x+B_4sin\beta|x|\}$$
$$-\frac{I_4(x)}{\Omega_4}ln\frac{\ell_4{}^2-x^2}{\ell_4{}^2}$$
$$-\frac{1}{\Omega_4}\int_{-\ell_1}^{\ell_1}\frac{I_1(\xi)e^{-j\beta r_4}-I_4(x)}{r_4}d\xi$$
$$-\frac{1}{\Omega_4}\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I_2(\xi)e^{-j\beta r_4}-I_4(x)}{r_4}d\xi$$
$$-\frac{1}{\Omega_4}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I_3(\xi)e^{-j\beta r_4}-I_4(x)}{r_4}d\xi$$
$$-\frac{1}{\Omega_4}\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_4}-I_4(x)}{r_4}d\xi$$
$$\cdots\cdots\cdots\cdots(12)$$

단, $\Omega_4=2ln\frac{2\ell_4}{\rho_4}$

다음에

$$\left.\begin{aligned}I_1'(x)&=I_1(x)-I_1(\ell_1)\\I_2'(x)&=I_2(x)-I_2(\ell_2)\\I_3'(x)&=I_3(x)-I_3(\ell_3)\\I_4'(x)&=I_4(x)-I_4(\ell_4)=I_4(x)\end{aligned}\right\}\cdots\cdots(13)$$

라고 노면 電流의 境界條件에 의하여

$I_1(\ell_1)=I_2(\ell_1),\ I_2(\ell_2)=I_3(\ell_2),$

$I_3(\ell_3)=I_4(\ell_3),\ I_4(\ell_4)=0$

(9)~(12)식을 (13)식에 대입하면

$$I_1'(x)=\frac{j4\pi\omega\varepsilon}{\Omega_1}\{A_1(\cos\beta x-\cos\beta\ell_1)+B_1(\sin\beta|x|-\sin\beta\ell_1)\}$$
$$-\frac{I_1'(x)}{\Omega_1}\ln\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$-\frac{1}{\Omega_1}\Big[\int_{-\ell_1}^{\ell_1}\frac{I_1'(\xi)e^{-j\beta r_1}-I_1'(x)}{r_1}d\xi$$
$$-\int_{-\ell_4,\ell_1}^{-\ell_1,\ell_4}\frac{I_1'(x)}{r_1}d\xi\Big]_{\ell_1}^{x}$$
$$-\frac{1}{\Omega_1}\Big[I_2'(\ell_1)\ln\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I_2'(\xi)e^{-j\beta r_1}-I_2'(\ell_1)}{r_1}d\xi$$
$$+I_2'(\ell_1)\int_{-\ell_1}^{\ell_1}\frac{e^{-j\beta r_1}-1}{r_1}d\xi$$
$$-I_2'(\ell_1)\int_{-\ell_4,\ell_2}^{-\ell_2,\ell_4}\frac{d\xi}{r_1}\Big]_{\ell_1}^{x}$$
$$-\frac{1}{\Omega_1}\Big[\{I_3'(\ell_2)\ell n\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I'_3(\xi)e^{-j\beta r_1}-I_3(\ell_2)}{r_1}d\xi$$
$$+I'_3(\ell_2)\int_{-\ell_2}^{\ell_2}\frac{e^{-j\beta r_1}-1}{r_1}d\xi$$
$$-I_3'(\ell_2)\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{d\xi}{r_1}\Big]_{\ell_1}^{x}$$
$$-\frac{1}{\Omega_1}\Big[\{I_4(\ell_3)\ln\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_1}-I_4(\ell_3)}{r_1}d\xi$$
$$+I_4(\ell_3)\int_{-\ell_3}^{\ell_3}\frac{e^{-j\beta r_1}-1}{r_1}d\xi\Big]_{\ell_1}^{x}\cdots\cdots(14)$$

$$I_2'(x)=\frac{j4\pi\omega\varepsilon}{\Omega_2}\{A_2(\cos\beta x-\cos\beta\ell_2)+B_2(\sin\beta|x|-\sin\beta\ell_2)\}$$
$$-\frac{1}{\Omega_2}\Big[\int_{-\ell_1}^{\ell_1}\frac{I_1'(\xi)e^{-j\beta r_2}}{r_2}d\xi\Big]_{\ell_2}^{x}$$
$$-\frac{1}{\Omega_2}\Big[I_2'(x)\ln\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_1}^{\ell_1}\frac{I'_2(\ell_1)e^{-j\beta r_2}-I'_2(x)}{r_2}d\xi$$
$$+\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I'_2(\xi)e^{-j\beta r_2}-I'_2(x)}{r_2}d\xi$$
$$-\int_{-\ell_4,\ell_2}^{-\ell_2,\ell_4}\frac{I'_2(x)}{r_2}d\xi\Big]_{\ell_2}^{x}$$
$$-\frac{1}{\Omega_2}\Big[\{I'_3(\ell_2)\ln\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I'_3(\xi)e^{-j\beta r_2}-I'_3(\ell_2)}{r_2}d\xi$$
$$+I'_3(\ell_2)\int_{-\ell_2}^{\ell_2}\frac{e^{-j\beta r_2}-1}{r_2}d\xi$$
$$-I'_3(\ell_2)\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{d\xi}{r_2}\Big]_{\ell_2}^{x}$$
$$-\frac{1}{\Omega_2}\Big[I_4(\ell_3)\ell n\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_3}-I_4(\ell_3)}{r_2}d\zeta$$
$$+I_4(\ell_3)\int_{-\ell_3}^{\ell_3}\frac{e^{-j\beta r_2}-1}{r_2}d\xi\Big]_{\ell_2}^{x}\cdots\cdots(15)$$

$$I'_3(x)=\frac{j4\pi\omega\varepsilon}{\Omega_3}\{A_3(\cos\beta x-\cos\beta\ell_3+B_3(\sin\beta|x|-\sin\beta\ell_3))\}$$
$$-\frac{1}{\Omega_3}\Big[\int_{-\ell_1}^{\ell_1}\frac{I'_1(\xi)e^{-j\beta r_3}}{r_3}d\xi\Big]_{\ell_3}^{x}$$
$$-\frac{1}{\Omega_3}\Big[I'_2(\ell_1)\int_{-\ell_1}^{\ell_1}\frac{e^{-j\beta r_3}}{r_3}d\xi$$
$$+\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I'_2(\xi)e^{-j\beta r_3}}{r_3}d\xi\Big]_{\ell_3}^{x}$$
$$-\frac{1}{\Omega_3}\Big[I'_3(x)\ln\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_2}^{\ell_2}\frac{I'_3(\ell_2)e^{-j\beta r_3}-I'_3(x)}{r_3}d\xi$$
$$+\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I'_3(\xi)e^{-j\beta r_3}-I'_3(x)}{r_3}d\xi$$
$$-\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I'^3(x)}{r_3}d\xi\Big]_{\ell_3}^{x}$$
$$-\frac{1}{\Omega_3}\Big[I_4(\ell_3)\ln\frac{\ell_4^2-x^2}{\ell_4^2}$$
$$+\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_3}-I_4(\ell_3)}{r_3}d\xi$$
$$+I_4(\ell_3)\int_{-\ell_3}^{\ell_3}\frac{e^{-j\beta r_3}-1}{r_3}d\xi\Big]_{\ell_3}^{x}\cdots\cdots(16)$$

$$I'_4(x)=\frac{j4\pi\omega\varepsilon}{\Omega_4}\{A_4(\cos\beta x-\cos\beta\ell_4)+B_4(\sin\beta|x|-\sin\beta\ell_4)\}$$
$$-\frac{1}{\Omega_4}\Big[\int_{-\ell_1}^{\ell_1}\frac{I'_1(\xi)e^{-j\beta r_4}}{r_4}d\xi\Big]_{\ell_4}^{x}$$
$$-\frac{1}{\Omega_4}\Big[I'_2(\ell_1)\int_{-\ell_1}^{\ell_1}\frac{e^{-j\beta r_4}}{r_4}d\xi$$
$$+\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{I'_2(\xi)e^{-j\beta r_4}}{r_4}d\xi\Big]_{\ell_4}^{x}$$

$$-\frac{1}{\Omega_4}\Big[I'_3(\ell_2)\int_{-\ell_2}^{\ell_2}\frac{e^{-j\beta r_4}}{r_4}d\xi + \int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{I'_3(\xi)e^{-j\beta r_4}}{r_4}d\xi\Big]_{x4}$$
$$-\frac{1}{\Omega_4}\Big[I_4(x)\ ln\frac{\ell_4^2-x^2}{\ell_4^2} + \int_{\ell_3}^{-\ell_3}\frac{I_4(\ell_3)e^{-j\beta r_4}-I_4(x)}{r_4}d\xi + \int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{I_4(\xi)e^{-j\beta r_4}-I_4(x)}{r_4}d\xi\Big]_{x4}$$

....(17)

윗式들에서 第1項을 電流의 Zero次 近似式이라고 생각할 수 있고 이 값들을 각각 남어지 項에 代入해줌으로서 電流의 1次 近似式이 얻어진다. 이와같은 代入을 계속 하여 가면 n次近似式도 얻을 수 있을 것이다. 이 過程을 간단히 數學的으로 나타내기 위하여 다음과 같은 Operator를 定義한다.

$$P_{11}\{F_1(x)\}=-\frac{1}{\Omega_1}\{F_1(x)-F_1(\ell_1)\}\ ln\frac{\ell_1^2-x^2}{\ell_4^2} -\frac{1}{\Omega_1}\int_{-\ell_1}^{\ell_1}\frac{\{F_1(\xi)-F_1(\ell_1)\}e^{-j\beta r_1}-\{F_1(x)-F_1(\ell_1)\}}{r_1}d\xi$$
$$P_{12}\{F_2(x)\}=-\frac{1}{\Omega_1}\int\{F_2(\ell_1)-F_2(\ell_2)\}\{\ ln\frac{\ell_2^2-x^2}{\ell_4^2}+\int_{-\ell_1}^{\ell_1}\frac{e^{-j\beta r_1}-1}{r_1}d\xi\} -\frac{1}{\Omega_1}{}_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{\{F_2(\xi)-F_2(\ell_2)\}e^{-j\beta r_1}-\{F_2(\ell_1)-F_2(\ell_2)\}}{r_1}d\xi$$
$$P_{12}\{F_3(x)\}=-\frac{1}{\Omega_1}\{F_3(\ell_2)-F_3(\ell_3)\}\{\ ln\frac{\ell_3^2-x^2}{\ell_4^2}+\int_{-\ell_2}^{\ell_2}\frac{e^{-j\beta r_1}-1}{r_1}d\xi\} -\frac{1}{\Omega_1}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{\{F_3(\xi)-F_3(\ell_3)\}e^{-j\beta r_1}-\{F_3(\ell_2)-F_3(\ell_3)\}}{r_1}d\xi$$
$$P_{14}\{F_4(x)\}=-\frac{1}{\Omega_1}\{F_4(\ell_3)-F_4(\ell_4)\}\{\ ln\frac{\ell_4^2-x^2}{\ell_4^2}+\int_{-\ell_3}^{\ell_3}\frac{e^{-j\beta r_1}-1}{r_1}d\xi\} -\frac{1}{\Omega_1}{}_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{\{F_4(\xi)-F_4(\ell_4)\}e^{-j\beta r_1}-\{F_4(\ell_3)-F_4(\ell_4)\}}{r_1}d\xi$$

..............(18)

$$P_{21}\{F_1(x)\}=-\frac{1}{\Omega_2}\int_{-l_1}^{l_1}\frac{\{F_1(\xi)-F_1({}_1\ell)\}e^{-j\beta r_2}}{r_2}d\xi$$
$$P_{22}\{F_2(x)\}=-\frac{1}{\Omega_2}\int\{F_2(x)-F_2(\ell_2)\}\ ln\frac{\ell_2^2-x^2}{l_4^2} -\frac{1}{\Omega_2}\int_{-\ell_1}^{\ell_1}\frac{\{F_2(\ell_1)-F_2({}_2)\}e^{-j\beta r_2}\ell_2-\{F_2(x)-{}_2F(\ell_2)\}}{r_2}d\xi -\frac{1}{\Omega_2}\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_1}\frac{\{F_2(\xi)-F_2(\ell_2)\}e^{-j\beta r_2}-\{F_2(x)-F_2(\ell_2)\}}{r_2}d\xi$$
$$P_{23}\{F_3(x)\}=-\frac{1}{\Omega_2}\{F_3(\ell_2)-F_3(\ell_3)\}\{\ell n\frac{\ell_3^2-x^2}{\ell_4^2}+\int_{-\ell_2}^{\ell_2}\frac{e^{-j\beta r_2}-1}{r_2}d\xi\} -\frac{1}{\Omega_2}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{F_3(\xi)-F_3(\ell_3)\}e^{-j\beta r_2}-\{F_3(\ell_2)-F_3(\ell_3)\}}{r_2}d\xi$$
$$P_{24}\{F_4(x)\}=-\frac{1}{\Omega_2}\{F_4(\ell_3)-F_4(\ell_4)\}\ ln\frac{\ell_4^2-x^2}{\ell_4^2}+\int_{-\ell_3}^{\ell_3}\frac{e^{-j\beta r_4}-1}{r_2}d\xi -\frac{1}{\Omega_2}\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{\{F_4(\xi)-F_4(\ell_4)\}e^{-j\beta r_2}-\{F_4(\ell_3)-F_4(\ell_4)\}}{r_2}d\xi$$

..............(19)

$$P_{31}\{F_1(x)\}=-\frac{1}{\Omega_3}\int_{-\ell_1}^{\ell_1}\frac{\{F_1(\xi)-F_1(\ell_1)\}e^{-j\beta r_2}}{r_3}d\xi$$
$$P_{32}\{F_2(x)\}=-\frac{1}{\Omega_3}\int_{-\ell_1}^{\ell_1}\frac{\{F_2(\ell_1)-F_2(\ell_2)e^{\ j\beta r_3}}{r_3}d\xi$$

$$-\frac{1}{\Omega_3}\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{\{F_2(\xi)-F_2(\ell_2)\}e^{-j\beta r_3}}{r_3}d\xi$$

$$P_{33}\{F_3(x)\}=-\frac{1}{\Omega_3}\{F_3(x)-F_3(\ell_3)\}\ ln\frac{\ell_3^2-x^2}{\ell_4^2}$$

$$-\frac{1}{\Omega_3}\int_{-\ell_2}^{\ell_2}\frac{\{F_3(\ell_2)-F_3(\ell_3)\}e^{-j\beta r_3}-\{F_3(x)-F_3(\ell_3)\}}{r_3}d\xi$$

$$-\frac{1}{\Omega_3}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{\{F_3(\xi)-F_3(\ell_3)\}e^{-j\beta r_3}-\{F_3(x)-F_3(\ell_3)\}}{r_3}d\xi$$

$$P_{34}\{F_4(x)\}=-\frac{1}{\Omega_3}\{F_4(\ell_3)-F_4(\ell_4)\}\{\ ln\frac{\ell_4^2-x^2}{\ell_4^2}+\int_{-\ell_3}^{\ell_3}\frac{e^{-j\beta r_3}-1}{r_3}d\xi\}$$

$$-\frac{1}{\Omega_3}\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{\{F_4(\xi)-F_4(\ell_4)\}e^{-j\beta r_3}-\{F_4(\ell_3)-F_4(\ell_4)\}}{r_3}d\xi \quad \cdots\cdots(20)$$

$$P_{41}\{F_1(x)\}=-\frac{1}{\Omega_4}\int_{-\ell_1}^{\ell_1}\frac{\{F_1(\xi)-F_1(\ell_1)\}e^{-j\beta r_4}}{r_4}d\xi$$

$$P_{42}\{F_2(x)\}=-\frac{1}{\Omega_4}\int_{-\ell_1}^{\ell_1}\frac{\{F_2(\ell_1)-F_2(\ell_2)\}e^{-j\beta r_4}}{r_4}d\xi$$

$$-\frac{1}{\Omega_4}\int_{-\ell_2,\ell_1}^{-\ell_1,\ell_2}\frac{\{F_2(\xi)-F_2(\ell_2)\}e^{-j\beta r_4}}{r_4}d\xi$$

$$P_{43}\{F_3(x)\}=-\frac{1}{\Omega_4}\int_{-\ell_2}^{\ell_2}\frac{\{F_3(\ell_2)-F_3(\ell_3)e^{-j\beta r_4}}{r_4}d\xi$$

$$-\frac{1}{\Omega_4}\int_{-\ell_3,\ell_2}^{-\ell_2,\ell_3}\frac{\{F_3(\xi)-F_3(\ell_3)\}e^{-j\beta r_4}}{r_4}d\xi$$

$$P_{44}\{F_4(x)\}=-\frac{1}{\Omega_4}\{F_4(x)-F_4(\ell_4)\}\ ln\frac{\ell_4^2-x^2}{\ell_4^2}$$

$$-\frac{1}{\Omega_4}\int_{-\ell_3}^{\ell_3}\frac{\{F_4(\ell_3)-F_4(\ell_4)\}e^{-j\beta r_4}-\{F_4(x)-F_4(\ell_4)\}}{r_4}d\xi$$

$$-\frac{1}{\Omega_4}\int_{-\ell_4,\ell_3}^{-\ell_3,\ell_4}\frac{\{F_4(\xi)-F_4(\ell_4)\}e^{-j\beta r_4}-F_4(x)-F_4(\ell_4)\}}{r_4}d\xi \quad \cdots\cdots(21)$$

(18)~(21) 식을 (14)~(17)식에 대입하면

$$I_1'(x)=\Big[\frac{j4\pi\omega\varepsilon}{\Omega_1}\{A_1cos\beta x+B_1sin\beta|x|\}+P_{11}\{I_1'(x)\}+P_{12}\{I_2'(x)\}+P_{13}\{I_3'(x)\}+P_{14}\{I_4(x)\}\Big]_{\ell_1}^{x}$$

$$I_2'(x)=\Big[\frac{j4\pi\omega\varepsilon}{\Omega_2}\{A_2cos\beta x+B_2sin\beta|x|\}+P_{21}I_1'(x)\}+P_{22}\{I_2'(x)\}+P_{23}\{I_3'(x)\}+P_{24}\{I_4(x)\}\Big]_{\ell_2}^{x}$$

$$I_3'(x)=\Big[\frac{j4\pi\omega\varepsilon}{\Omega_3}\{A_3cos\beta x+B_3sin\beta|x|\}+P_{31}\{I_1'(x)\}+P_{32}\{I_2'(x)\}+P_{33}\{I_3'(x)\}+P_{34}\{I_4(x)\}\Big]_{\ell_3}^{x}$$

$$I_4'(x)=\Big[\frac{j4\pi\omega\varepsilon}{\Omega_4}\{A_4cos\beta x+B_4sin\beta|x|\}+P_{41}\{I_1'(x)\}+P_{42}\{I_2'(x)\}+P_{43}\{I_3'(x)\}+P_{44}\{I_4(x)\}\Big]_{\ell_4}^{x} \quad \cdots\cdots(22)$$

윗式에서 r_1, r_2,, 및 r_3 대신에 $|x-\xi|$를 앞에서 설명한바와 같이 近似的으로 사용할 수 있다.

위의 定義에 따라 다음의 관계가 있음을 잘 알 수 있으며 이것들은 나중에 계산에 이용된다.

$$\left.\begin{aligned}
&\Omega_1[P_{11}\{F_1(x)\}]_{\ell_1}=\Omega_2[P_{21}\{F_1(x)\}]_{\ell_1}=\Omega_3[P_{31}\{F_1(x)\}]_{\ell_1}=\Omega_4[P_{41}\{F_1(x)\}]_{\ell_1}\\
&\Omega_1[P_{12}\{F_2(x)\}]_{\ell_1}=\Omega_2[P_{22}\{F_2(x)\}]_{\ell_1}\\
&\Omega_2[P_{22}\{F_2(x)\}]_{\ell_2}=\Omega_3[P_{32}\{F_2(x)\}]_{\ell_2}=\Omega_4[P_{42}\{F_2(x)\}]_{\ell_2}\\
&\Omega_1[P_{13}\{F_3(x)\}]_{\ell_2}=\Omega_2[P_{23}\{F_3(x)\}]_{\ell_2}=\Omega_3[P_{33}\{F_3(x)\}]_{\ell_2}\\
&\Omega_1[P_{14}\{F_4(x)\}]_{\ell_3}=\Omega_2[P_{24}\{F_4(x)\}]_{\ell_3}=\Omega_3[P_{34}\{F_4(x)\}]_{\ell_3}=\Omega_4[P_{44}\{F_4(x)\}]_{\ell_3}
\end{aligned}\right\}\cdots\cdots(23)$$

이제 여기서 계산의 편의상

$$F_{10}(x)=\frac{\cos\beta x}{\Omega_1},\ H_{10}(x)=0,\ J_{10}(x)=0,$$
$$K_{10}(x)=0,\ L_{10}(x)=0,\ M_{10}(x)=0,$$
$$N_{10}(x)=0$$
$$F_{20}(x)=0,\ H_{20}(x)=\frac{\cos\beta x}{\Omega_2},$$
$$J_{20}(x)=\frac{\sin\beta|x|}{\Omega_2},\ K_{20}(x)=0,\ L_{20}(x)=0,$$
$$M_{20}(x)=0,\ N_{20}(x)=0,$$
$$F_{30}(x)=0,\ H_{30}(x)=0,\ J_{30}(x)=0,$$
$$K_{30}(x)=\frac{\cos\beta x}{\Omega_3},\ L_{30}(x)=\frac{\sin\beta|x|}{\Omega_3},$$
$$M_{30}(x)=0,\ N_{30}(x)=0$$
$$F_{40}(x)=0,\ H_{40}(x)=0,\ J_{40}(x)=0,\ K_{40}(x)=0,$$
$$L_{40}(x)=0,\ M_{40}(x)=\frac{\cos\beta x}{\Omega_4},$$
$$N_{40}(x)=\frac{\sin\beta|x|}{\Omega_4}\cdots\cdots(24)$$

라하고 또 $n=1,2,3\cdots\cdots$에 대해서

$$\left.\begin{aligned}
&F_{1,n+1}(x)=P_{11}\{F_{1,n}(x)\}+P_{12}\{F_{2,n}(x)\}+P_{13}\{F_{3,n}(x)\}+P_{14}\{F_{4,n}(x)\}\\
&F_{2,n+1}(x)=P_{21}\{F_{1,n}(x)\}+P_{22}\{F_{2,n}(x)\}+P_{23}\{F_{3,n}(x)\}+P_{24}\{F_{4,n}(x)\}\\
&F_{3,n+1}(x)=P_{31}\{F_{1,n}(x)\}+P_{32}\{F_{2,n}(x)\}+P_{33}\{F_{3,n}(x)\}+P_{34}\{F_{4,n}(x)\}\\
&F_{4,n+1}(x)=P_{41}\{F_{1,n}(x)\}+P_{42}\{F_{2,n}(x)\}+P_{43}\{F_{3,n}(x)\}+P_{44}\{F_{4,n}(x)\}
\end{aligned}\right\}\cdots\cdots(25)$$

로 나타내며 $H_{r,s}(x)$, $J_{r,s}(x)\cdots\cdots N_{r,s}(x)$도 같은 요령으로 나타냈다고 하면 이 函數들을 사용하여서 다음의 네 式을 얻는다.

$$I'_{r,n}=j4\pi\omega\varepsilon\sum_{s=1}^{n-1}\Big[A_1F_{r,s}(x)+A_2H_{r,s}(x)+B_2J_{r,s}(x)+A_3K_{r,s}(x)+B_3L_{r,s}(x)+A_4M_{r,s}(x)+B_4N_{r,s}(x)\Big]_{\ell_1}^{x}\cdots\cdots(26)$$

단, $0<|x|<\ell_1$에 대해서는 $r=1$, $\ell_1<|x|<\ell_2$에는 $r=2$, $\ell_2<|x|<\ell_3$에는 $r=3$, $\ell_3<|x|<\ell_4$에는 $r=4$이며 $I'_{4,n}=I_{4,n}$임.

(26)式의 실계를 (22)式에 代入하면 $r=1\sim4$에 대해서 각각

$$(22)\text{式의 右邊}=j4\pi\omega\varepsilon\sum_{s=0}^{n}\Big[A_1F_{r,s}(x)+A_2H_{r,s}(x)+B_2J_{r,s}(x)+A_3K_{r,s}(x)+B_3L_{r,s}(x)+A_4M_{r,s}(x)+B_4N_{r,s}(x)\Big]_{\ell_1}^{x}=I'_{r,n+1}(x)\cdots\cdots(27)$$

윗式에서 Σ 記號內의 s=n 項을 생략하므로서 (26)式으로 주워지는 $I'_{1,n}(x)$, $I_2'{}_{,n}(x)$, 및 $I_3'{}_{,n}(x)$가 (22)式을 만족함을 곧 알 수 있다. 따라서 (22)式이 電流分布의 (n−1)次 近似式임을 알 수 있다. 그러나 여기서 우리는 (22)式의 級數가 準收斂을 한다고 생각하고 Hallèn의 理論처럼 처음 몇項만을 取한다.

이제 積分常數 A_n, B_n을 定하기 위하여 (9)～(12)式에 Operator를 사용하여서 다음 관계式을 얻는다.

$$I_r(x)=\frac{j4\pi\omega\varepsilon}{\Omega_r}\{A_r\cos\beta x+B_r\sin\beta|x|\}+P_{r1}\{I_1'(x)\}+P_{r2}\{I_2'(x)\}+P_{r3}\{I_3'(x)\}+P_{r4}\{I_4(x)\}\cdots\cdots(28)$$

단, $r=1,2,3,4$

윗式에서 얻어지는 $I_1(x)$, $I_2(x)$, $I_3(x)$, 및 $I_4(x)$와 $I_1(\ell_1)=I_2(\ell_1)$, $I_2(\ell_2)=I_3(\ell_2)$, $I_3(\ell_3)=I_4(\ell_3)$, $I_4(\ell_4)=0$의 境界條件에 의하여

$$\sum_{s=0}^{n}\big[A_1\{F_{2s}(\ell_1)-F_{1s}(\ell_1)\}+A_2\{H_{2s}(\ell_1)-H_{1s}(\ell_1)\}+B_2\{J_{2s}(\ell_1)-J_{1s}(\ell_1)\}+A_3\{K_{2s}(\ell_1)\}+B_3\{L_{2s}(\ell_1)-L_{1s}(\ell_1)\}+A_4\{M_{2s}(\ell_1)-M_{1s}(\ell_1)\}+B_4\{N_{2s}(\ell_1)-N_{1s}(\ell_1)\}\big]=0\cdots\cdots(29)$$

$$\sum_{s=0}^{n}\big[A_1\{F_{3s}(\ell_1)-F_{2s}(\ell_2)\}+A_2\{H_{3s}(\ell_1)-H_{3s}(\ell_1)\}+B_2\{J_{3s}(\ell_1)-J_{2s}(\ell_1)\}+A_3\{K_{3s}(\ell_1)-K_{2s}(\ell_1)\}+B_3\{L_{3s}(\ell_1)-L_{2s}(\ell_1)\}+A_4\{M_{3s}(\ell_1)-M_{2s}(\ell_1)\}+B_4\{N_{3s}(\ell_1)-N_{2s}(\ell_1)\}\big]=0\cdots\cdots(30)$$

$$\sum_{s=0}^{n}\big[A_1\{F_{4s}(\ell_1)-F_{3s}(\ell_2)\}+A_2\{H_{4s}(\ell_1)$$

$$-H_{3s}(\ell_1)\}+B_2\{J_{4s}(\ell_1)-{}_{3s}(\ell_1)\}$$
$$+A_3\{K_{4s}(\ell_1)-K_{3s}(\ell_1)\}+B_3\{L_{4s}(\ell_1)$$
$$-L_{3s}(\ell_1)\}+A_4\{M_{4s}(\ell_1)-M_{4s}(\ell_1)\}$$
$$+B_4\{N_{4s}(\ell_1)-N_{3s}(\ell_1)\}=0 \quad \cdots\cdots\cdots(31)$$

$$\sum_{s=0}^{n}[A_1F_{4s}(\ell_4)+A_2H_{4s}(\ell_4)+B_2J_{4s}(\ell_4)$$
$$+A_3K_{4s}(\ell_1)+B_3L_{4s}(\ell_4)+A_4M_{4s}(\ell_4)$$
$$+B_4N_{4s}(\ell_4)]=0 \quad \cdots\cdots\cdots(32)$$

윗式과 (6)式으로부터 係數 A_n B_n을 決定된다.

3. 電流分布의 近似式

係數 A_n B_n을 결정하기 위하여 편의상 다음의 函數들을 定義한다.

$$f_1=-\sin\beta\ell_1,\ h_1=\sin\beta\ell_1,\ j_1=-\cos\beta\ell_1,$$
$$k_1=0,\ \ell_1=0,\ m_1=0,\ n_1=0$$
$$f_2=0,\ k_2=-\sin\beta\ell_2,\ j_2=\cos\beta\ell_2,$$
$$k_2=\sin\beta\ell_2,\ \ell_2=-\cos\beta\ell_2,\ m_2=0,\ n=0$$
$$f_3=0,\ h_3=0,\ j_3=0,\ k_3=-\sin\beta\ell_3,$$
$$\ell_3=\cos\beta\ell_3,\ m_3=\sin\beta\ell_3,\ n_3=-\cos\beta\ell_3$$
$$f_4=\sum_{s=0}^{n}F_{4s}(\ell_4),\ h_4=\sum_{s=0}^{n}H_{4s}(\ell_4),$$
$$j_4=\sum_{s=0}^{n}J_{4s}(\ell_4),\ k_4=\sum_{0}^{n}K_{4s}(\ell_4),$$
$$\ell_4=\sum_{0}^{n}L_{4s}(\ell_4),\ m_4=\sum_{s=0}^{n}M_{4s}(\ell_4),$$
$$n_4=\sum_{s=0}^{n}N_{4s}(\ell_4) \quad \cdots\cdots\cdots(33)$$

r=5, 6, 7에 대해서

$$f_r=\sum_{0}^{n}\{F_{r-3,s}(\ell_{r-4})-F_{r-4,s}(\ell_{r-4})\}$$
$$h_r=\sum_{0}^{n}H_{r-4,s}(\ell_{r-4})-H_{r-4,s}(\ell_{r-4})\}$$
$$j_r=\sum_{0}^{n}\{J_{r-3,s}(\ell_{s-4})-J_{r-4,s}(\ell_{r-4})\}$$
$$k_r=\sum_{0}^{n}\{K_{r-3,s}(\ell_{r-4})-K_{r-4,s}(\ell_{r-4})\}$$
$$\ell_r=\sum_{0}^{n}\{L_{r-3,s}(\ell_{r-4})-L_{r-4,4}(\ell_{r-4}\},$$
$$m_r=\sum_{0}^{n}\{M_{r-3,s}(\ell_{r-3})-M_{r-4,s}(\ell_{r-4})\}$$
$$n_r=\sum_{0}^{n}\{N_{r-3,s}(\ell_{r-4})-N_{r-4,s}(\ell_{r-4})\}$$
$$\cdots\cdots\cdots(34)$$

그러면 (6)式과 (29)～(32)式으로 부터

$$\left.\begin{aligned}
&f_1A_1+h_1A_2+j_1B_3=\frac{E_1}{B}\\
&h_2A_2+j_2B_2+k_2A_3+\ell_2B_3=\frac{E_2}{B}\\
&k_3A_3+\ell_3B_3+m_3A_4+n_3B_4=\frac{E_3}{B}\\
&f_4A_1+h_4A_2+j_4B_2+k_4A_3\\
&\quad+\ell_4B_3+m_4A_4+n_4B_4=0\\
&f_5A_1+h_5A_2+j_5B_2+k_5A_3\\
&\quad+\ell_5B_3+m_5A_4+n_5B_4=0\\
&f_6A_1+h_6A_2+j_6B_2+k_6A_3\\
&\quad+\ell_6B_3+m_6A_4+n_6B_4=0\\
&f_7A_1+h_7A_2+j_7B_2+k_7A_3\\
&\quad+\ell_7B_3+m_7A_4+n_7B_4=0
\end{aligned}\right\}\cdots(35)$$

(35)式에서

$$\left.\begin{aligned}
A_1&=\frac{\triangle_1E_1-\triangle_2E_2+\triangle_3E_3}{\beta\triangle_0}\\
A_2&=\frac{-\triangle_4E_1+\triangle_5E_2-\triangle_6E_3}{\beta\triangle_0}\\
B_2&=\frac{\triangle_7E_1-\triangle^8E_2+\triangle cE_3}{\beta\triangle_0}\\
A_3&=\frac{-\triangle_{10}E_1+\triangle_{11}E_2-\triangle_{12}E_3}{\beta\triangle_0}\\
B_3&=\frac{\triangle_{13}E_1-\triangle_{14}E_2+\triangle_{15}E_3}{\beta\triangle_0}\\
A_4&=\frac{-\triangle_{16}E_1+\triangle_{17}E_2-\triangle_1{}^8E_3}{\beta\triangle_0}\\
B_4&=\frac{\triangle_{19}E_1-\triangle_{20}E_2+\triangle_{21}E_3}{\beta\triangle_0}
\end{aligned}\right\}\cdots(36)$$

단,

$$\triangle_0=\begin{vmatrix}
f_1 & h_1 & j_1 & k_1 & \ell_1 & m_1 & n_1\\
f_2 & h_2 & j_2 & k_2 & \ell_2 & m_2 & n_2\\
f_3 & h_3 & j_3 & k_3 & \ell_3 & m_3 & n_3\\
f_4 & h_4 & j_4 & k_4 & \ell_4 & m_4 & n_4\\
f_5 & h_5 & j_5 & k_5 & \ell_5 & m_5 & n_5\\
f_6 & h_6 & j_6 & k_6 & \ell_6 & m_6 & n_6\\
f_7 & h_7 & j_7 & k_7 & \ell_7 & m_7 & n_7
\end{vmatrix}$$

$\triangle_1=\triangle'_{11}\equiv\triangle_0$에서 제1行, 제1列에 대한 餘因子 行列式$/(-1)^{1+1}$

$\triangle_2=\triangle'_{21},\ \triangle_3=\triangle'_{31},\ \triangle_4=\triangle'_{12},$

$\triangle_5=\triangle'_{22},\ \triangle_6=\triangle'_{32}$

$\triangle_7=\triangle'_{13},\ \triangle^8=\triangle'_{23},\ \triangle_9=\triangle'_{33},$

$\triangle_{10}=\triangle'_{14}$, $\triangle_{11}=\triangle'_{24}$

$\triangle_{12}=\triangle'_{34}$, $\triangle_{13}=\triangle'_{15}$, $\triangle_{14}=\triangle'_{25}$

$\triangle_{15}=\triangle'_{35}$, $\triangle_{16}=\triangle'_{16}$

$\triangle_{17}=\triangle'_{26}$, $\triangle_{18}=\triangle'_{36}$, $\triangle_{19}=\triangle'_{17}$

$\triangle_{20}=\triangle'_{27}$, $\triangle_{21}=\triangle'_{37}$

(36)식에서 알 수 있는바와 같이 안테나 電流는 E_1만에 비례하는 成分, E_2만에 비례하는 成分, 및 E_3 만에 비례하는 成分으로 되고 있으므로 (29)~(32)式에서 Σ中의 s=n項을 생략한 것과 (26)式으로부터 E_1 에의한 各部分(그림3 (a)의 電流 $I_1^a(x)$, $I_2^a(x)$, $I_3^a(x)$ 및 $I_4^a(x)$는

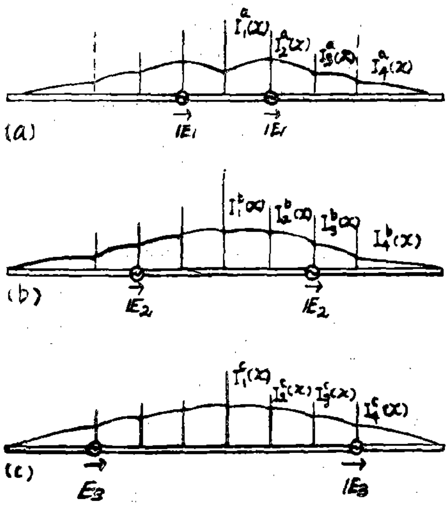


그림3. 六點給電線形안테나 上의 電流分布

$$I_r^a(x)=\frac{jE_1}{30\triangle_0}\sum_{s=0}^{n-1}\left[\triangle_1F_{rs}(x)-\triangle_4H_{rs}(x)+\triangle_7J_{rs}(x)-\triangle_{10}K_{rs}(x)+\triangle_{13}L_{rs}(x)-\triangle_{16}M_{rs}(x)+\triangle_{19}N_{rs}(x)\right]\cdots\cdots\cdots\cdots(37)$$

단, r=1, 2, 3, 4

똑같이 하여 E_2에 의한 各部分의 電流(그림 3(b))는

$$I_r^b(x)=\frac{jE_2}{30\triangle_0}\sum_{s=0}^{n-1}\left[\triangle_2F_{rs}(x)+\triangle_5H_{rs}(x)-\triangle_8J_{rs}(x)+\triangle_{11}K_{rs}(x)-\triangle_{14}L_{rs}(x)+\triangle_{17}M_{rs}(x)\right]\cdots\cdots\cdots\cdots(38)$$

단, r=1,2,3,4

E_3에 의한 各部分의 電流(그림 3의 (c)는

$$I_r^c(x)=\frac{jE_3}{30\triangle_0}\sum_{s=0}^{n-1}\left[\triangle_3F_{rs}(x)-\triangle_6H_{rs}(x)+\triangle_9J_{rs}(x)-\triangle_{12}K_{rs}(x)+\triangle_{15}L_{rs}(x)-\triangle_{18}M_{rs}(x)+\triangle_{21}N_{rs}(x)\right]\cdots\cdots\cdots\cdots(39)$$

단, r=1,2,3,4

이 級數의 主體를 계산 하기 위하여 다음의 函數들을 定義한다.

$F_{11}(0)=\frac{f_{10}}{\Omega_1^2}$, $H_{11}(0)=\frac{h_{10}}{\Omega_1\Omega_2}$

$J_{11}(0)=\frac{j_{10}}{\Omega_1\Omega_2}$, $K_{11}(0)=\frac{k_{10}}{\Omega_1\Omega_3}$,

$L_{11}(0)=\frac{\ell_{10}}{\Omega_1\Omega_3}$, $M_{11}(0)=\frac{m_{10}}{\Omega_1\Omega_4}$,

$N_{11}(0)=\frac{n_{10}}{\Omega_1\Omega_4}$,

$F_{11}(\ell_1)=\frac{f_{11}}{\Omega_1^2}$, $H_{11}(\ell_1)=\frac{h_{11}}{\Omega_1\Omega_2}$,

$J_{11}(\ell_1)=\frac{j_{11}}{\Omega_1\Omega_2}$, $K_{11}(\ell_1)=\frac{k_{11}}{\Omega_1\Omega_3}$,

$L_{11}(\ell_1)=\frac{\ell_{11}}{\Omega_1\Omega_3}$, $M_{11}(\ell_1)=\frac{m_{11}}{\Omega_1\Omega_4}$,

$N_{11}(\ell_1)=\frac{n_{11}}{\Omega_1\Omega_4}$

$F_{21}(\ell_1)=\frac{f_{21}}{\Omega_1\Omega_2}$, $H_{21}(\ell_1)=\frac{h_{21}}{\Omega_2^2}$,

$J_{21}(\ell_1)=\frac{j_{21}}{\Omega_2^2}$, $K_{21}(\ell_1)=\frac{k_{21}}{\Omega_2\Omega_3}$,

$L_{21}(\ell_1)=\frac{\ell_{21}}{\Omega_2\Omega_3}$, $M_{21}(\ell_1)=\frac{m_{21}}{\Omega_2\Omega_4}$,

$N_{21}(\ell_1)=\frac{n_{21}}{\Omega_2\Omega_4}$,

$F_{21}(\ell_2)=\frac{f_{23}}{\Omega_1\Omega_2}$, $H_{21}(\ell_2)=\frac{h_{23}}{\Omega_2^2}$,

$J_{21}(\ell_2)=\frac{j_{23}}{\Omega_2^2}$, $K_{21}(\ell_2)=\frac{k_{23}}{\Omega_2\Omega_3}$,

$L_{21}(\ell_2)=\frac{\ell_{23}}{\Omega_2\Omega_3}$, $M_{21}(\ell_2)=\frac{m_{23}}{\Omega_2\Omega_4}$,

$N_{21}(\ell_2)=\frac{n_{23}}{\Omega_2\Omega_4}$,

$F_{31}(\ell_2)=\frac{f_{32}}{\Omega_1\Omega_3}$, $H_{31}(\ell_2)=\frac{h_{32}}{\Omega_2\Omega_3}$,

$J_{31}(\ell_2)=\frac{j_{32}}{\Omega_2\Omega_3}$, $K_{31}(\ell_2)=\frac{k_{32}}{\Omega_3^2}$,

$L_{31}(\ell_2)=\frac{\ell_{32}}{\Omega_3^2}$, $M_{31}(\ell_2)=\frac{m_{32}}{\Omega_3\Omega_4}$,

$N_{31})\ell_2)=\frac{n_{32}}{\Omega_3\Omega_4}$

$F_{31}(\ell_3)=\frac{f_{34}}{\Omega_1\Omega_3}$, $H_{31}(\ell_3)=\frac{h_{34}}{\Omega_2\Omega_3}$,

$J_{31}(\ell_3)=\frac{j_{34}}{\Omega_2\Omega_3}$, $K_{31}(\ell_3)=\frac{k_{34}}{\Omega_3^2}$,

$L_{31}(\ell_3)=\frac{\ell_{34}}{\Omega_3^2}$, $M_3(\ell_3)=\frac{m_{34}}{\Omega_3\Omega_4}$,

$$N_{31}(\ell_3)=\frac{n_{34}}{\Omega_3\Omega_4}$$

$$F_{41}(\ell_3)=\frac{f_{43}}{\Omega_1\Omega_4},\quad H_{41}(\ell_3)=\frac{h_{43}}{\Omega_2\Omega_4},$$

$$J_{41}(\ell_3)=\frac{j_{43}}{\Omega_2\Omega_4},\quad K_{41}(\ell_3)=\frac{k_{43}}{\Omega_3\Omega_4},$$

$$L_{41}(\ell_3)=\frac{\ell_{43}}{\Omega_3\Omega_4},\quad M_{41}(\ell_3)=\frac{m_{43}}{\Omega_4{}^2},$$

$$N_{41}(\ell_3)=\frac{n_{43}}{\Omega_4{}^2}$$

$$F_{41}(\ell_4)=\frac{f_{44}}{\Omega_1\Omega_4},\quad H_{41}(\ell_4)=\frac{h_{44}}{\Omega_2\Omega_4},$$

$$J_{41}(\ell_4)=\frac{j_{44}}{\Omega_2\Omega_4},\quad K_{41}(\ell_4)=\frac{\ell_{44}}{\Omega_3\Omega_4},$$

$$L_{41}(\ell_4)=\frac{\ell_{44}}{\Omega_3\Omega_4},\quad M_{41}(\ell_4)=\frac{m_{44}}{\Omega_4{}^2},$$

$$N_{41}(\ell_4)=\frac{n_{44}}{\Omega_4{}^2},\ \cdots\cdots\cdots\cdots\cdots (40)$$

윗函數들에 의하여 (33), (34)式은 다음과 같이 주워진다.

$$f_4=\frac{f_{44}}{\Omega_1\Omega_4},\quad h_4=\frac{h_{44}}{\Omega_1\Omega_4},\quad j_4=\frac{j_{44}}{\Omega_2\Omega_4},$$

$$k_4=\frac{k_{44}}{\Omega_3\Omega_4},\quad \ell_4=\frac{\ell_{44}}{\Omega_3\Omega_4},$$

$$m_4=\frac{\cos\beta\ell_4}{\Omega_4}+\frac{m_{44}}{\Omega_4{}^2},\quad n_4=\frac{\sin\beta\ell_4}{\Omega_4}+\frac{n_{44}}{\Omega_4{}^2},$$

$$f_5=-\frac{\cos\beta\ell_1}{\Omega_1}+\frac{f_{21}}{\Omega_1\Omega_2}-\frac{f_{31}}{\Omega_1{}^2},$$

$$h_5=\frac{\cos\beta\ell_1}{\Omega_2}+\frac{h_{21}}{\Omega_2{}^2}-\frac{h_{12}}{\Omega_1\Omega_2},$$

$$j_5=\frac{\sin\beta\ell_1}{\Omega_3}+\frac{j_{21}}{\Omega_2{}^2},\quad k_5=\frac{k_{21}}{\Omega_2\Omega_3}-\frac{k_{12}}{\Omega_1\Omega_3},$$

$$\ell_5=\frac{\ell_{\ 1}}{\Omega_2\Omega_3}-\frac{\ell_{12}}{\Omega_1\Omega_3},\quad m_5=\frac{m_{21}}{\Omega_2\Omega_4}-\frac{m_{12}}{\Omega_1\Omega_4},$$

$$n_5=\frac{n_{21}}{\Omega_2\Omega_4}-\frac{n_{12}}{\Omega_1\Omega_4}$$

$$f_6=-\frac{f_{23}}{\Omega_1\Omega_2}+\frac{f_{32}}{\Omega_1\Omega_3},$$

$$h_6=-\frac{\cos\beta\ell_2}{\Omega_2}-\frac{h_{23}}{\Omega_2{}^2}+\frac{h_{32}}{\Omega_2\Omega_3}$$

$$j_6=-\frac{\sin\beta\ell_2}{\Omega_2}-\frac{j_{23}}{\Omega_2{}^2}+\frac{j_{32}}{\Omega_2\Omega_3}$$

$$k_6=\frac{\cos\beta\ell_2}{\Omega_3}-\frac{k_{23}}{\Omega_2\Omega_3}+\frac{k_{32}}{\Omega_{32}},$$

$$\ell_6=\frac{\sin\beta\ell_2}{\Omega_3}-\frac{\ell_{23}}{\Omega_2\Omega_3}+\frac{\ell_{32}}{\Omega_3{}^2},$$

$$m_6=-\frac{m_{23}}{\Omega_2\Omega_4}+\frac{m_{32}}{\Omega_3\Omega_4},\quad n_6=-\frac{n_{32}}{\Omega_2\Omega_4}+\frac{n_{32}}{\Omega_3\Omega_4}$$

$$f_7=-\frac{f_{34}}{\Omega_1\Omega_3}+\frac{f_{43}}{\Omega_1\Omega_4},\quad h_7=-\frac{h_{34}}{\Omega_2\Omega_3}+\frac{h_{43}}{\Omega_1\Omega_4},$$

$$j_7=-\frac{j_{34}}{\Omega_2\Omega_3}+\frac{j_{43}}{\Omega_2\Omega_4}$$

$$k_7=-\frac{\cos\beta\ell_3}{\Omega_3{}^2}-\frac{k_{34}}{\Omega_2{}^2}+\frac{k_{43}}{\Omega_3\Omega_4}$$

$$\ell_7=\frac{\sin\beta\ell_3}{\Omega_3}-\frac{\ell_{34}}{\Omega_3{}^2}+\frac{\ell_{43}}{\Omega_3\Omega_4},$$

$$m_7=\frac{\cos\beta\ell_3}{\Omega_4}-\frac{m_{34}}{\Omega_3\Omega_4}+\frac{m_{43}}{\Omega_4{}^2},$$

$$n_7=\frac{\sin\beta\ell_3}{\Omega_4}-\frac{n_{34}}{\Omega_3\Omega_4}+\frac{n_{43}}{\Omega_4{}^2}\cdots\cdots (41)$$

윗式에서 간편을 위하여 $\rho_1=\rho_2=\rho_3=\rho_4=\rho$, 따라서 $\Omega_1=\Omega_2=\Omega_3=\Omega_4=\Omega\gg 1$ 이라 假定하고 $\frac{1}{\Omega}$에 關한 級數中에서 그 主體部分만을 取한다면[3]

$$I_1{}^a(x)=j\frac{E_1}{30}\ \frac{\sin\beta(\ell_4-\ell_1)\cos\beta x}{\Omega\cos\beta\ell_4+f_{44}+h_{44}+m_{44}+k_{44}}$$

$$I_2{}^a(x)=j\frac{E_1}{30}\ \frac{\cos\beta\ell_1\sin\beta(\ell_4-|x|)}{''}$$

$$I_3{}^a(x)=j\frac{E_1}{30}\ \frac{\cos\beta\ell_1\sin\beta(\ell_4-|x|)}{''}$$

$$I_4{}^a(x)=j\frac{E_1}{30}\ \frac{\cos\beta\ell_1\sin\beta(\ell_4-|x|)}{''}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (42)$$

$$I_1{}^b(x)=j\frac{E_2}{30}\ \frac{\sin\beta(\ell_4-\ell_2)\cos\beta x}{\Omega\cos\beta\ell_4+f_{44}+h_{44}+k_{44}+m_{44}}$$

$$I_2{}^b(x)=j\frac{E^{z}}{30}\ \frac{\sin\beta(\ell_4-\ell_2)\cos\beta x}{''}$$

$$I_3{}^b(x)=j\frac{E_2}{30}\ \frac{\cos\beta\ell_2\sin\beta(\ell_4-|x|)}{''}$$

$$I_4{}^b(x)=j\frac{E_2}{30}\ \frac{\cos\beta\ell_2\sin\beta(\ell_4-|x|)}{''}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (43)$$

$$I_1{}^c(x)=-j\frac{E_3}{30}\ \frac{\sin\beta(\ell_4-\ell_3)\cos\beta x}{\Omega\cos\beta\ell_4+f_{44}+h_{44}+k_{44}+m_{44}}$$

$$I_2{}^c(x)=-j\frac{E_3}{30}\ \frac{\sin\beta(\ell_4-\ell_3)\cos\beta x}{''}$$

$$I_3{}^c(x)=-j\frac{E_3}{30}\ \frac{\sin\beta(\ell_4-\ell_3)\cos\beta x}{''}$$

$$I_4{}^c(x)=-j\frac{E_3}{30}\ \frac{\cos\beta\ell_3\sin\beta(\ell_4-|x|)}{''}$$

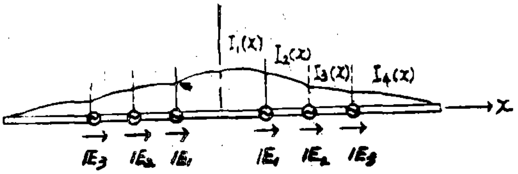
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (44)$$

여기서 $f_{44}+h_{44}+k_{44}+m_{44}$는 Hallèn의 안테나 理論에 나오는 α_1이 같으며 ℓ_4만에 관한 函數로서 다음과 같다.[4]

$$\alpha_1(\ell_4)=\frac{1}{2}\{\{\cos\beta\ell_4\{C(4\beta\ell_4)-2C(2\beta\ell 4)\}$$
$$-\sin\beta\ell_4 S(4\beta\ell_4)+j[\cos\beta\ell_4\{S(4\beta\ell 4)$$
$$-2S\}2\beta\ell_4)\}+\sin\beta\ell_4 C(4\beta\ell_4)]\}\}\cdots (45)$$

4. 計 算 制

그림 4와 같이 $\ell_1=\frac{\lambda}{4}$, $\ell_2=\frac{\lambda}{2}$, $\ell_3=\frac{3}{4}\lambda$, $\ell_4=\frac{5}{4}\lambda$ 이며 $\rho_1=\rho_4\doteqdot\rho_2=\rho_3=\rho$ 인 경우에 대한 電流分布를 (42)~(44) 式에 의하여 計算해 본다.



原 안테나

그림 4 計算된 電流分布例

$$\beta\ell_1=\frac{\pi}{2},\ \beta\ell_2=\pi,\ \beta\ell_3=\frac{3}{2}\pi,\ \beta\ell_4=\frac{5}{2}\pi$$

이므로

$$I_r^a(x)=0 \quad\cdots\cdots(46)$$

$$I_1^b(x)=j\frac{E_2}{30\alpha_1}\sin\frac{3\pi}{2}\cos\beta x=-\mathrm{I}\cos\beta x=\mathrm{I}_2^b(x)$$

$$\mathrm{I}_3^b(x)=j\frac{E_2}{30\alpha_1}\cos\pi\sin\beta(\ell_4-|x|)=-\mathrm{I}\sin\beta(\ell_4-|x|)=\mathrm{I}_4^b(x)\quad\cdots\cdots(47)$$

단, $\mathrm{I}=j\frac{E_2}{30\alpha_1}$

$$\mathrm{I}_r^c(x)=0\quad\cdots\cdots(48)$$

따라서 이 경우의 電流分布는 $|\mathrm{I}|$를 그 최대값으로 하는 그림 4와 같은 分布로 나타난다.

(42)~(44)式이 1次近似式이기 때문에 node 點에 給電된 E_1 및 E_3에 의한 電流分布가 零이 되고 있다고 생각되는데 만약 式의 近似度를 높인다면 이 경우라 하더라도 큰 E_1 및 E_3에 대한 電流分布가 相當한 크기로 나타날 것이 예상된다. 여기서 그림 2와 같은 안테나의 경우를 생각할때 실제로 給電하는것은 E_2 뿐이지만 부수적으로 나타나는 E_1과 E_3이 E_2의 20倍 정도에 까지 이르므로 (5), (6) 이러한 경우의 電流分布는 그림 4와 달라질 것이라 생각된다.

또한 만약 6點給電안테나의 칫수가 위와 같은 極端의 경우가 아니고 안테나 各區間의 길이가 $\frac{\lambda}{4}\times n$이 아닐때(가령 電源의 周波數가 共振周波數에서 약간 벗어난 周波數인 경우)에는 위의 (42)~(44) 式에 의해서 E_1 및 E_3에 의한 電流分布가 계산될것이 틀림 없을 것으로 본다.

5. 結 論

本硏究에 의하여 六點給電線形안테나系各部의 電流分布式이 誘導되었으며 이로서 대체적인 電流分布의 計算을 할 수 있게 되었다.

뿐만 아니라 이 電流分布式으로 부터 各給電임피이던스 및 相互임피이던스의 計算式을 얻을 수 있을 것이며 포울딛드다이폴 안테나 素子를 中央에 대해서 對稱으로 여러 段 取付한 多段안테나의 解析에 많은 도움이 된것으로 믿는다.

끝으로 本硏究는 1969年度 文敎部學術硏究助成費에 의한것의 1部이며 本硏究에 있어서 複雜한 積分計算에 많은 支援을 하여준 本校數學科 李起安 敎授와 多元聯立方程式을 푸는데 필요했던 많은 多元行列式의 計算을 도와준 本校電子科 姜元赫, 朴成漢君에게 깊이 感謝한다.

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<附 錄>

$$f_{44}=\Omega^2F_{41}(\ell_4)$$

$$=(\cos\beta\ell_4-\cos\beta\ell_1)\ln\frac{\ell_4-\ell_1}{\ell_4+\ell_1}$$

$$-\cos\beta\ell_1\{E(\beta(\ell_4+\ell_1)-E(\beta(\ell_4-\ell_1))\}$$

$$+\frac{1}{2}e^{j\beta}r_4\ell_4\{E(2\beta(\ell_4+\ell_1))-E(2\beta(\ell_4-\ell_1))\}$$

$$h_{44}=\Omega^2H_{41}(\ell_4)$$

$$=-\cos\beta\ell_1\ln\frac{\ell_4+\ell_1}{\ell_4-\ell_1}+\cos\beta\ell_2\ln\frac{\ell_4+\ell_2}{\ell_4-\ell_2}$$

$$+\cos\beta\ell_4\ln\frac{\ell_4-\ell_2}{\ell_4+\ell_2}\ \frac{\ell_4+\ell_1}{\ell_4-\ell_1}$$

$$+\cos\beta\ell_1\{E(\beta(\ell_4+\ell_1))-E(\beta(\ell_4-\ell_1))\}$$

$$-\mathrm{con}\beta\ell_2\{E(\beta(\ell_4+\ell_2))-E(\beta(\ell_4-\ell_2))\}$$

$$-\frac{1}{2}e^{j\beta 14}\{E(2\beta(\ell_4+\ell_1))-E(2\beta(\ell_4+\ell_2))$$

$$+E(2\beta(\ell_4-\ell_2))-E(2\beta(\ell_4-\ell_1))\}$$

$$k_{44}=\Omega^2K_{41}(\ell_4)$$

$$=-\cos\beta\ell_2\ln\frac{\ell_4+\ell_2}{\ell_4-\ell_2}+\cos\beta\ell_3\ln\frac{\ell_4-\ell_3}{\ell_4+\ell_3}$$

$$+\cos\beta\ell_4\ln\frac{\ell_4-\ell_3}{\ell_4+\ell_3}\ \frac{\ell_4+\ell_2}{\ell_4-\ell_2}$$

$$+\cos\beta\ell_2\{E(\beta(\ell_4+\ell_2))-E(\beta(\ell_4-\ell))\}$$

$$-\cos\beta\ell_3\{E(\beta(\ell_4+\ell_3))-E(\beta(\ell_4-\ell_3))\}$$

$$-\frac{1}{2}e^{j\beta}r_4\{E(2\beta(\ell_4+\ell_2))-E(2\beta(\ell_4+\ell_3))$$

$$+E(2\beta(\ell_4-\ell_3))-E(2\beta(\ell_4-\ell_2))\}$$

$$m_{44}=\Omega^2M_{41}(\ell_4)$$

$$=\cos\beta\ell_3\ln\frac{\ell_4-\ell_3}{\ell_4+\ell_3}-\cos\beta\ell_3\ln\frac{\ell_4-\ell_3}{\ell_4+\ell_3}$$

$$+\cos\beta\ell_3\{E(\beta(\ell_4+\ell_3))-E(\beta(\ell_4-\ell_3))\}$$

$$-\cos\beta\ell_4\cdot E(2\beta\ell_4)$$

$$+\frac{1}{2}e^{j\beta 14}\{E(4\beta\ell_4)+E(2\beta(\ell_4-\ell_3))$$

$$-2\beta(\ell_4+\ell_3))\}$$

$$E(x)=C(x)+jS)x)$$

$$=\int_0^x\frac{1-\cos\mu}{\mu}d\mu+j\int_0^x\frac{\sin\mu}{\mu}d\mu$$