The Rate of Internal Energy Increase of a Star Cluster Caused by the Tidal Attraction of the Galaxy

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ABSTRACT

The gain of internal energy of a star cluster caused by the tidal attraction of our Galaxy is examined. Expressions are derived which include the effects of a two-body orbit and internal motions of the cluster. These formulae are compared with previous results based on (i) uniform rectilinear motion and (ii) neglect of internal motions induced by cluster gravitation(i.e., impulsive approximation), and it is found that these simplifying assumptions generally introduce significant uncertainties.

I. INTRODUCTION

If the relative velocity of an encounter between two stellar systems is much larger than their internal motions, the approximation of uniform rectilinear orbital motion is generally adopted for studies of tidally-induced energy transfer (e.g., Spitzer, 1958). It is often the case, however, that the mutual gravitational attraction between the bodies cannot be ignored and the actual relative orbital motion must be taken into account. This was done by Alladin(1565) for the special case in which the internal motions induced by cluster gravitation are neglected(i.e., the impulsive approximation).

The purpose of the present paper is to derive expressions for the tidally-induced energy gain allowing for *both* the relative orbit of a two-body system and internal motions.

In this paper, for definiteness, the system consisting of our Galaxy and a star cluster is discussed. At the outset, we make the following simplifying assumptions: (1) The Galaxy is approximated by a point mass. (2) There is two-body motion. This is valid when (i) the radius of the cluster is small compared with the distance separating the two systems, and (ii) the change of orbital energy in one revolution is small. (3) The cluster suffers no mass loss. A refined analysis of the dynamical evolution of a cluster may have to take into account the orbital energy changes and mass loss, because these effects reduce the orbital size and alter the structure of the cluster.

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In this paper, the equations of motion of a star in a cluster with two-body orbital motion are derived in simple form (Section II). They are applied to the derivation of expressions for the tidally-induced energy gain of a cluster. These equations are specialized to the case of the impulsive approximation, and compared with results for uniform rectilinear motion(Section III). The internal motion of the cluster is then taken into account, and numerical values of internal energy gain are tabulated to show the characteristics of slow energy transfer in elliptical orbital motion(Section IV).

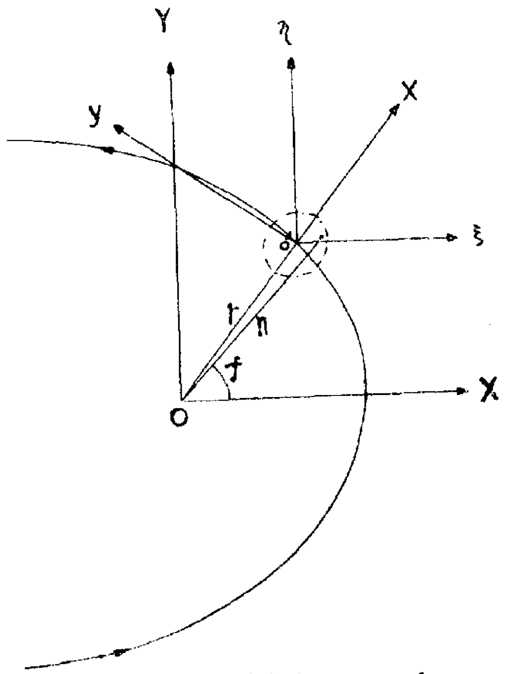

FIG. The orbit of a globular star cluster. The Galactic center is at 0, the cluster center at 0′. The fixed coordinate axes (X, Y, Z) and (ξ, η, ζ), and the moving coordinate axes(x, y, z) are shown.

II. EQUATIONS OF MOTION

Place a fixed Cartesian coordinate system(X,Y,Z) at the Galactic center. Place a moving coordinate system(x, y, z) at the center of the cluster(See Fig.). The axes z and Z are perpendicular to the orbital plane. For an elliptical cluster orbit, the following relations apply to a star in the cluster;

$$X=(r+x)\cos f-y\sin f$$
$$Y=(r+x)\sin f+y\cos f \tag{1}$$
$$Z=z$$

where r is the distance of the cluster center from the Galactic center, and f is the true anomaly of the cluster in its orbit. It follows that the kinetic energy per unit mass of an axially symmetric cluster.

$$T=\frac{1}{2}(\dot{X}^2+\dot{Y}^2+\dot{Z}^2)$$
$$=\frac{1}{2}(\dot{r}^2+r^2w_c^2)+\frac{1}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2) \tag{2}$$
$$+\frac{1}{2}w_c^2(x^2+y^2)+w_c(x\dot{y}-y\dot{x})$$

where ω_c is the orbital angular velocity,

$$w_c=\frac{n\sqrt{1-e^2}}{(1-e\cos E)^2} \tag{3}$$

E is the eccentric anomaly, e is the eccentricity of the orbit, and n is the mean orbital angular velocity of the cluster.

The total gravitational potential of a star in the cluster, V, is given by the sum of the contributions from the cluster itself, V_C, and the Galaxy, V_G,

$$V=V_c+V_G \tag{4}$$

where

$$V_G=-\frac{GM_G}{r}+\frac{GM_Gx}{r^2}$$
$$-\frac{1}{2}\frac{GM_G}{r^2}(2x^2-y^2-z^2) \tag{5}$$

Eq. (5) is obtained from a series expansion of V_G about the center of the cluster, where terms of orders higher than the second in the partial derivatives are neglected. The last two terms in Eq.(5)

result from the tidal field across the cluster. M_G is the mass of the Galaxy.

Applying Eqs. (2), (4), and (5) to the Lagrangian relations, we obtain the equations of motion of a cluster star in the moving coordinate system,

$$\begin{aligned} \ddot{x}-2w_c\dot{y}-w_c^2x-\dot{w}_cy &= -\frac{\partial V_c}{\partial x}+\frac{2GM_G}{r^3}x \\ \ddot{y}+2w_c\dot{x}-w_c^2y+\dot{w}_cx &= -\frac{\partial V_c}{\partial y}-\frac{GM_G}{r^3}y \qquad (6) \\ \ddot{z} &= -\frac{\partial V_c}{\partial z}-\frac{GM_G}{r^3}z \end{aligned}$$

where $\dot{\omega}_c$ is the orbital angular acceleration.

The equations of motion in the fixed coordinate system(X,Y,Z) are, from Eqs. (5) and (1),

$$\begin{aligned} \ddot{X} &= \ddot{X}_0+\left(-\frac{\partial V_c}{\partial x}+\frac{2GM_Gx}{r^3}\right)\cos f \\ &\quad -\left(-\frac{\partial V_c}{\partial y}-\frac{GM_Gy}{r^3}\right)\sin f \\ \ddot{Y} &= \ddot{Y}_0+\left(-\frac{\partial V_c}{\partial x}+\frac{2GM_Gx}{r^3}\right)\sin f \\ &\quad +\left(-\frac{\partial V_c}{\partial y}-\frac{GM_Gy}{r^3}\right)\cos f \qquad (7) \\ \ddot{Z} &= -\frac{\partial V_c}{\partial z}-\frac{GM_Gz}{r^3} \end{aligned}$$

where $\ddot{X}_0$ and $\ddot{Y}_0$ are the orbital accelerations of the centroid of the cluster. The equations of motion of a cluster star in a coordinate system(ξ, η, ζ) paralled to(X, Y, Z) but with origin at the cluster center(see Fig.) are given by

$$\begin{aligned} &\ddot{\xi}+\frac{\partial V_c}{\partial \xi} \\ &=\frac{GM_G}{r^3}[\xi(2-3\sin^2 f)+3\eta\sin f-\cos f] \\ &\ddot{\eta}+\frac{\partial V_c}{\partial \eta} \qquad (8) \\ &=\frac{GM_G}{r^3}[\eta(2-3\cos^2 f)+3\xi\sin f-\cos f] \\ &\ddot{\zeta}+\frac{\partial V_c}{\partial \zeta}=\frac{-GM_G}{r^2}\zeta \end{aligned}$$

where

$$\xi=X-X_0$$
$$\eta=Y-Y_0$$
$$\zeta=Z$$

Ⅲ. RATE OF ENERGY INCREASE ON THE IMPULSIVE APPROXIMATION

In the impulsive approximation, the effect of the gravitation of the cluster on the motion of a member star is neglected. That is,

$$\frac{\partial V_c}{\partial \xi}=\frac{\partial V_c}{\partial \eta}=\frac{\partial V_c}{\partial \zeta}=0 \qquad (10)$$

When the cluster moves in its elliptical orbit over the time interval from $-t$ to $t(t=0$ at pericentron passage), the tidally-induced increment increases in the velocity components of a member star obtained from integration of Eqs.(8) are,

$$\begin{aligned} \Delta\dot{\xi} &= \left[\frac{GM_G}{a^3(1-e^2)^3}\right]^{\frac{1}{2}}\xi(f+4e\sin f \\ &\quad +\tfrac{3}{2}\sin 2f-2e\sin^3 f) \\ \Delta\dot{\eta} &= \left[\frac{GM_G}{a^3(1-e^2)^3}\right]^{\frac{1}{2}}\eta(f+4e\sin f \qquad (11) \\ &\quad -\tfrac{3}{2}\sin 2f+2e\sin^3 f) \\ \Delta\dot{\zeta} &= \left[\frac{GM_G}{a^3(1-e^2)^3}\right]^{\frac{1}{2}}\zeta(2f+2e\sin f) \end{aligned}$$

Here a is the semi-major axis, e is the eccentricity, and f is the true anomaly at time t of the cluster in its orbit. Integration over an entire orbital period gives.

$$\begin{aligned} \Delta\dot{\xi} &= \pi\left[\frac{GM_G}{a^3(1-e^2)^3}\right]\xi \\ \Delta\dot{\eta} &= \pi\left[\frac{GM_G}{a^3(1-e^2)^3}\right]\eta \qquad (12) \\ \Delta\dot{\zeta} &= 2\pi\left[\frac{GM_G}{a^3(1-e^2)^3}\right]^{\frac{1}{2}}\zeta \end{aligned}$$

From Eqs. (12), we derive the increase of total kinetic energy of the cluster.

By the impulsive approximation, this equals the gain of internal energy ΔU.

$$\Delta U=\pi^2\frac{GM_GM_cR_c^2}{a^3(1-e^2)^3} \quad \text{(entire orbit)} \quad (13)$$

Where R_c^2 is the mean square of the distance of the stars from the cluster center, and M_c is the mass of the cluster. Over the pericentron passage time(defined as the time for the cluster to move from one latus rectum of the orbit to the other through pericentron), the increase of internal energy [from Eq. (11)] is

$$\Delta U_{\pm\frac{\pi}{2}}=\frac{\pi^2}{4}\frac{GM_GM_cR_c^2}{a^3(1-e^2)^3}\left(1+\frac{8e}{\pi}+\frac{88e^2}{3\pi^2}\right) \quad (14)$$

For the orbital region, $f=45°$ to $45°$,

$$\Delta U_{\pm\frac{\pi}{4}}=\frac{GM_GM_cR_c^2}{a^3(1-e^2)^3}\left(\frac{\pi^2}{16}+\frac{3}{4}+\frac{19e^2}{6}+\sqrt{2}\pi e-\sqrt{2}e\right) \quad (15)$$

Similarly, in the case of hyperbolic orbital motion:

$$\Delta U=\frac{22}{3}\cdot\frac{GM_GM_cR_c^2e^2}{a^3(e^2-1)^3}\cdot D \quad \text{(entire orbit)}$$

$$\Delta U_{\pm\frac{\pi}{2}}=\frac{22}{3}\cdot\frac{GM_GM_cR_c^2e^2}{a^3(e^2-1)^3}\left(1+\frac{3}{11e}+\frac{3\pi^2}{88e^2}\right)$$

$$\Delta U_{\pm\frac{\pi}{4}}=\frac{19}{6}\cdot\frac{GM_GM_cR_c^2e^2}{a^3(e^2-1)^3}\left(1+\frac{3\sqrt{2}\pi}{19e}-\frac{3\sqrt{2}}{19e}+\frac{3\pi^2}{152e}+\frac{8}{38e^2}\right) \quad (16)$$

In Eq. (16),

$$D=1+\frac{6}{11e}\left(1-\frac{1}{e^2}\right)^{\frac{1}{2}}+\frac{1}{e^2}\left(\frac{3\pi^2}{22}-\frac{9}{11}\right)+\frac{3}{22e^2}\left(\cos^{-1}\frac{1}{e}\right)^2-\frac{6}{11e}\left(1-\frac{1}{e^2}\right)^{\frac{1}{2}}\cos^{-1}\frac{1}{e}-\frac{\pi}{11e^2}\cos^{-1}\frac{1}{e}-\frac{3}{22e^4}-\frac{9}{22e^6} \quad (17)$$

In the case of parabolic motion:

$$\Delta U=\frac{\pi^2}{8}\frac{GM_GM_cR_c^2}{q^3} \quad \text{(entire orbit)}$$

$$\Delta U_{\pm\frac{\pi}{2}}=\frac{11}{12}\frac{GM_GM_cR_c^2}{q^3}\left(1+\frac{6\pi}{22}+\frac{3\pi^2}{88}\right)$$

$$\Delta U_{\pm\frac{\pi}{4}}=\frac{GM_GM_cR_c^2}{q^3}\left(\frac{\pi^2}{128}+\frac{47}{96}+\frac{\sqrt{2}\pi}{16}-\frac{\sqrt{2}}{10}\right) \quad (18)$$

Here q is the pericentron distance.
In the case of uniform rectilinear orbital motion, Spitzer's (1958) expression is

$$\Delta U_r=\frac{4}{3}\cdot\frac{G^2M_G^2M_cR_c^2}{p^4v_c^2} \quad \text{(entire orbit)} \quad (19)$$

Table I. Ratio of internal energy increase for two-body motion to that for uniform rectilinear motion, on the impulsive approximation.

e	$\frac{\Delta U}{\Delta U_r}$ (entire orbit)	$\frac{\Delta U_{\pm\frac{\pi}{2}}}{\Delta U_r}$	$\frac{\Delta U_{\pm\frac{\pi}{4}}}{\Delta U_r}$
5.76	4.69	4.63	1.89
2	3.89	3.70	1.42
1.41	3.30	3.35	1.26
1.15	2.65	3.14	1.19
1	1.85	3.02	1.13
0.8	2.28	2.82	0.97
0.5	3.29	2.48	0.83
0.2	5.14	2.09	0.72
0.05	6.71	1.90	0.71

In Table I, the internal energy gain based on two-body motion is compared with that based on uniform rectilinear motion, where the relative velocity at pericentron is adopted for v_c. We note that the uniform rectilinear motion approximation may lead to either an over-estimate or an under-estimate of the internal energy gain, depending on the value of the eccentricity and the time interval considered. The error is less than a factor of about 3 over the range of eccentricities appropriate to globular clusters, $0.5 \lesssim e \lesssim 1$. This result is independent of the dynamical parameters M_c, M_G, R_c, and a.

Ⅳ. SLOW TIDAL ENERGY TANSFER

On the impulsive approximation, the stars in the cluster are assumed to be displaced a negligible fraction of a cluster radius(in the ξ, η, ζ, coordinate system) over the time interval during which the tidal energy transfer effectively occurs. Although this approximation simplifies the derivation of the internal energy gain, it is unrealistic that encounters are often "slow". Spitzer (1958) persented an analysis which takes account of actual internal cluster motion. He has applied it to the case of uniform rectilinear orbital motion. We apply Spitzer's procedures to the case of elliptical orbital motion of the cluster. We assume that (i) the tidal force on a star in the cluster is small compared to the gravitational force produced by the cluster, and (ii) the motion of a star is simple harmonic oscillation around the cluster center(this is equivalent to assuming a homogeneous spherical cluster). That is, for a star oscillating in the rotational plane of the cluster,

$$\frac{\partial V_c}{\partial \xi}=\omega^2\xi \qquad \frac{\partial V_c}{\partial \eta}=\omega^2\eta \tag{20}$$

And for a star oscillating in the meridian plane,

$$\frac{\partial V_c}{\partial \zeta}=\omega^2\zeta \tag{21}$$

Application of the method of variation of parameters(used by Spitzer) to Eqs. (8), (20), and (21), gives

$$\Delta U=\frac{1}{4}\frac{G^2M_G^2M_cR_c^2}{P^6}(I_c^2+3I_{cc}+3I_{ss}) \tag{22}$$

where

$$\begin{aligned} I_c &= \int_{-t}^{t} F(\tau)\cos 2\omega\tau \cdot d\tau \\ I_{cc} &= \int_{-t}^{t} F(\tau)\cos 2\omega\tau \cdot \cos 2f \cdot d\tau \\ I_{ss} &= \int_{-t}^{t} F(\tau)\sin 2\omega\tau \cdot \sin 2f \cdot d\tau \end{aligned} \tag{23}$$

and $F(\tau)$ is the non-dimensional function of time,

$$F(\tau)=\frac{P^3}{r^3} \tag{24}$$

where $p/(1+e)$ is the pericentron distance of the orbit. In Eq. (23) ω is the rotational angular velocity of a typical cluster star. For an elliptical orbit, the Eq. (23) integrations can be carried out by the series expansions involving $F(\tau)$ (Smart, 1953).

Setting $p=a(1-e^2)$, the energy gain Eq. (22) becomes

$$\Delta U=\frac{GM_GM_cR_c^2}{a^3(1-e^2)^3}\,4\pi^2\cdot\kappa^2\cdot L(\alpha\kappa) \tag{25}$$

where

$$L(\alpha,\ \kappa)=I_c^{\prime 2}+\frac{3}{4}(I_{cc}^{\prime 2}+I_{ss}^{\prime 2}) \tag{26}$$

and

$$I_c'=\frac{\sin 4\pi\alpha}{4\pi\alpha}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{m,n}\left[\frac{(n+1)(n+2)\beta^n}{2}+\frac{3(n+2)\beta^{n+2}}{1-\beta^2}+\frac{6\beta^{n+4}}{(1-\beta^2)^2}\right](S_-+S_+)$$

$$I'_{cc}=\left[\frac{e}{2(1-e^2)}\sum_{m=1}^{\infty}A_{1,m}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{n+1,m}(A'_{n-1}+B'_{n+3})\right]\cdot(S_--S_+)$$

$$I'_{ss}=\left[\frac{e}{2\sqrt{1-e^2}}\sum_{m=1}^{\infty}B_{1,m}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}B_{n+1,m}(A'_{n-1}-B'_{n+3})\right](S_--S_+) \tag{27}$$

$$S_-=\frac{\sin[2\pi(2\alpha-m\kappa)]}{2\pi(2\alpha+m\kappa)},$$

$$S_+=\frac{\sin[2\pi(2\alpha+m\kappa)]}{2\pi(2\alpha+m\kappa)}$$

$$A_{n,m}=\frac{n}{m}[J_{m-n}(me)-J_{m+n}(me)],$$

$$B_{n,m}=\frac{n}{m}[J_{m+n}(me)+J_{n-m}(me)$$

$$A'_n=(1-\beta^2)^2\left[\frac{(n+1)(n+2)(n+3)(n+4)}{24}\right.$$
$$+\frac{(n+2)(n+3)(n+4)\beta^2}{6(1-\beta^2)}$$
$$+\frac{(n+3)(n+4)\beta^4}{2(1-\beta^2)^2}+\frac{(n+4)\beta^6}{(1-\beta^2)^3}$$
$$\left.+\frac{\beta^8}{(1-\beta^2)^4}\right]\cdot\beta^n$$

$$B_n'=\frac{\beta^n}{(1-\beta^2)^2}$$

$$\beta=\frac{1}{e}(1-\sqrt{1-e^2})$$

Here α is the ratio of the time since pericentron passage t, to the the rotational period of a typical culster star P_s;

$$\alpha=\frac{t}{P_s} \tag{28}$$

and κ is the ratio of t to the orbital period of the cluster P_c,

$$\kappa=\frac{t}{P_c} \tag{29}$$

In Eqs. (27), $J_{m-n}(me)$ is the $(m-n)$th order Bessel function with argument me, where e is the eccentricity of the orbit and m is a positive integer.

From Eq. (25), the total energy gain over one full orbit period($t=\frac{1}{2}P_c$, $\kappa=\frac{1}{2}$) is

$$\Delta U=\pi^2\frac{GM_GM_cR_c^2}{a^2(1-e^2)^3}\cdot L\left(\alpha,\frac{1}{2}\right) \tag{30}$$

For the limiting case of the impulsive approximation (i.e., $\alpha=0$) Eq. (30) reduces to Eq. (13).

Applying the virial theorem and the total gravitational potential energy of the cluster $\left(\Omega=-\frac{3}{5}\frac{GM_c^2}{R_c}\right)$ to Eq. (25), we find that the fractional increase in internal energy is

$$\frac{\Delta U}{|U|}=\frac{(3+e)}{(1-e)^3}\frac{M_G}{M_c}\left(\frac{R_c}{a}\right)^3\cdot Q \tag{31}$$

where

$$Q=\frac{40\pi^2}{3}\cdot\frac{(1-e)^3}{(3+e)}\kappa^2(1-e^{2-3})L(\alpha,\kappa) \tag{32}$$

The tidally-determined limiting radius at pericentron is (King, 1962),

$$R_{LIM}\text{ (at pericentron)}=\left(\frac{M_c}{M_G}\right)^{\frac{1}{3}}\left(\frac{1}{3+e}\right)^{\frac{1}{3}}\cdot a(1-e) \tag{33}$$

It follows that Q is $\Delta U/|U|$ for $R_c=R_{LIM}$ (at pericentron). Table II gives values of Q for several values of e, P_c/P_s, and $\kappa=t/P_c$. The κ-values are arranged in order of decreasing t. κ_{orb} repersents a full orbit, κ_m corresponds to the half orbit from $E=-90°$ to $90°$, κ_p is for the orbital region between the ends of the latus rectum through pericentron, $\kappa_{0.5}=0.5\kappa_p$, and $\kappa_{0.1}=0.1\kappa_p$.

Table II shows the following characteristics for slow tidal energy transfer along an elliptical orbit:

(i) In general,the impulsive approximation($P_c/P_s=0$) leads to either an overestimate or an under-estimate of the actual tidal energy transfer, depending on the value of e and P_c/P_s.

(ii) Over an entire orbital period, the maximum energy gain for a given orbital eccentricity occurs at $P_c/P_s=1$(entire period). For $P_c/P_s>1$, the energy gain decreases with increasing P_c/P_s. For $P_c/P_s<1$, the energy gain converges rapidly to values appropriate to the impulsive approximation($P_c/P_s=0$). The convergence is essentially complete at $P_c/P_s=0.001$.

(iii) For clusters with large random

Table II. Q-values: Fractional energy gain of a cluster for $R_c = R_{LIM}$(pericentron).

e	κ_{orb}	κ_m	κ_p	$\kappa_{0.5}$	$\kappa_{0.1}$	P_c/P_s	range of m,n
0.2	.000	.000	.000	.000	.000	100	1-50
0.5	.000	.000	.000	.000	.000		1-50
0.2	.000	.000	.000	.000	.000	50	1-50
0.5	.000	.000	.000	.001	.001		1-50
0.2	.000	.001	.001	.002	.004		1-50
0.5	.000	.000	.004	.004	.001	20	1-50
0.8	.180	.190	.216	.171	.082		1-50
0.2	.000	.003	.005	.010	.007		1-30
0.5	.000	.004	.003	.026	.072	10	1-30
0.8	.776	.826	.855	.416	.052		1-30
0.2	.000	.017	.011	.052	.128		1-15
0.5	.158	.183	.091	.351	.158	5	1-15
0.8	.711	.769	1.044	.464	.028		1-20
0.2	.064	.067	.101	.488	.133		1-15
0.5	1.209	1.321	1.729	1.251	.190	2.5	1-15
0.8	14.082	18.960	1.064	.530	.028		1-20
0.2	8.902	.565	.619	.497	.094		1-15
0.5	8.982	2.228	1.727	.713	.187	1	1-15
0.8	33.076	13.279	.923	.532	.029		1-20
0.2	5.466	2.394	2.235	1.323	.133		1-15
0.5	2.724	1.935	1.658	1.041	.202	0.1	1-15
0.8	1.716	1.712	1.028	.553	.029		1-20
0.2	5.945	2.364	2.202	1.323	.133		1-15
0.5	2.784	1.843	1.607	1.040	.202	0.01	1-15
0.8	1.487	1.371	1.028	.553	.029		1-20
0.2	5.950	2.364	2.202	1.323	.133		1-15
0.5	2.785	1.842	1.607	1.040	.202	0.01	1-15
0.8	1.485	1.365	1.028	.553	.029		1-20

motion or rapid rotation ($P_c/P_s \gtrsim 20$), the net energy gain over an entire orbit is negligibly small, and the impulsive approximation yields large over-estimates. Such high density clusters are favored for survival against tidal disruption.

V. SUMMARY AND CONCLUSIONS

(i) On the impulsive approximation, the use of uniform rectilinear motion gives either an over-estimate or an under-estimate of the two-body internal energy gain of a cluster, depending on the values of the eccentricity and the time interval considered(See Table I). The difference is less than a factor of about 3 over the range of eccentricity $0.5 \lesssim e \lesssim 1.0$.

(ii) The tidally-induced internal energy gain of a cluster depends strongly upon its internal motions and the orbital eccentricity(See Table II). For example, in one revolution, a cluster with $P_c \gg P_s$ [an orbital period(P_c) much larger than a characteristic rotational period of a member star(P_s)] gains a negligibly small amount of energy. In contrast, in one revolution, a cluster with $e \gtrsim 0.5$ and $P_c/P_s \lesssim 2$ gains an amount of energy sufficient for disruption. The case $P_c/P_s \approx 10^8$ yrs/10^7yrs$=10$ lies intermediate between the above two extremes.